A Simplified Dual-Porosity Model for Two-Phase Flow[1]

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ABSTRACT

A model for two-phase, incompressible, immiscible fluid flow in a highly fractured porous medium is derived as a simplification of a much more detailed dual-porosity model. This simplified model has a nonlinear matrix-fracture interaction, and it is more general than similar existing "transfer function" models. It is computationally less complex than the detailed model, and simulation results are presented which assess any loss in accuracy. It is shown that the new model approximates capillary effects quite well, and better than similar existing models.

1. INTRODUCTION

The flow of fluids through highly fractured porous media is often modeled as a dual-porosity system in which the network of fractures and the matrix rock are viewed as distinct, interacting porous structures. Several strategies have been proposed to model the matrix-fracture interaction. Among these are, one, to directly compute the internal flow within the matrix blocks as it is affected at the blocks' surfaces by the external fracture flow [6, 7], and, two, to define the matrix-fracture interaction by a "transfer function" [10, 5, 8, 6]. The first approach should be considered to be the most accurate, as the equations are derived by homogenization (i.e., averaging) of the equations describing the flow on a microscopic level (see, e.g., [6]). We pay for this accuracy through computational complexity.

In this paper we mathematically simplify the equations for the first, more detailed model for two-phase, immiscible, incompressible flow. The result is a new model of the second type that has a nonlinear transfer function, with well defined physical parameters. It is more general than similar existing models, and it is better able to capture the important capillary effects [9] between the two porous media, yet it remains computationally

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about as simple. The model is also extended to cover groundwater flow.

Computational results are presented which assess the validity of the new model by comparing it to the first, more detailed one, and which demonstrate the unique features of the new model over the existing transfer function models.

2. DERIVATION OF THE SIMPLIFIED MODEL

We begin by presenting the homogenized dual-porosity model [6]. Let x denote a point in the medium Ω, and t denote time. For wetting and nonwetting phases $\xi = \mathrm{w}, \mathrm{n}$, respectively, let $p_{\xi,\mathrm{f}}(x,t)$, $\Phi_{\xi,\mathrm{f}}(x,t)$, $s_{\xi,\mathrm{f}}(x,t)$, $\phi_{\mathrm{f}}(x)$, $k_{\mathrm{f}}(x)$, $p_{\mathrm{c,f}}(s_{\mathrm{w,f}})$, and $\lambda_{\xi,\mathrm{f}}(s_{\mathrm{w,f}})$ denote the fracture system's pressure, flow potential, saturation (so $s_{\mathrm{w,f}} + s_{\mathrm{n,f}} = 1$), porosity, permeability, capillary pressure, and relative mobility (i.e., relative ξ-permeability divided by ξ-viscosity). (These are *macroscopic* quantities defined on the scale of the fracture *spacing*, not on the scale of the fracture *width*.) We denote the corresponding quantities for the matrix system similarly, but we must account for the complicated topology of a dual-porosity system. First replace subscript f by m and then replace x by (x,y), where x determines the matrix block $\mathcal{Q}(x)$ and y determines the point within that block.

Flow in the fracture system is governed by the usual equations of two-phase incompressible, immiscible flow, except for the addition of two "matrix source" terms $q_{\xi,\mathrm{m}}$. For $\xi = \mathrm{w}, \mathrm{n}$,

$$\left.\begin{array}{ll} (a) & v_{\xi,\mathrm{f}} = -k_{\mathrm{f}}\lambda_{\xi,\mathrm{f}}(s_{\mathrm{w,f}})\nabla\Phi_{\xi,\mathrm{f}} \quad \text{for } x \in \Omega,\ t > 0, \\ (b) & \phi_{\mathrm{f}}\partial_t s_{\xi,\mathrm{f}} + \nabla\cdot\, v_{\xi,\mathrm{f}} = q_{\xi} + q_{\xi,\mathrm{m}}, \quad \text{for } x \in \Omega,\ t > 0, \\ (c) & p_{\mathrm{c,f}}(s_{\mathrm{w,f}}) = p_{\mathrm{n,f}} - p_{\mathrm{w,f}}, \\ (d) & \Phi_{\xi,\mathrm{f}} = p_{\xi,\mathrm{f}} - \gamma_{\xi}x_3, \end{array}\right\} \tag{1}$$

