Spring School in Probability

IUC, Dubrovnik, Croatia

April 23-27, 2012

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Lecturers

Jean Jacod (Université Paris VI, France)
Panki Kim (Seoul National University, South Korea)
Takashi Kumagai (RIMS, Kyoto University, Japan)
Vlada Limić (Université de Provence, Marseille, France)
René L. Schilling (Technische Universität Dresden, Germany)

Organising committee

René L. Schilling (Technische Universität Dresden, Germany)
Renming Song (University of Illinois, United States)
Zoran Vondraček (University of Zagreb, Croatia)

Contents

Useful information

Spring School Site: All lectures and short talks will take place in classroom 2 on the second floor of the Inter-University Centre. The IUC is located at Don Frana Bulića 4 (2 on the map on page 32). The classroom is equipped with a laptop, an overhead projector, as well as a blackboard.

Registration: **Sunday, April 22, 18:00-20:30** in the lobby of Hotel Lero, and **Monday, April 23, 8:15-8:50** at the IUC. The registration fee of 40 € (alternatively 300 kn) is payable in cash at the IUC office (first floor) every day of the week. The IUC office will provide the confirmation.

Computer/E-mail Access (at the IUC): Participants will have a computer room with internet connection and printer at their disposal every work day from 8:00 to 19:00. Photocopying of work material and handouts is also available. In the building (including the courtyard) there is a free EDUROAM access which allows you to log on to your computer account if your computer is set up for this by your home institution. Additionally, there are two wireless networks in the building. Login names and passwords will be provided at the site.

Tea and Coffee Breaks: Refreshments will be served each morning at 10:15 in the courtyard (most likely). On the ground floor of the IUC building there is a coffee shop Atrium serving drinks and sandwiches (not for free).

Lunch: We do not provide lunchtime catering. A generous lunch break is scheduled from 12:00 (or 13:05, depending on the day) until 15:30 which should be plenty of time to enjoy the lunch in and around the Old City. A map and a short guide of possible places for lunch in the Old City can be found on pages 32 and 36 of this booklet.

Dinner: For those staying in Hotel Lero, dinner is available as a part of half-board. There are also plenty of possibilities to eat out in the Old City and elsewhere in Dubrovnik.

Spring School Dinner: This will take place in the Restaurant *Orhan* (address: Od Tabakarije 1, 3 on the map on page 32) on Tuesday, April 24, from 20:00. The restaurant is situated at the foot of Tower Lovrjenac, 200 m away from the Old City with a view at the fort Bokar. The dinner is free to all participants. There will be two choices of menu – seafood and meat menu (vegetarian menu available upon request). Drinks included (not in unlimited quantities).

Old City Tour: The two-hour guided tour through the Old City will be organized on Wednesday afternoon (April 25) from 15:30 until 17:30. The tour is in English and is free except for the entrance fee to museums.

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
08:15-08:50	Registration				
08:50-09:00	Opening				
09:00-10:15	Kumagai	Schilling	Kim	Jacod	Limić
10:15-10:45	Coffee break	Coffee break	Coffee break (10:15-10:30)	Coffee break	Coffee break (10:15-10:30)
10:45-12:00	Kim	Jacod	Limić (10:30-11:45)	Kumagai	Schilling (10:30-11:45)
		Berschneider (12:05-12:25)	Schilling (11:50-13:05)		Jacod (11:50-13:05)
					Closing
15:30-16:45	Limić	Kumagai	City Tour (15:30-17:30)	Kim	
16:45-17:00	Break				
17:00-17:20	Szczypkowski	Li (17:00-17:45)		Valba	
17:25-17:45	Lee			Milošević	
17:50-18:10	Lupaşcu	Campese		Spangenberg	
18:15-18:35	Wojciechowski	Cygan		Sandrić	
20:00-		Dinner			

Tuesday evening – school dinner at Restaurant Orhan; Address: Od Tabakarije 1

Wednesday afternoon – guided old city tour

Schedule by day

Monday, April 23

08:15-08:50	Registration
08:50-09:00	Opening
09:00-10:15	**Takashi Kumagai**, RIMS, Kyoto University *Random walks on graphs and applications to random media 1*
10:15-10:45	Coffee break
10:45-12:00	**Panki Kim**, Seoul National University *Probabilistic potential theory for jump processes and heat kernel estimates 1*
12:00-15:30	Lunch break
15:30-16:45	**Vlada Limić**, Université de Provence, Marseille *Reinforced random walks: a guide through martingales and coupling 1*
16:45-17:00	Break
17:00-17:20	**Karol Mateusz Szczypkowski**, Wrocław University of Technology *Gradient perturbations of transition densities*
17:25-17:45	**Yunju Lee**, Seoul National University *Oscillation of harmonic functions for subordinate Brownian motion and its applications*
17:50-18:10	**Oana Lupaşcu**, Romanian Academy of Sciences *Subordination of L^p-semigroups and right processes*
18:15-18:35	**Łukasz Wojciechowski**, University of Wrocław *Lévy system and its application to Fourier multiplier*

