p-ellipticity

Oliver Dragičević

Univerza v Ljubljani

For $p > 1$ define $\mathcal{J}_p : \mathbb{C}^n \to \mathbb{C}^n$ by $\mathcal{J}_p(\alpha + i\beta) = \alpha/p + i\beta/q$, where $1/p + 1/q = 1$. Given an open set $\Omega \subset \mathbb{R}^n$ and a complex matrix function $A : \Omega \to \mathbb{C}^{n,n}$ with L^∞ entries, introduce

$$\Delta_p(A) := 2 \operatorname*{ess\,inf}_{x \in \Omega} \min_{|\xi|=1} \Re \left\langle A(x)\xi, \mathcal{J}_p \xi \right\rangle_{\mathbb{C}^n}.$$

We say that A satisfies the *p-ellipticity* condition if $\Delta_p(A) > 0$. That is, if there exists $C > 0$ such that a.e. $x \in \Omega$ we have

$$\Re \left\langle A(x)\xi, \mathcal{J}_p \xi \right\rangle \geq C|\xi|^2, \qquad \forall \xi \in \mathbb{C}^n.$$

Clearly, $\Delta_2(A) > 0$ is a reformulation of the uniform strict ellipticity of A. Let $L_A u = -\mathrm{div}(A \nabla u)$ be the elliptic operator associated to A.

In the talk we shall explain that p-ellipticity lies at the junction of several phenomena in analysis and PDE which may occur in the presence of complex accretive matrices. They are:

(1) convexity of power functions,
(2) dimension-free bilinear embeddings,
(3) contractivity of semigroups $\exp(-tL_A)$,
(4) holomorphic functional calculus, and
(5) regularity theory of elliptic PDE with complex coefficients.

The talk is based on a joint work with Andrea Carbonaro [1], with the exception of part (5) which is due to Dindoš and Pipher [2].

Riferimenti bibliografici

[1] A. Carbonaro, O. Dragičević: *Convexity of power functions and bilinear embedding for divergence-form operators with complex coefficients*, preprint (2016), http://arxiv.org/abs/1611.00653

[2] M. Dindoš, J. Pipher: *Regularity theory for solutions to second order elliptic operators with complex coefficients and the L^p Dirichlet problem*, preprint (2016), http://arxiv.org/abs/1612.01568