where $v_{\xi,\mathrm{f}}(x,t)$ is the Darcy phase velocity, $\partial_t = \partial/\partial t$, γ_ξ is the gravity-density term, x_3 is the vertical coordinate, and $q_\xi(s_{\mathrm{w},f}, x, t)$ is the external volumetric source. To these we add boundary and initial conditions.

The matrix system satisfies on each matrix block $\mathcal{Q}(x)$ the usual equations of porous media flow written in terms of the flow potential [2]. Hence, for each fixed $x \in \Omega$ and $\xi = \mathrm{w}, \mathrm{n}$,

$$\left.\begin{array}{ll} (a) & v_{\xi,\mathrm{m}} = -k_{\mathrm{m}}\lambda_{\xi,\mathrm{m}}(s_{\mathrm{w,m}})\nabla_y\Phi_{\xi,\mathrm{m}}, \quad \text{for } y \in \mathcal{Q}(x),\ t > 0, \\ (b) & \phi_{\mathrm{m}}\partial_t s_{\xi,\mathrm{m}} + \nabla_y\cdot\, v_{\xi,\mathrm{m}} = 0, \quad \text{for } y \in \mathcal{Q}(x),\ t > 0, \\ (c) & p_{\mathrm{c,m}}(s_{\mathrm{w,m}}) = p_{\mathrm{n,m}} - p_{\mathrm{w,m}}, \\ (d) & \Phi_{\xi,\mathrm{m}} = p_{\xi,\mathrm{m}} - \gamma_{\xi}(x_3 + y_3). \end{array}\right\} \tag{2}$$

Here, the boundary conditions are very important, since the fracture system influences the matrix flow through them. For $x \in \Omega$ and $\xi = \mathrm{w}, \mathrm{n}$

$$\Phi_{\xi,\mathrm{m}}(x,y,t) = \Phi_{\xi,\mathrm{f}}(x,t) \quad \text{for } y \in \partial\mathcal{Q}(x),\ t > 0. \tag{3}$$

Note that the scale of the matrix blocks is so much finer than that of the fracture system that these right-hand sides are constant in space over $\partial\mathcal{Q}(x)$. For the initial condition, assume equilibrium with the fracture system.

Let $\bar{\phi}_{\mathrm{m}}(x) = \dfrac{1}{|\mathcal{Q}(x)|}\displaystyle\int_{\mathcal{Q}(x)} \phi_{\mathrm{m}}(x,y)\,dy$. Define the average matrix w-saturation at $x \in \Omega$ to be

$$\bar{s}_{\mathrm{w,m}}(x,t) = -\frac{1}{|\mathcal{Q}(x)|}\int_{\mathcal{Q}(x)} \frac{\phi_{\mathrm{m}}(x,y)}{\bar{\phi}_{\mathrm{m}}(x)} s_{\mathrm{w,m}}(x,y,t)\,dy. \tag{4}$$

Finally, define the matrix w-source function at $x \in \Omega$ to be

$$q_{\mathrm{w,m}}(x,t) = -\bar{\phi}_{\mathrm{m}}(x)\,\partial_t \bar{s}_{\mathrm{w,m}}(x,t) \tag{5}$$

and set $q_{\mathrm{n,m}} = -q_{\mathrm{w,m}}$ by incompressibility.

This model as stated must necessarily give rise to a fairly complicated numerical approximation scheme [7]. We now derive a computationally more tractable simplification of the model. We assume that imbibition is the dominant physical process affecting the movement of fluids between the matrix and the fractures, and we neglect all other processes.