Tuesday, April 24

09:00-10:15	**René L. Schilling**, Technische Universität Dresden *Coupling methods for Lévy processes 1*
10:15-10:45	Coffee break
10:45-12:00	**Jean Jacod**, Université Paris VI *High-frequency statistics for jump processes 1*
12:05-12:25	**Georg Berschneider**, Technische Universität Dresden *Spectral representation of intrinsically stationary random fields*
12:25-15:30	Lunch break
15:30-16:45	**Takashi Kumagai**, RIMS, Kyoto University *Random walks on graphs and applications to random media 2*
16:45-17:00	Break
17:00-17:45	**Wenbo Li**, University of Delaware *Fastest rate of convergence for Brownian motion with jump boundary*
17:50-18:10	**Simon Campese**, Université du Luxembourg *Increasing the speed of convergence in the normal approximation of functionals of Gaussian fields*
18:15-18:35	**Wojciech Cygan**, University of Wrocław *Green function asymptotic for subordinated random walk*
20:00-	Dinner

Wednesday, April 25

09:00-10:15	**Panki Kim**, Seoul National University *Probabilistic potential theory for jump processes and heat kernel estimates 2*
10:15-10:30	Coffee break
10:30-11:45	**Vlada Limić**, Université de Provence, Marseille *Reinforced random walks: a guide through martingales and coupling 2*
11:50-13:05	**René L. Schilling**, Technische Universität Dresden *Coupling methods for Lévy processes 2*
13:05-15:30	Lunch break
15:30-17:30	City Tour

Thursday, April 26

09:00-10:15	**Jean Jacod**, Université Paris VI *High-frequency statistics for jump processes 2*
10:15-10:45	Coffee break
10:45-12:00	**Takashi Kumagai**, RIMS, Kyoto University *Random walks on graphs and applications to random media 3*
12:00-15:30	Lunch break
15:30-16:45	**Panki Kim**, Seoul National University *Probabilistic potential theory for jump processes and heat kernel estimates 3*
16:45-17:00	Break
17:00-17:20	**Olga Valba**, Université Paris Sud *On exclusivity of alphabets with four nucleotide types*
17:25-17:45	**Marija Milošević**, University of Niš *Numerical solution of highly nonlinear neutral stochastic differential equations with time-dependent delay*
17:50-18:10	**Felix Spangenberg**, Technische Universität Braunschweig *ARMA Processes in Banach spaces*
18:15-18:35	**Nikola Sandrić**, University of Zagreb *Recurrence and transience property for stable-like Markov chains*

Friday, April 27

09:00-10:15	**Vlada Limić**, Université de Provence, Marseille *Reinforced random walks: a guide through martingales and coupling 3*
10:15-10:30	Coffee break
10:30-11:45	**René L. Schilling**, Technische Universität Dresden *Coupling methods for Lévy processes 3*
11:50-13:05	**Jean Jacod**, Université Paris VI *High-frequency statistics for jump processes 3*
13:05	Closing

Abstracts of lectures

Jean Jacod, Université Paris VI
http://www.proba.jussieu.fr/pageperso/jacod
High-frequency statistics for jump processes

This course will be in the setting of high-frequency statistics, that is, one observes a process at regularly spaced times over a finite time interval, and the mesh of the observation scheme is small and eventually goes to 0.

We will consider the case where the observed process is a discontinuous Lévy process, or more generally a discontinuous Itô semimartingale: although technically more difficult, the second case is essentially the same as the Lévy case, on which emphasis will be put in the whole course.

First, we examine what quantities, related to the Lévy measure of the process (that is what governs the law of the jumps), are "identifiable" in the following sense: a parameter is identifiable if it can be estimated in a consistent way as the observation mesh goes to 0. this reduces basically to the so-called successive Blumenthal-Getoor indices and the associated intensities, and we will spend some time explaining what those objects are. A relatively surprising fact is that the identifiable parameters do not depend whether there is a Gaussian part or not. On the other hand the Lévy measure itself is never identifiable, even in the simplest case where the process is compound Poisson.

Second, and in the case of Lévy processes only, we will try to establish bounds on the rate of convergence of any possible sequence of estimators for the indices and the corresponding intensities: this will be done for the first two indices only, on the basis of the behavior of the Fisher information.

Third, we will give methods to estimate these identifiable parameters, with emphasis on the first Blumenthal-Getoor index which indeed drives the "jump activity": the biggest is the index, the most active are the jumps. For the estimation, the situation varies greatly, according to whether the Lévy process has a Gaussian part (or, in the semimartingale setting, the continuous martingale part is not vanishing), or when it has not. In particular the rates of convergence of the estimators will depend on this fact.

This program is probably rather ambitious, but may be adapted to the audience. Depending on the wishes of the participants, emphasis can be put more on the proof of the basic results, or more on the statistical applications.

Panki Kim, Seoul National University
http://www.math.snu.ac.kr/∼pkim

Probabilistic potential theory for jump processes and heat kernel estimates

A symmetric α-stable process X on $\mathbb{R}^d$ is a Lévy process whose transition density $p(t, x-y)$ relative to Lebesgue measure is uniquely determined by its Fourier transform $\int_{\mathbb{R}^d} e^{ix\cdot\xi} p(t,x)dx = e^{-t|\xi|^\alpha}$. Here α must be in the interval (0, 2]. Brownian motion is the symmetric 2-stable process, which has been intensively studied due to its central role in modern probability theory and its numerous important applications in other scientific areas including many other branches of mathematics.

Recently there has been intense interest in studying non-Gaussian stable processes ($0 < \alpha < 2$), due to their importance both in theory and in application. It is well-known that many physical and economic systems should be and in fact have been successfully modeled by non-Gaussian stable processes.

Although a lot is known about symmetric stable processes and their potential theory, little is known until very recently about the counterparts to some of the deep results for Brownian motion, such as sharp estimates on transition density of symmetric stable processes in open sets. This course will treat some of recent developments of this topic.

