UNIC Mathematics Workshop 2023

University of Zagreb, May 22–25

BOOK OF ABSTRACTS

Sponsors

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Participating universities

UCC University College Cork, Ireland Coláiste na hOllscoile Corcaigh

KOÇ UNIVERSITY

UNIVERSITY OF LODZ

UNIVERSITY OF OULU

University of Zagreb

Location: Faculty of Science, Department of Mathematics, Bijenička cesta 30, Zagreb

Webpage: https://web.math.pmf.unizg.hr/unic2023

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Workshop description

The UNIC Mathematics Workshop 2023 aims at bringing together mathematicians from several universities under the umbrella of the UNIC network. The workshop then seeks to provide space for collaboration and cooperation among participating mathematicians to build a network and advance institutional capacities. Within the context of the workshop, participants will present possible avenues to develop joint study programs and possible joint PhD supervision, among other forms of collaboration. The workshop serves to foster collaborative research efforts that can benefit the wider mathematical community. It will feature a presentation of the future study program Theoretical Mathematics, that is under discussion, several mathematical lectures on diverse topics, a discussion table, and possibilities for networking. The workshop is open to mathematicians from all UNIC institutions who are interested in the future study program or fostering collaboration with UNIC colleagues, as well as to researchers and PhD students who will benefit from the workshop scientific program.

Basic information

The workshop is organized at the University of Zagreb, Faculty of Science, Department of Mathematics, Bijenička cesta 30, Zagreb; see the map.

Lectures will be held in classroom A101 on the first floor.

Talks last 50 minutes + 5 minutes for questions + 5 minutes for smooth transitions.

Lunches will be served in room A318 on the third floor.

The conference dinner will be held in Restaurant Vinodol, Teslina 10, Zagreb; see the map.

For *wifi* you can use Eduroam with the same settings and credentials as at your home institution; it should connect you automatically. If you do not have Eduroam, then use:

- network: `kongres_2022`,
- password: `kongres_2022`.

Contacts of the organizers:

- Luka Grubišić, `luka.grubisic@math.hr`,
- Vjekoslav Kovač, `vjekoslav.kovac@math.hr`.

Workshop schedule

Tuesday, May 23rd	Wednesday, May 24th
08:50–09:00 Opening	
Morning chairperson: Pekka Salmi	*Morning chairperson*: Mine Caglar
09:00–10:00 Ville Suomala, *Conformal dimension of fractal percolation*	09:00–10:00 Emre Mengi, *Estimation of the dominant poles of a large-scale descriptor system*
10:00–11:00 Esa Järvenpää, *Visible part conjecture*	10:00–11:00 Luka Grubišić, *Randomized POD-Beyn algorithm for nonlinear eigenvalue problems — analysis and perspectives*
11:00–11:20 Coffee break	11:00–11:20 Coffee break
11:20–12:20 Spyridon Dendrinos, *A restricted 2-plane transform associated to surfaces of codimension 2*	11:20–12:20 Erna Begović Kovač, *Tensor diagonalization algorithms*
12:20–14:00 Lunch break, informal discussions, Team Leaders Meeting	12:20–14:00 Lunch break, informal discussions
Afternoon chairperson: Ville Suomala	*Afternoon chairperson*: Adam Różycki
14:00–15:00 Maja Resman, *Fractal data in reading invariants of dynamical systems*	14:00–15:00 Andrei Mustata, *From representations of reductive groups to geometric invariants*
15:00–16:00 Vjekoslav Kovač, *Large copies of large configurations in large sets*	15:00–16:00 Stanisław Spodzieja, *Effective Bertini theorem and formulas for multiplicity and the local Łojasiewicz exponent*
16:00–16:30 Coffee break	16:00–16:20 Coffee break
16:30–18:00 Souad Ali Osseiran, *Presentation of the joint UNIC Theoretical Mathematics programme*, Open table on UNIC Theoretical Mathematics, Q&A	16:20–17:20 Goran Radunović, *An overview of the theory of fractal zeta functions*
19:00 Workshop dinner	

Abstracts

Erna Begović Kovač (University of Zagreb, Faculty of Chemical Engineering and Technology)

Tensor diagonalization algorithms

For a tensor $\mathcal{A}$ of order $d \geq 3$ we study two closely related problems, the SVD-like tensor decomposition and the approximate tensor diagonalization. We develop two Jacobi-type algorithms, one that maximizes the squares of the diagonal entries of $\mathcal{A}$ and the other that maximizes the trace of $\mathcal{A}$. For a general tensor these are the alternating least squares algorithms. The rotation matrices are chosen in each mode one-by-one to maximize the corresponding objective function. We prove the convergence of our algorithms and discuss different initializations, as well as the special case of symmetric tensors. Finally, we present several numerical examples.

Spyridon Dendrinos (University College Cork)

A restricted 2-plane transform associated to surfaces of codimension 2

I will draw a connection between the affine invariant surface measures constructed by P. Gressman and the boundedness of the 2-plane transform associated to surfaces of codimension 2. The measures will be used to determine whether a surface is well-curved or not. The operators are related to Fourier Restriction and other problems for such surfaces. I will present an application to a question in Geometric Measure Theory and will also give an equivalent characterisation of well-curvedness whose proof uses techniques coming from Geometric Invariant Theory.