Following [4], we omit the $\gamma_\xi y_3$ gravitational term in (2d), and rewrite (2) in terms of the global pressure

$$p_{\mathrm{m}} = p_{\mathrm{n,m}} - \int_{s_{\mathrm{wr,m}}}^{s_{\mathrm{w,m}}} \left(\frac{\lambda_{\mathrm{w,m}} p'_{\mathrm{c,m}}}{\lambda_{\mathrm{w,m}} + \lambda_{\mathrm{n,m}}}\right)(s)\,ds, \tag{6}$$

where $s_{\mathrm{wr,m}}$ is the residual wetting saturation in the matrix. By adding together the two equations in (2b), we obtain for p_{m} an elliptic equation with a constant Dirichlet boundary condition; hence, p_{m} is constant in space over $\mathcal{Q}(x)$ and $v_{\mathrm{w,m}} + v_{\mathrm{n,m}} = 0$. Consequently, we ignore the n-equation in (2b) and rewrite the w-equation as

$$\phi_{\mathrm{m}}\partial_t s_{\mathrm{w,m}} + \nabla_y \cdot \left[k_{\mathrm{m}}\left(\frac{\lambda_{\mathrm{w,m}}\lambda_{\mathrm{n,m}} p'_{\mathrm{c,m}}}{\lambda_{\mathrm{w,m}} + \lambda_{\mathrm{n,m}}}\right)(s_{\mathrm{w,m}})\nabla_y s_{\mathrm{w,m}}\right] = 0. \tag{7}$$

Let $\sigma = p_{\mathrm{c,m}}^{-1} \circ p_{\mathrm{c,f}}$ relate fracture w-saturations to matrix ones through continuity of the phase pressures. Linearization now yields the system

$$\left.\begin{array}{ll} (a) & \phi_{\mathrm{m}}\partial_t s_{\mathrm{w,m}} - \nabla_y \cdot (\kappa_{\mathrm{m}} \nabla_y s_{\mathrm{w,m}}) = 0 \quad \text{for } y \in \mathcal{Q}(x),\ t > 0, \\ (b) & s_{\mathrm{w,m}} = \sigma(s_{\mathrm{w,f}}) \quad \text{for } y \in \partial\mathcal{Q}(x),\ t > 0, \end{array}\right\} \tag{8}$$

for $x \in \Omega$ and for some $\kappa_{\mathrm{m}}(x,y)$ uniformly positive in y, defined in the next section. This system is easily solved by means of a Green's function. In fact, for $x \in \Omega$, let $\omega(x,y,t)$ be defined by

$$\left.\begin{array}{ll} (a) & \phi_{\mathrm{m}}\partial_t \omega - \nabla_y \cdot (\kappa_{\mathrm{m}} \nabla_y \omega) = 0 \quad \text{for } y \in \mathcal{Q}(x),\ t > 0, \\ (b) & \omega = 1 \quad \text{for } y \in \partial\mathcal{Q}(x),\ t > 0, \\ (c) & \omega = 0 \quad \text{for } y \in \mathcal{Q}(x),\ t = 0. \end{array}\right\} \tag{9}$$

Then,

$$s_{\mathrm{w,m}}(x,y,t) = \int_0^t \omega(x,y,t-\tau)\,\partial_t \sigma(s_{\mathrm{w,f}}(x,\tau))\,d\tau + \sigma(s_{\mathrm{w,f}}(x,0)) \quad (10)$$

is a convolution in time (as in the case of single phase flow [1]).

