Random geometry, graphs and extremes

Book of abstracts

Organizers: Bojan Basrak, Bartek Blaszczyszyn, Ilya Molchanov

Dubrovnik, October 27-31, 2025

Contents

Sample-Path Large Deviations for Functions of Poisson Cluster Processes

Fabien Baeriswyl

TU Wien

In this talk, we discuss sample-path large deviation principles for the centered cumulative functional of marked Poisson cluster processes in the Skorokhod space equipped with the M1 topology, under joint regular variation assumptions on the marks and the offspring distributions governing the propagation mechanism. These findings can also be interpreted as hidden regular variation of the cluster processes' functionals, extending the results in Dombry et al. (2022) to cluster processes with heavy-tailed characteristics, including mixed Binomial Poisson cluster processes and Hawkes processes. Notably, by restricting to the adequate subspace of measures on D([0, 1], R+), and applying the correct normalisation and scaling to the paths of the centered cumulative functional, the limit measure is shown to concentrate on paths with multiple large jumps. Joint work with Valérie Chavez-Demoulin and Olivier Wintenberger.

Stick-Breaking Distributions and Pitman-Yor theorem

Bojan Basrak

University of Zagreb

Pitman and Yor in 1992 proved that the self-normalized jumps of α-stable subordinators, with $\alpha \in (0, 1)$, have identical distributions when observed over temporal or spatial intervals. This is somewhat counterintuitive because only in the latter case such an observation includes one special, incomplete jump. The result has a number of interesting implications, particularly for the behaviour of Brownian excursions. Later, they showed that the interval partitions generated in this way fit within a broader family of Poisson-Dirichlet processes, which have a wide range of mathematical applications.

We give an intuitive and constructive proof of the Pitman-Yor theorem based only on the elementary properties of Poisson processes. Moreover, employing the same approach, we present a new and simple proof for the stick-breaking distribution of length biased orderings of general Poisson-Dirichlet processes, recovering the classical results of McCloskey and Perman.

Limiting spectral laws for random circulant matrices

Adrian Beker
University of Zagreb

A well-known conjecture in random matrix theory asserts that the empirical spectral distributions (ESDs) of random d-regular digraphs converge to the so-called oriented Kesten–McKay law. In this talk, we will discuss a circulant analogue of this conjecture. Specifically, we will be concerned with circulant matrices defined with respect to arbitrary finite abelian groups and having a fixed number d of non-zero entries in each row/column. Equivalently, these can be viewed as adjacency matrices of random d-regular directed Cayley graphs. We will give exact criteria for convergence in expectation and in probability of ESDs in terms of the sequence of underlying groups. As a consequence, a log-asymptotic for the determinant will be derived.

Quantitative CLT for number of extreme points in Delaunay tessellations

Chinmoy Bhattacharjee
University of Hamburg

In this talk, we will consider three examples of Laguerre tessellations, namely, the β, β' and Gaussian Delaunay tessellations based on a Poisson point process in $\mathbb{R}^d \times \mathbb{R}$. We prove a quantitative central limit theorem with presumably optimal rates for the Gaussian convergence of number of extreme points (points that form a cell) in the tessellations with their spatial coordinates falling within a growing window. The proofs rely on the notion of region-stabilization which allows us to deal with some long-range interactions. The talk is based on an ongoing joint work with Anna Gusakova.

Ranges of Extremal Processes and Heavy-Tailed Random Walks in Spaces of Growing Dimension

Jin Bochen
University of Bern

We consider extremal processes and random walks generated by heavy-tailed random vectors. We assume that these vectors take values in $\mathbb{R}^d$, and establish limit theorems for their paths in the triangular array setting when both the number of steps n and the dimension d grow to infinity. For this, the paths are interpreted as finite metric spaces with the ℓ_p metric for $p \in [1, \infty]$ and the convergence is understood as convergence in distribution on the family of metric spaces equipped with the Gromov–Hausdorff distance.