1. Basic concepts of Lévy processes and symmetric stable process.
2. Symmetric stable process in open subsets
3. Harmonic functions, Harnack inequalities, and boundary Harnack inequality
4. Dirichlet heat kernel estimates

Prerequisites: Graduate probability and measure theory

References

[1] K. Bogdan, T. Byczkowski, T. Kulczycki, M. Ryznar, R. Song and Z. Vondraček, *Potential analysis of stable processes and its extesions.* Lecture Notes in Math, **1980**, Springer, 2009.

[2] K. Bogdan, T. Grzywny and M. Ryznar, Heat kernel estimates for the fractional Laplacian with Dirichlet conditions. *Ann. Probab.* **38** (2010), 1901–1923.

[3] Z.-Q. Chen, Multidimensional symmetric stable processes. *Korean J. Comput. Appl. Math.* **6** (1999), 227–266.

[4] Z.-Q. Chen, Symmetric jump processes and their heat kernel estimates. *Sci. China Ser. A,* **52** (2009), 1423-1445.

[5] Z.-Q. Chen, P. Kim, and R. Song, Heat kernel estimates for Dirichlet fractional Laplacian. *J. European Math. Soc.* **12 (5)** (2010), 1307–1329

[6] Z.-Q. Chen and R. Song, Estimates on Green functions and Poisson kernels of symmetric stable processes, *Math. Ann.*, **312** (1998), 465-601.

[7] Z.-Q. Chen and J. Tokle: Global heat kernel estimates for fractional Laplacians in unbounded open sets. *Probab. Theory Relat. Fields* **149** (2011), 373–395.

Takashi Kumagai, RIMS, Kyoto University
http://www.kurims.kyoto-u.ac.jp/∼kumagai

Random walks on graphs and applications to random media

The main theme of these lectures is to analyze heat conduction on disordered media such as fractals and percolation clusters using both probabilistic and analytic methods, and to study scaling limits of Markov chains on the media.

The problem of random walk on a percolation cluster 'the ant in the labyrinth' has received much attention both in the physics and the mathematics literature. In 1986, H. Kesten showed an anomalous behavior of a random walk on a percolation cluster at critical probability for trees and for $\mathbb{Z}^2$. – To be precise, the critical percolation cluster is finite, so the random walk is considered on an incipient infinite cluster (IIC), namely a critical percolation cluster conditioned to be infinite.– Partly motivated by this work, analysis and diffusion processes on fractals have been developed since the late eighties. As a result, various new methods have been produced to estimate heat kernels on disordered media, and these turn out to be useful to establish quenched estimates on random media. Recently, it has been proved that random walks on IICs are subdiffusive on $\mathbb{Z}^d$ when d is high enough, on trees, and on the spread-out oriented percolation for $d > 6$.

Throughout the lectures, I will survey the above mentioned developments in a compact way. In the first part of the lectures, I will briefly summarize potential theory for symmetric Markov chains on weighted graphs. I will also discuss heat kernel estimates and their stability under perturbations of operators and spaces. In the latter part of the lectures, I will give various examples of disordered media and obtain heat kernel estimates for Markov chains on them. In some models, I will also discuss scaling limits of the Markov chains. Examples of disordered media include fractals, percolation clusters, random conductance models and random graphs.

The following is a concrete plan of each lecture.

Lecture 1: First, I will briefly summarize general potential theory for symmetric Markov chains on weighted graphs. Then I will explain some results concerning the stability of heat kernel estimates (such as the Nash inequality and Gaussian heat kernel estimates) under perturbations of the weights. I will also mention that effective resistance is a convenient tool to estimate Green functions, exit times from balls etc.

Lecture 2: I will discuss random walk on an incipient infinite cluster (IIC) for a critical percolation. I will give some sufficient condition for the sharp on-diagonal heat kernel bounds for random walk on random graphs. I then show that the heat kernel behaves anomalously for random walks on IICs of high dimensional critical bond percolations on $\mathbb{Z}^d$. If time permits, I will briefly discuss estimates for mixing times of Markov chains on random graphs, especially on the Erdős-Rényi random graph.

Lecture 3: In the last lecture, I will summarize recent results on the heat kernel estimates and quenched invariance principle for the random conductance model on $\mathbb{Z}^d$. Put *i.i.d.* conductance μ_e on each bond in $\mathbb{Z}^d$ and consider the Markov chain associated with the (random) weighted graph. I will discuss its scaling limit. The ideas of the proof (including the corrector method) will be given during the lecture.

References

[1] S. Andres, M. T. Barlow, J-D. Deuschel and B. M. Hambly. Invariance principle for the random conductance model. *Preprint 2011.* `http://people.maths.ox.ac.uk/hambly/PDF/Papers/rc.pdf`

[2] M. Biskup. Recent progress on the Random Conductance Model. *Probability Surveys,* **8** (2011), 294–373.

[3] G. Kozma and A. Nachmias. The Alexander-Orbach conjecture holds in high dimensions. *Invent. Math.* **178** (2009), 635–654.

[4] T. Kumagai. Random walks on disordered media and their scaling limits. *St. Flour Lecture Notes* (2010). `http://www.kurims.kyoto-u.ac.jp/~kumagai/StFlour-TK.pdf`

Vlada Limić, Université de Provence, Marseille
http://www.cmi.univ-mrs.fr/∼vlada

Reinforced random walks: a guide through martingales and coupling

The course will focus on strongly edge reinforced random walks, for which the reinforcement weight function W is reciprocally summable. In this setting, it can be easily seen that the walk eventually traverses (fixates on) a single edge of the graph, with positive probability.