As a final simplification, note that ω varies from 0 to 1 in time, so approximate it by a sum of m exponentials as follows: Choose $\alpha_i(x)$ and $\beta_i(x)$, $i = 1, ..., m$, such that

$$\frac{1}{|\mathcal{Q}(x)|}\int_{\mathcal{Q}(x)} \phi_{\mathrm{m}}(x,y)\omega(x,y,t)\,dy \approx \bar{\phi}_{\mathrm{m}}(x)\left(1 - \sum_{i=1}^{m} \beta_i(x) e^{-\alpha_i(x)t}\right), \quad (11)$$

where $\sum_i \beta_i(x) = 1$. Then the average matrix saturation in $\mathcal{Q}(x)$ is

$$\bar{s}_{\mathrm{w,m}}(x,t) = \sum_{i=1}^{m} \beta_i(x)\left\{\alpha_i(x)\int_0^t \sigma(s_{\mathrm{w,f}}(x,\tau))\,e^{-\alpha_i(x)(t-\tau)}\,d\tau \right. \\ \left. + \sigma(s_{\mathrm{w,f}}(x,0))e^{-\alpha_i(x)t}\right\}. \quad (12)$$

The simplified model is (1), (5), and (12), and it is somewhat more general than that proposed by deSwaan [10] and independently by Chen [5], where σ is a multiple of the identity and $m = 1$. This model is also more general than the "limit model" of Douglas and Paes Leme [8, 6], where simply

$$\bar{s}_{\mathrm{w,m}}(x,t) = \sigma(s_{\mathrm{w,f}}(x,t)); \quad (13)$$

this is our model with $m = 1$ and $\alpha_1 = +\infty$. The existence of a solution to the model, in fact (1), (5), and (10), has been shown by the author [3].

3. SELECTION OF PARAMETERS

It remains to define κ_{m}, α_i, and β_i, $i = 1, ..., m$. Clearly we must choose κ_{m} so that

$$\kappa_{\mathrm{m}} \approx -k_{\mathrm{m}}\left(\frac{\lambda_{\mathrm{w,m}}\lambda_{\mathrm{n,m}}p'_{\mathrm{c,m}}}{\lambda_{\mathrm{w,m}} + \lambda_{\mathrm{n,m}}}\right)(s) \quad (14)$$

for all values s of interest. Let us take the arithmetic average in both s and y, over the region of interest $s_{\mathrm{w}r,\mathrm{m}} \le s \le 1 - s_{\mathrm{n}r,\mathrm{m}}$ and $y \in \mathcal{Q}(x)$.

Now for the α_i and β_i, assume that the matrix blocks $\mathcal{Q}(x)$ are rectangular, of size $\ell_1 \times \ell_2 \times \ell_3$, and that the permeability is a diagonal tensor. Also replace $\phi_{\mathrm{m}}(x,y)$ by $\bar{\phi}_{\mathrm{m}}(x)$. In that case, separation of variables applied to (9) shows that

$$\frac{1}{|\mathcal{Q}|}\int_{\mathcal{Q}} \bar{\phi}_{\mathrm{m}}(x)\omega(x,y,t)\,dy = \bar{\phi}_{\mathrm{m}}\left\{1 - \sum_{m_1,m_2,m_3 \text{ odd}} \frac{512}{m_1^2 m_2^2 m_3^2 \pi^6} \right. \\ \left. \times \exp\left[-\left(\frac{m_1^2}{\ell_1^2}\kappa_{\mathrm{m},11} + \frac{m_2^2}{\ell_2^2}\kappa_{\mathrm{m},22} + \frac{m_3^2}{\ell_3^2}\kappa_{\mathrm{m},33}\right)\frac{\pi^2}{\bar{\phi}_{\mathrm{m}}}t\right]\right\}. \quad (15)$$

We may truncate (15) as we wish. For this paper, assume that $\ell_j = \ell$ and $\kappa_{\mathrm{m},jj} = \kappa_\mathrm{m}$, $j = 1, 2, 3$, and take $m = 2$. Then $\alpha_1 = 3\kappa_\mathrm{m}\pi^2/\bar{\phi}_\mathrm{m}\ell^2$. Let $\alpha_2 = 0$; then, $\beta_2 = 1 - \beta_1$ represents the fraction of essentially immobile fluid over the time-scale of interest.