Luka Grubišić (University of Zagreb, Faculty of Science)

Randomized POD-Beyn algorithm for nonlinear eigenvalue problems — analysis and perspectives

We propose a method to accelerate the solution of 3D FEM-discretized nonlinear eigenvalue problems by utilizing a reduced order model (ROM) via a randomized projection onto a suitable subspace, with eigenpairs identical to the full problem in a region of the complex plane (Beyn approach). The subspace is automatically constructed by solving the full problem at a few random points inside the region of interest. The obtained method is suitable for any nonlinear eigenvalue problem given in the separable (Affine like) form. We test our theory on a family of thermoacoustic application, and show how does the method generalize to applications dealing with other vibrational problems.

Esa Järvenpää (University of Oulu)

Visible part conjecture

I give a survey talk on the visible part conjecture according to which typical visible parts of a compact set $K \subset \mathbb{R}^n$ are $(n-1)$-dimensional provided the dimension of K is at least $n-1$.

Vjekoslav Kovač (University of Zagreb, Faculty of Science)

Large copies of large configurations in large sets

We will discuss a sub-branch of geometric measure theory that studies patterns in positive-measure subsets of the unit cube $[0,1]^d$ or positive-density subsets of the Euclidean space $\mathbb{R}^d$. The proofs typically require results and tools from harmonic analysis (e.g., multilinear singular integrals, oscillatory integrals) and additive combinatorics (e.g., Szemerédi's theorem, the Gowers norms). We will present one possible scheme of approach, several recent results, and several open problems.

Emre Mengi (Koç University)

Estimation of the dominant poles of a large-scale descriptor system

The dominant poles of the transfer function of a descriptor system are those poles that can cause large frequency response. They and corresponding eigenvectors can be used to form a reduced-order approximation to the system. In the talk, I will describe a subspace framework to estimate a prescribed number of dominant poles of a large-scale descriptor system. The framework applies Petrov-Galerkin projections to the original system, then computes the dominant poles of the projected small-scale system, for instance by the QZ algorithm, and expands the subspaces so that the projected system after the subspace expansion interpolates the original system at these dominant poles. I will explain why the subspace framework converges at a quadratic rate, and report numerical results illustrating the rapid convergence, and accuracy of the approach.

Andrei Mustata (University College Cork)

From representations of reductive groups to geometric invariants

Geometric Invariant Theory provides a way to construct a quotient of a variety by the action of a group and it is classically used to construct moduli spaces in algebraic geometry. There is an alternative version of constructing quotients introduced by Mikhail Kapranov called the Chow quotient, which can also be used to construct moduli spaces. We will discuss how birational transformations of a variety with a group action can be used to understand the Chow quotient and the implications of this method for studying the intersection theory of moduli spaces. This presentation is based on work in progress with Anca Mustata.

Goran Radunović (University of Zagreb, Faculty of Science)

An overview of the theory of fractal zeta functions

In this talk I will give an overview of the theory of fractal zeta function and complex dimensions of subsets of Euclidean spaces. The theory has been developed in a series of papers and in a research monograph "Fractal Zeta Functions and Fractal Drums: Higher-Dimensional Theory of Complex Dimensions" coauthored by M. L. Lapidus, G. Radunovic and D. Zubrinic. It is a far-reaching generalization of the one-dimensional theory for fractal strings developed by M. L. Lapidus, M. van Frankenhuijsen as well as by their numerous collaborators. The complex dimensions of a set are defined as poles of the corresponding fractal zeta function and generalize the notion of Minkowski dimension. Although the complex dimensions are defined analytically, they have an explicit geometric meaning revealed in the Steiner-like fractal tube formula of the volume of its ϵ-parallel set. We will illustrate the theory by interesting examples and also reflect on application to dynamical systems.

Maja Resman (University of Zagreb, Faculty of Science)

Fractal data in reading invariants of dynamical systems

We give a short overview of how we can use fractal data (box dimension, tube function or fractal zeta function) to read some intrinsic properties of dynamicals systems, using only one realization (orbit). In particular, we are interested in normalizations and classifications of vector fields and diffeomorphisms close to the singular points, related to problems of cyclicity (number of closed periodic orbits) in planar vector fields. I will give an overview of joint work with M. Klimes, P. Mardesic, G. Radunovic, J. P. Rolin, V. Zupanovic.

Stanisław Spodzieja (University of Łódź)

Effective Bertini theorem and formulas for multiplicity and the local Łojasiewicz exponent (joint with T. Rodak and A. Rozycki)

The classical Bertini theorem on generic intersection of an algebraic set with hyperplanes states the following:

Let X be a nonsingular closed subvariety of $\mathbb{P}^n_k$, where k is an algebraically closed field. Then there exists a hyperplane $H \subset \mathbb{P}^n_k$ not containing X and such that the scheme $H \cap X$ is regular at every point. Furthermore, the set of hyperplanes with this property forms an open dense subset of the complete linear system $|H|$ considered as a projective space.

We show that one can effectively indicate a finite family of hyperplanes H such that at least one of them satisfies the assertion of the Bertini theorem, provided the characteristic of the field k is equal to zero. As an application of the method used in the proof we give effective formulas for the multiplicity and the Łojasiewicz exponent of polynomial mappings.

Ville Suomala (University of Oulu)

Conformal dimension of fractal percolation

We investigate a family of random sets in $\mathbb{R}^d$, the fractal percolation, and show that almost surely, the Hausdorff dimension of the fractal percolation set may be decreased via a quasiconformal map $f\colon \mathbb{R}^d \to \mathbb{R}^d$.