However, a lot more effort is needed to show that the tail event "the walk eventually fixates on one edge" has probability one. The first goal of the course is to explain the two main techniques used in this analysis on general graphs, through a detailed study of the strong W-urns, which are the basic examples of reinforced processes. These techniques also prove to be useful in various, similar but different, reinforcement settings. It is surprising that the parity of the underlying graph has such an important influence on the difficulty of the above study. For example, the behavior is completely understood on any bipartite graph due to a slick argument of Sellke, but if the underlying graph is just a triangle, the best known theorem is not completely general. For W increasing and reciprocally summable, the aboce attracting edge property is understood on any graph of bounded degree. The course will next address the properties of the law of the fixation time in this restricted setting, and several interesting related open questions will be mentioned. If time permits, further open questions on general reinforced random walks will be discussed.

A few references in chronological order:

References

[1] T. Sellke. Reinforced random walks on the d-dimensional integer lattice, Technical report 94-26, Purdue University (1994)

[2] V. Limic. Attracting edge property for a class of reinforced random walks. Ann. Probab., Vol. 31, 1615–1654, 2003.

[3] V. Limic and P. Tarres. Attracting edge and strongly edge rein-

forced walks. Ann. Probab., Vol. 35, No. 5, 1783–1806, 2007.

[4] V. Limic and P. Tarres. What is the difference between a square and a triangle? In and out of equillibrium 2. Series: Progress in Probability, Vol. 60, 481–496, Birkhauser, 2008.

[5] C. Cotar and V. Limic. Attraction time for strongly reinforced walks. Ann. Appl. Probab. Vol. 19, No. 5, 1972-2007, 2009.

René L. Schilling, Technische Universität Dresden
http://www.math.tu-dresden.de/sto/schilling
Coupling methods for Lévy processes

The aim of these lectures is to give a concise introduction to the (Doeblin) coupling of Lévy processes.

We start with a short introduction to coupling, some consequences and applications. We will, in particular, investigate couplings for simple random walks (Mineka coupling) and Brownian motion (reflection coupling). Then we give a very brief introduction to Lévy processes and their construction from compound Poisson processes. Finally we show how we can prove the existence of (optimal) couplings for Lévy and Lévy-driven Ornstein-Uhlenbeck processes. The last part is based on a series of joint papers with Jian Wang (Fujian Normal University, Fouzou, China).

References

General texts on Coupling

[1] Mu-Fa Chen: *Eigenvalues, Inequalities, and Ergodic Theory*. Springer, London 2004.

[2] Torgny Lindvall: *Lectures on the Coupling Method*. Dover, Mineola (NY) 2002. Reprint of the edition Wiley, New York 1992.

[3] Hermann Thorisson: *Coupling, Stationarity and Regeneration*. Springer, New York 2000.

Lévy Processes

[4] Ken-iti Sato: *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press, Cambridge 1999.

Coupling of Lévy Processes

These papers and preprints are available from my homepage
http://www.math.tu-dresden.de/sto/schilling/research/forschung.html

[5] B. Böttcher, R.L. Schilling, J. Wang: Constructions of Coupling Processes for Lévy Processes and Their Applications. *Stoch. Proc. Appl.* **121** (2011), 1201–1216.

[6] R.L. Schilling and J. Wang: On the coupling property of Lévy processes. *Ann. Inst. Henri Poincaré: Probab. & Stat.* **47** (2011), 1147–1159.

[7] R.L. Schilling and J. Wang: On the coupling property and the Liouville theorem for Ornstein-Uhlenbeck processes. *J. Evol. Eq.* **12** (2012), 119–140.

[8] R.L. Schilling, P. Sztonyk and J. Wang: Coupling property and gradient estimates for Lévy processes via the symbol. To appear in: *Bernoulli*.

Abstracts of contributed talks

Georg Berschneider, Technische Universität Dresden
Spectral representation of intrinsically stationary random fields

Requiring for a random field to have (certain) weakly stationary generalized increments one obtains the class of intrinsically stationary random fields. Processes of this type have been investigated by A. M. Yaglom, M. S. Pinsker, N. Wiener and G. Matheron. They can be used to model nonstationary phenomena in d-dimensional Euclidean space and generalize stationary stochastic processes and processes with stationary increments.

We use H. Cramér's characterization of stationary random fields as Fourier transforms of orthogonal random measures to show a spectral representation of intrinsically stationary random fields. The resulting representation resembles the Lévy-Khinchine formula for the characteristic exponents of Lévy processes.

Simon Campese, Université du Luxembourg
Increasing the speed of convergence in the normal approximation of functionals of Gaussian fields

Using Edgeworth expansions, one can increase the speed of convergence of normalized iid-sums in the classical central limit theorem. We will show how this approach can be generalized to cover random vectors whose components are functionals of Gaussian fields (such as, for example, the fractional Brownian motion). Our main ingredients will be Stein's method and Malliavin calculus.

Wojciech Cygan, University of Wrocław
Green function asymptotic for subordinated random walk

I consider a process S_n^ψ in $\mathbb{Z}^d$, $d \geq 3$, obtained by subordinating simple random walk according to the concept of discrete subordination. The function ψ, which is the Laplace exponent of subordinator is assumed to be a complete Bernstein function. With an additional assumption on the behaviour of ψ at zero, which is prescribed in the realm of regularly varying functions, I establish the asymptotic of the Green function at infinity. Further, by virtue of the Green function asymptotic, I present Wiener's test of massiveness.