Of course, if possible we should adjust the α_i and β_i to match empirical data; (14)–(15) is merely for theoretical guidance.

4. DISCRETIZATION OF THE MODEL

Apart from the $q_{\xi,\mathrm{m}}$ term, there are many ways to discretize (1). This author chose to use a fully implicit, backward Euler time approximation coupled to a standard cell-centered finite difference scheme with upstream weighted relative mobilities, solved by a Newton iteration. The approximation of the average matrix saturation (12) at any grid cell j at time t^N can be given as follows:

$$\bar{s}^N_{\mathrm{w,m},j} = \sum_{i=1}^{m} \beta_{i,j}\tilde{s}^N_{\mathrm{w,m},i,j} \tag{16}$$

$$\begin{aligned}
\tilde{s}^N_{\mathrm{w,m},i,j} &= \left\{ \alpha_{i,j} \sum_{n=1}^{N} \int_{t^{n-1}}^{t^n} \left[\sigma(s^{n-1}_{\mathrm{w,f},j}) \right.\right. \\
&\quad \left.\left. + \frac{\sigma(s^n_{\mathrm{w,f},j}) - \sigma(s^{n-1}_{\mathrm{w,f},j})}{t^n - t^{n-1}}(\tau - t^{n-1}) \right] e^{\alpha_{i,j}\tau}\, d\tau + \sigma(s^0_{\mathrm{w,f},j}) \right\} e^{-\alpha_{i,j}t^N} \\
&= \left[\tilde{s}^{N-1}_{\mathrm{w,m},i} - \sigma(s^{N-1}_{\mathrm{w,f},j}) + \frac{\sigma(s^N_{\mathrm{w,f},j}) - \sigma(s^{N-1}_{\mathrm{w,f},j})}{\alpha_{i,j}(t^N - t^{N-1})} \right] e^{-\alpha_{i,j}(t^N - t^{N-1})} \\
&\quad + \sigma(s^N_{\mathrm{w,f},j}) - \frac{\sigma(s^N_{\mathrm{w,f},j}) - \sigma(s^{N-1}_{\mathrm{w,f},j})}{\alpha_{i,j}(t^N - t^{N-1})}.
\end{aligned} \tag{17}$$

where $\tilde{s}^0_{\mathrm{w,m},i,j} = \sigma(s^0_{\mathrm{w,f},j})$. This definition does *not* require that the entire past history of the solution be saved in memory.

Finally, over the time interval t^{N-1} to t^N (cf. (5)),

$$-q^N_{\mathrm{n,m},j} = q^N_{\mathrm{w,m},j} = -\bar{\phi}_{\mathrm{m},j}\frac{\bar{s}^N_{\mathrm{w,m},j} - \bar{s}^{N-1}_{\mathrm{w,m},j}}{t^N - t^{N-1}}. \tag{18}$$

5. SOME COMPUTATIONAL RESULTS

To assess the loss in accuracy due to the simplifications we made to the original, homogenized model (1)–(5), a two dimensional, vertical water-flood problem was simulated. In this problem, oil plus residual water initially fills the reservoir, water is injected along one vertical face, and fluids are collected along the opposite face. The reservoir is water-wet, so imbibition is the dominant physical force on the fluids.

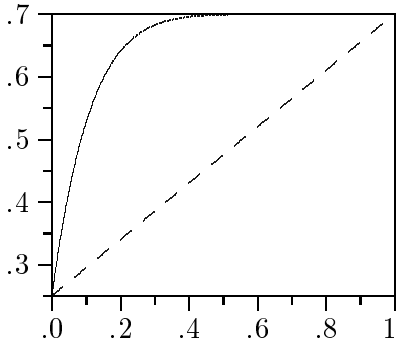

Fig. 1. A graph of $\sigma(s)$.