Yunju Lee, Seoul National University

Oscillation of harmonic functions for subordinate Brownian motion and its applications

In this talk, we establish an oscillation estimate of nonnegative harmonic functions for a large class of integro-differential operators. Such operators are the infinitesimal generators of pure-jump subordinate Brownian motion. As an application, we give a probabilistic proof of relative Fatou theorem for harmonic functions for the integro-differential operators in bounded κ-fat open set. That is, if u is a positive harmonic function in a bounded κ-fat open set D and h is a positive harmonic function in D vanishing on D^c, then the non-tangential limit of u/h exists almost everywhere with respect to the Martin-representing measure of h. Under the gaugeability assumption, relative Fatou theorem is true for operators obtained from the generator of pure-jump subordinate Brownian motion in bounded κ-fat open set D through non-local Feynman-Kac transforms.

Wenbo Li, University of Delaware

Fastest rate of convergence for Brownian motion with jump boundary

"Consider" a family of probability measures $\{\nu_y : y \in \partial D\}$ on a bounded open domain $D \subset \mathbb{R}^d$ with smooth boundary. For any starting point $x \in D$, we run a standard d-dimensional Brownian motion $B(t) \in \mathbb{R}^d$ until it first exits D at time τ, at which time it jumps to a point in the domain D according to the measure $\nu_{B(\tau)}$ at the exit time, and starts the Brownian motion afresh. The same evolution is repeated independently each time the process reaches the boundary. The resulting diffusion process is called Brownian motion with jump boundary (BMJ).

The spectral gap of non-self-adjoint generator of BMJ, which describes the exponential rate of convergence to the invariant measure, is studied. In particular, we prove the so-called 2/3-conjecture on the largest spectral gap (fastest rate of convergence) among all possible jump measures in one-dimensional setting.

Oana Lupaşcu, Romanian Academy of Sciences
Subordination of L^p-semigroups and right processes

We show that the subordination induced by a convolution semigroup (subordination in the sense of Bochner) of a C_0-semigroup of sub-Markovian operators on an L^p space is actually associated to the subordination of a right process. It turns out that an enlargement of the base space is necessary. A main step in the proof is the preservation under such a subordination of the property of a Markov process to be a (Borel) right process. We use several analytic and probabilistic potential theoretical tools.

Marija Milošević, University of Niš
Numerical solution of highly nonlinear neutral stochastic differential equations with time-dependent delay

The evolution of a physical system depending only on its present state and some random input can often be described by a stochastic ordinary differential equation. However, in many physical situations the rate of change of the system depends not only on the present but also on its past states. In such cases, neutral stochastic differential delay equations provide an important tool for describing and analyzing such systems. The fact that stochastic differential equations in most cases cannot be solved explicitly has been the main motivation for the development of different approximate methods.

The subject of this talk are discrete time approximations for solutions of a class of highly nonlinear neutral stochastic differential equations with time-dependent delay. The convergence in probability of the approximate solution to the exact solution is established under generalized

Khasminskii-type conditions. Also, some stability results are obtained for exact and the approximate solution.

Nikola Sandrić, University of Zagreb
Recurrence and transience property for stable-like Markov chains

Let $\{S_n\}_{n\geq 0}$ be a random walk on $\mathbb{R}^d$, $d \geq 1$. The random walk $\{S_n\}_{n\geq 0}$ is said to be recurrent if

$$\mathbb{P}\left(\liminf_{n\longrightarrow\infty} |S_n| = 0\right) = 1,$$

and transient if

$$\mathbb{P}\left(\lim_{n\longrightarrow\infty} |S_n| = \infty\right) = 1.$$

It is well known that every random walk is either recurrent or transient. In the class of α-stable random walks on $\mathbb{R}$, a symmetric α-stable random walk is recurrent if and only if $\alpha \geq 1$. We generalize one-dimensional symmetric α-stable random walk in the way that the index of stability of jump distribution depends on the current position and we study the recurrence and transience property of the generalization. In other words, we consider the recurrence and transience problem for a temporally homogeneous Markov chain on the real line with transition function $p(x, dy) = f_x(y - x)dy$, where the density functions $f_x(y)$, for large $|y|$, have a power-law decay with exponent $\alpha(x) + 1$, where $\alpha(x) \in (0, 2)$.

Under a uniformity condition on the densities $f_x(y)$ and some mild technical conditions, we prove that when $\liminf_{|x|\longrightarrow\infty} \alpha(x) > 1$, the chain is recurrent, while when $\limsup_{|x|\longrightarrow\infty} \alpha(x) < 1$, the chain is transient.

Furthermore, if $f_x(y)$ are densities of symmetric distributions such that the function $x \longmapsto f_x$ is periodic and the set $\{x : \alpha(x) = \alpha_0 := \inf_{x\in\mathbb{R}} \alpha(x)\}$ has positive Lebesgue measure, then, under some mild technical conditions on the densities $f_x(y)$, the chain is recurrent if and only if $\alpha_0 \geq 1$.

Finally, if $f_x(y)$ is the density of a symmetric α-stable distribution for negative x and the density of a symmetric β-stable distribution for nonnegative x, where $\alpha, \beta \in (0, 2)$, then the chain is recurrent if and only if $\alpha + \beta \geq 2$.

As a special case of these results we give a new proof for the recurrence and transience property of a symmetric α-stable random walk on $\mathbb{R}$ with the index of stability $\alpha \in (0, 2)$.

Karol Mateusz Szczypkowski, Wrocław University of Technology
Gradient perturbations of transition densities

I will present upper and lower bounds of perturbation series for transition densities, corresponding to additive gradient perturbations satisfying certain space-time integrability conditions. The talk will be based on my joint work with Tomasz Jakubowski, Ph.D.

Felix Spangenberg, Technische Universität Braunschweig
ARMA Processes in Banach spaces

We wish to study the existence and other properties of solutions of ARMA equations in separable complex Banach spaces. We will mainly investigate strictly stationary solutions of ARMA equations, where the white noise is a series of Banach-space-valued i.i.d. random variables, in contrast to Bosq [1] who deals with weakly stationary solutions.

Our first goal is to generalise results by Brockwell and Lindner [2] and Brockwell, Lindner and Vollenbröcker [3]. They characterise necessary and sufficient conditions in the multivariate ARMA(1, q) case in terms of the eigenvalues of the matrix of the AR part. Our first attempt is to generalise this by investigating the spectrum of the operator in the AR(1) case. We arrive at partial results for necessary and sufficient conditions by excluding the unit circle and zero from the spectrum. In this case, we have a $\log^+$-moment condition for our white noise. We extend these results to the AR(p) case and also show uniqueness.

We further look at what can happen in the case when zero is in the spectrum or the intersection of the spectrum and the unit circle is nonempty. We give two examples of AR(1) equations whose operators have the closed unit disc as spectrum but where one equation admits a nondeterministic solution while the other does not. We also provide an example of an AR(1) equation given by a quasinilpotent operator that provides a sufficient condition being weaker than the usual $\log^+$-moment condition.

References

[1] D. Bosq *Linear Processes in Function Spaces* Springer-Verlag, New York 2000

[2] P. J. Brockwell, A. Lindner *Strictly Stationary Solutions of Autoregressive Moving Average Equations*. Biometrika 97, 765-772 2010

[3] P. J. Brockwell, A. Lindner and B. Vollenbröker *Strictly stationary solutions of multivariate ARMA equations with i.i.d. noise* Ann. Inst. Statist. Math. 2011

Olga Valba, Université Paris Sud

On exclusivity of alphabets with four nucleotide types

Genetic information in all life cells is kept within the primary sequences of DNA and RNA molecules. Both of them are heteropolymers consisting of four different nucleotide types. It seems reasonable to ask why nature uses exactly four letters to encode genetic information. This eternal question has been discussed since the role of DNA and RNA in storing and transmission of genetic information is understood. Typically, the attempts to answer it are based on the chemistry of interacting nucleotides [1], or deal with the conjectures lying in the general information theory [2].

Here we present an observation concerning the statistics of RNA secondary structures, which, for the best of our knowledge, was never discussed before, and which may, in our opinion, be a new contribution to the problem of "why only four?" Using the methods of statistical physics we demonstrate the existence of a specific phase transition which occurs in a toy model of random RNA–like chains.

The free energy of random heteropolymers which can form hierarchical cactus–like secondary structure, typical for RNA molecules, is analyzed. In particular, the fraction f of nucleotides involved in the formation of the secondary structure as a function of the number c of different monomer species is investigated. It is shown, that with changing c the secondary structures of random heteropolymers undergo a phase transition [3]. Namely, for $c \leq c_{\rm cr} = 4$ the fraction of "active" nucleotides f tends to 1 as the chain length n goes to infinity, signaling the formation of a virtually "perfect" gapless secondary structure. In turn, for $c > 4$ the value of f tends to some self–averaged value $\bar{f}(c) < 1$, meaning that a non-perfect structure with gaps is formed. The value $c_{cr} = 4$ is critical: f approaches 1 logarithmically for $n \to \infty$. Such a critical behavior allows one to speculate about exclusivity of a four–letter alphabet used in natural RNAs with complex cactus–like secondary structures.

Joint work with M. Tamm and S. Nechaev.

References

[1] E. Szathmary (2003) Why are there four letters in the genetic alphabet? Nature Reviews Genetics 4: 995-1001.

[2] A. Patel (2001) Why Genetic Information Processing could have a Quantum Basis. Journal of Biosciences 26: 145-151.

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Lévy system and its application to Fourier multiplier

We are interested in Lévy systems and their applications to Fourier multipliers.

Single and double Lévy systems may be applied to construct martingales

$$t \mapsto \sum_{0<v\le t} F(v, Y_{v-}, Y_v) - \int_0^t \int_{\mathbb{R}^d} F(v, Y_v, Y_v + z)\nu(dz)dv,$$

and identify their quadratic variation as $\sum\limits_{0<v\le t} F^2(v, Y_{v-}, Y_v)$, under some integrability conditions on F. Here Y is a pure jump Lévy process with Lévy measure ν.

We will apply the resulting stochastic calculus to construct a class of Fourier multipliers.

For each function $M : \mathbb{R}^d \to \mathbb{C}$ bounded by 1 there is a unique linear contraction $\mathcal{M}$ on $L^2(\mathbb{R}^d)$ defined in terms of the Fourier transform as follows,

$$\widehat{\mathcal{M}g} = M\hat{g}\,. \tag{1}$$

We are interested in *symbols* M for which the *Fourier multiplier* $\mathcal{M}$ extends to a bounded linear operator on $L^p(\mathbb{R}^d)$ for $p \in (1, \infty)$. Such symbols have been recently obtained in [1, 2] by transforming parabolic or heat martingales of Lévy process (including the Brownian motion). Burkholder-Wang inequalities for differentially subordinate continuous time martingales [3], were used in [1,2] to prove that the operator norm of these multipliers on $L^p(\mathbb{R}^d)$ does not exceed

$$p^* - 1 = \max\{p-1, \frac{1}{p-1}\}\,. \tag{2}$$

For example, we obtained multipliers for martingales driven by two linear transformations of Wiener processes, using the standard Itô calculus but the symbols obtained in this way are symmetric.