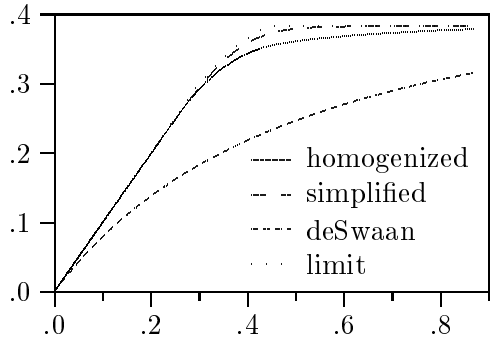

Fig. 2. Cumulative oil production vs. water injection (in pore-volumes).

The reservoir is rectangular, 1080 m long and 24 m in depth, and the matrix blocks are 2 m square. The fracture thickness is 100 micrometers (so $\phi_{\rm f} = .0001$). Also, $k_{\rm f} = 42.2\,{\rm md}$, $\phi_{\rm m} = .2$, $k_{\rm m} = 40\,{\rm md}$, $s_{{\rm wr,f}} = 0$, $s_{{\rm nr,f}} = 0$, $s_{{\rm wr,m}} = .25$, and $1 - s_{{\rm nr,m}} = .7$. Reasonable relative permeability and capillary pressure functions were used; $\sigma(s)$ is shown in Figure 1. Water density was $1\,{\rm g/cm^3}$ and viscosity was .5 cp, and oil density was $.9\,{\rm g/cm^3}$ and viscosity was 2 cp. Injection rate was .2 pore-volumes of water per year. In these simulations, it was appropriate to take $\beta_1 = .85$, since about 15% of the recoverable oil was essentially immobile over the span of a few years. From (15), $\alpha_1 = .012$/day.

A uniform fracture grid of size 25×6 was used, and, in the homogenized model, the matrix grid was 6×6. The final time is 4.3 years, and the time-step varied from .01 to 8 days.

Solutions were obtained for four models: the homogenized model (1)–(5); the simplified model (1), (5), and (12); the model of deSwaan [10, 5], where $\sigma(s) = s_{{\rm wr,m}} + (1 - s_{{\rm nr,m}} - s_{{\rm wr,m}})s$; and the limit model of Douglas and Paes Leme [8, 6] (1), (5), and (13). The latter two models were modified to account for the essentially immobile fluid.

Figure 2 shows the cumulative production of fluids from the reservoir. It is evident that β_1 is correctly chosen for the three simpler models. Note that breakthrough occurs too soon for the deSwaan model, while the models with the nonlinear $\sigma(s)$ provide much better results. This is understood from Figure 1, since the linearized $\sigma(s)$ clearly predicts too little imbibition of the w-saturation fracture front as the simulation proceeds. The proper nonlinear influence of capillary pressure is simulated by the simplified and limit models.

Figures 3 and 4 show water saturation contours for the simulation at one year. Water is injected from the left-hand side of the figure, and produced from the right-hand side. Clearly the simplified model of this paper provides the best match with the homogenized (most nearly correct) solution. Again, we see that the deSwaan model produces far too broad a

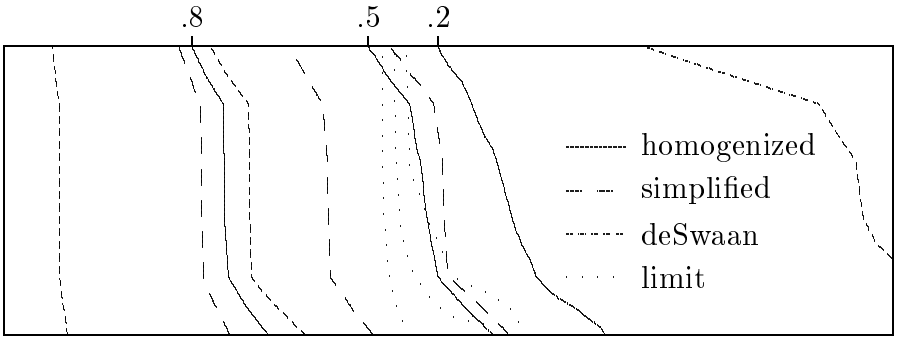

Fig. 3. Fracture w-saturation contours .8, .5, and .2.