References

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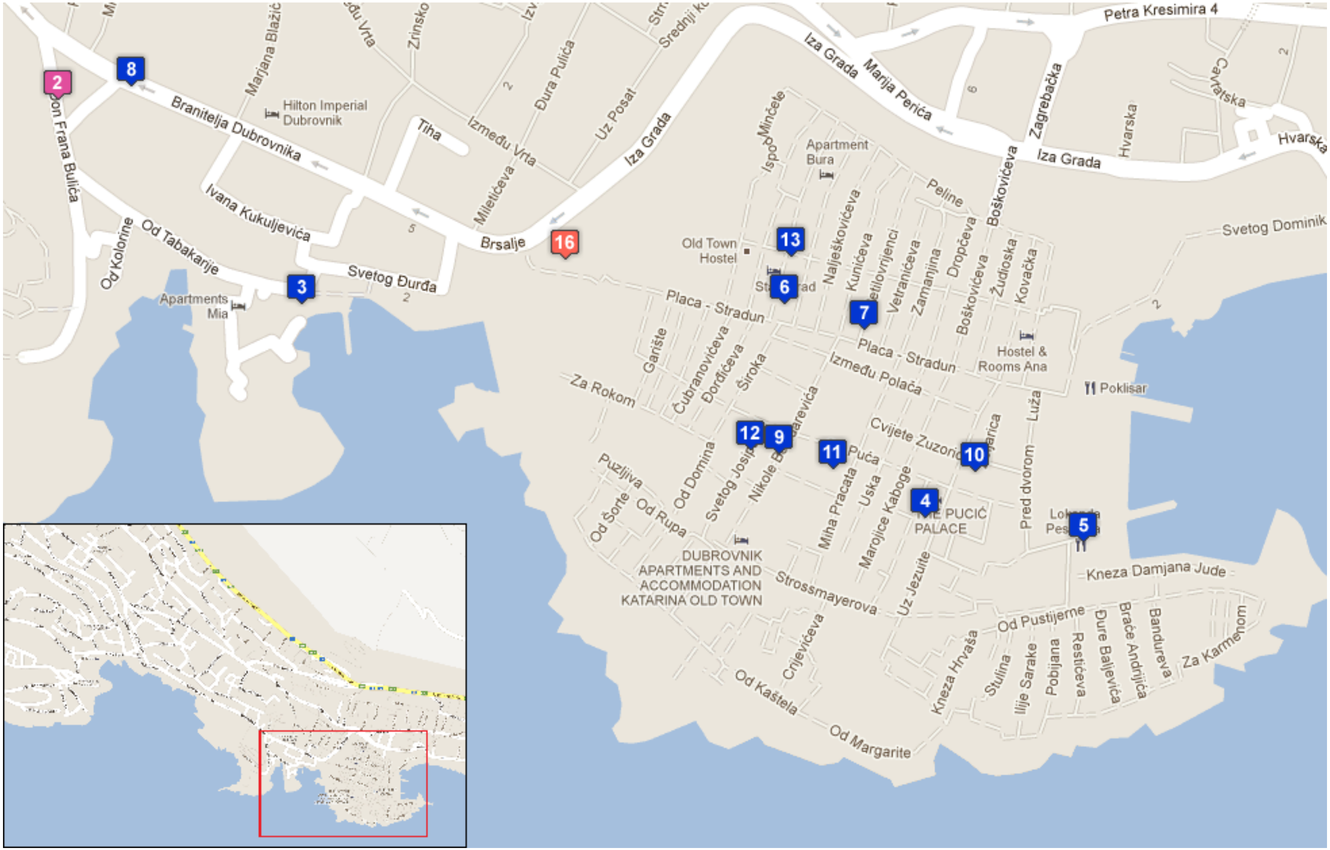
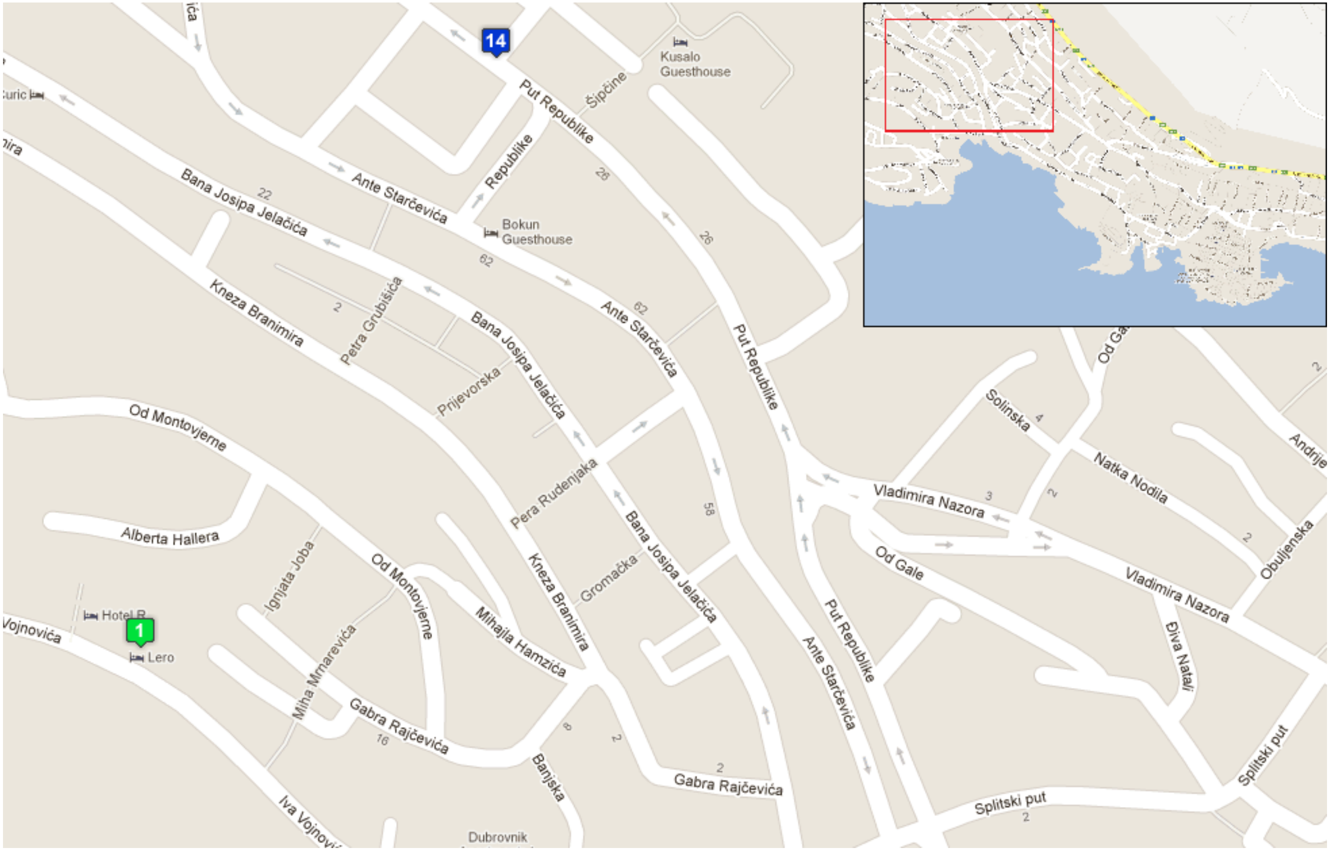
City map - IUC and Old City



City map - Around Hotel Lero



Getting around

From airport to town

The bus into town leaves after every scheduled flight (the price of a one-way ticket is 35 kn or 5 €). You should get off at Pile Gate (16 on the map on page 32). From there bus line 4 (see below) will take you to Hotel Lero. An alternative is to use a taxi. The price of a taxi ride from the airport to Hotel Lero is 200 - 210 kn.

Money

The Croatian currency is the kuna (usually listed as *kn*). As of March 2012, the exchange rate is 7,54 kn for 1 €, 5,73 kn for 1 \$. *Mjenjačnica* is the Croatian word for Exchange Office. There are lots of ATMs in the city and at the airport. The shops and restaurants accept only Croatian kunas.

Public transportation

The **orange buses** are managed by the local company Libertas Dubrovnik (http://libertasdubrovnik.hr) and run from 5 am until midnight. The price of a ticket (for journeys within the city) is 10 kn if you buy it from news kiosks (*Tisak*) and 12 kn if you buy it on the bus (from the driver). The ticket is validated by inserting it into the ticket-stamping machine on the bus. Popular bus lines are included in the table below.

Line	Connecting
1A, 1B, 1C	Mokošica and Pile Gate
3	Nuncijata and Pile Gate
4	Hotel Palace (Lapad) and Pile Gate
6	Babin Kuk (Lapad) and Pile Gate

You will probably find line 4 to be the most useful as one of its stops is close to Hotel Lero. The first departure from Hotel Palace (Lapad) is at 5:40, and the last departure from Pile Gate is at 0:05.

If you want to get a **taxi**, call 0800 09 70. Additionaly, taxis are waiting at the following designated taxi stations: Pile Gate, Main Bus Station, Gruž Harbour, Ploče Gate, central Lapad. The initial price is 25 kn, and then 8 kn per kilometre, 2 kn per baggage item (max five) and 80 kn per hour for waiting.

Where to eat

Name	Address	Symbol	Description
Orhan	Od Tabakarije 1	3	This is where the official dinner will take place
Kamenice	Gundulićeva poljana 8	4	Seafood - oysters, mussels, risottos, small fried fish
Lokanda Peskarija	Ribarnica	5	Simple fish dishes
Buffet Škola	Antuninska 1	6	Sandwiches with prosciutto, marinated cheese, sardines
Tovjerna Maro	Petilovrijenci 4	7	Small, diverse meals, meats, poultry, salad
Sesame	Danthe Alighieria b.b.	8	Mediterranean food, closest to the IUC (cca 50 m)
Baracuda	Nikole Božidarevića 10	9	Pizza
Oliva Pizzeria	Lučarica 5	10	Pizza and pasta
Taj Mahal	Nikole Gučetića 2	11	Bosnian, Turkish (not Indian)
Sugar & Spice	Sv. Josipa 5	12	Bakery, desserts
Nishta	Corner of Palmotićeva and Prijeko	13	Vegetarian
Konoba Tabak	Put Republike 32	14	*gableci*

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