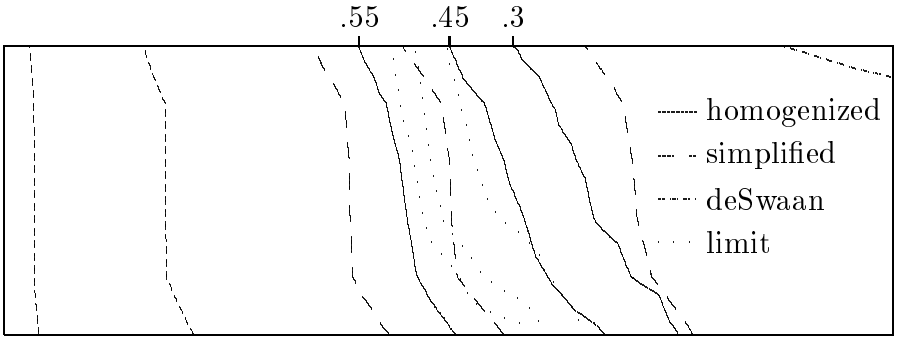

Fig. 4. Average matrix w-saturation contours .55, .45, and .3.

front. The limit model assumes that fluid flow into the matrix is immediate, subject only to the capillary force; consequently, it produces far too sharp a front. The delay in matrix-fracture fluid transfer associated with a finite α_1 results in a widening of the front; the figures show that the simplified model obtained approximately the correct front width.

The simplified model is limited in its ability to model relative permeability effects. Matrix-fracture fluid transfer is most rapid for moderate saturations and slowest for extreme saturations, since then the medium is much less permeable to one of the two fluids. Hence, the average matrix saturation front for the simplified model is advanced at low saturations, and lags at high saturations. The fractures lose water to the matrix due to excessive imbibition at the leading edge, causing the front to lag there. The full, homogenized model simulates these relative permeability effects, since then we obtain not just the average saturation, but the full saturation distribution within the matrix blocks. Perhaps a modification of $\sigma(s)$ in the simplified model would allow us to obtain a better empirical match.

6. EXTENSIONS TO GROUNDWATER FLOW

The extension of the model to groundwater flow is easily obtained. First, replace (1) by Richard's equation in the fractures:

$$\partial_t \theta(\psi_{\rm f}) - \nabla \cdot [K_{\rm f}(\psi_{\rm f}) \nabla(\psi_{\rm f} - x_3)] = q + q_{\rm m}, \quad \text{for } x \in \Omega,\ t > 0, \qquad (19)$$

where, in the fractures, $K_{\rm f}$ is the macroscopic hydraulic conductivity, $\psi_{\rm f}$ is the pressure head, and $\theta_{\rm f}$ is the moisture content. If $\theta_{\rm m}$ is the moisture content in the matrix, we replace (5) and (12) by

$$q_{\rm m}(x,t) = -\partial_t \overline{\theta_{\rm m}(\psi_{\rm f}(x,t))}, \tag{20}$$

$$\overline{\theta_{\rm m}(\psi_{\rm f}(x,t))} = \sum_{i=1}^{m} \beta_i(x) \Big\{ \alpha_i(x) \int_0^t \theta_{\rm m}(\psi_{\rm f}(x,\tau))\, e^{-\alpha_i(x)(t-\tau)}\, d\tau + \theta_{\rm m}(\psi_{\rm f}(x,0)) e^{-\alpha_i(x)t} \Big\}. \tag{21}$$

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