The minimum spanning tree and Brownian path

Nicolas Broutin
Sorbonne Université

Consider a complete graph on $[n]$ vertices, whose edges are weighted with iid random variables uniform on $[0, 1]$. Then, there is a unique spanning tree which minimizes the sum of the weight of the edges. It is known that this tree, seen as a measured metric space, converges if we rescale suitably the distances. I will explain how one can construct explicitly the limit metric space using a Brownian motion. This is based on joint work with J.-F. Marckert.

Compound Poisson approximation under beta-mixing and stabilization

Nicolas Chenavier
Université du Littoral Côte d'Opale

We introduce a result on compound Poisson approximation for the so-called point process of exceedances. This process consists of the set of points from an underlying process at which an extreme-type event occurs. The convergence associated with this approximation is quantified in terms of the Wasserstein distance, allowing for the study of various extreme value problems in random geometry. Two applications are provided: the first one concerns the nearest-neighbor distance for a non-Poisson process, and the second one deals with small angles in a Delaunay triangulation. Joint work with M. Otto.

Void probability and likelihood approximation for Gibbs processes

Ottmar Cronie
Chalmers University of Technology & University of Gothenburg

When fitting a model to data, ideally one would like to make use of maximum likelihood estimation due to the associated statistical properties. However, in the case of general Gibbs point processes, likelihood functions, which are represented by (families of local) Janossy densities, are typically not tractable as an effect of their associated normalising constants/partition functions. This has led to the development of a range of alternative statistical approaches, e.g. Takacs-Fiksel estimation, or its special case pseudolikelihood estimation and its generalisation Point Process Learning.

In this talk we present an approach to performing approximate likelihood estimation for Gibbs processes. We do so by leveraging a simple but highly useful representation of Janossy densities for Gibbs processes, consisting of two terms: a term which involves the (Papangelou) conditional intensity of the Gibbs process and a void probability term which can be expanded in terms of the conditional intensity.

Since the conditional intensity is typically known and tractable, this representation becomes tractable in the sense all of its components are known. From a statistical point of view, however, we still need to address how we can practically handle the void probability expansion. Having discussed different ways of carrying out void probability approximation, and thereby how to approximate the likelihood function, we proceed by discussing how the new approximate likelihood approach plays out in specific settings and by comparing it to the state-of-the-art.

This is joint work with Mathis Rost (Chalmers University of Technology & University of Gothenburg).

Topological data analysis for random sets

Vesna Gotovac Đogaš

University of Split

This work paves the way for a methodology for detecting outliers and testing the goodness-of-fit of random sets using topological data analysis. Our approach is based on building a filtration using the sublevel sets of the signed distance function and analyzing various summary functions derived from the persistence diagrams of the resulting persistent homology. Outlier detection is performed using functional depth measures applied to these summary functions. To evaluate goodness-of-fit, we use global envelope tests with summary statistics serving as test statistics. The methodology is supported by a simulation study based on random set germ-grain models and is also applied to real-world data from histological images of mastopathic and breast cancer tissue.

Evergreen and long-lasting leaves in preferential attachment trees

Andrii Ilienko

University of Bern

We study preferential attachment trees – that is, trees in which each new vertex attaches to an existing one with probability proportional to a given function of the target's out-degree. We give sufficient conditions for the existence and non-existence of evergreen leaves, i.e., leaves that never get any children.

For affine preferential attachment, evergreen leaves do not occur. In this case we consider long-lasting leaves, whose lifetime – the waiting time until the first attachment – is large. For the first $[nt]$ vertices, we prove a joint functional limit theorem for the index of the longest-lasting leaf and its lifetime; the limit can be called a two-dimensional beta-decorated Fréchet extremal process. For uniform trees, we go further and, using Stein's method and size-bias coupling, show asymptotic normality of the number of long-lasting leaves, regardless of how "long-lasting" is defined.

The talk is based on joint work with Oleksii Galganov (Kyiv, Ukraine).

On the tails of high- and infinite- dimensional Markov chains

Daniela Ivanković
University of Zagreb

The study of asymptotic tail behavior of the stationary distributions of Markov chains has a rich history, including the Lindley recursion in the context of GI/G/1 queues (Iglehart, 1972), stochastic recurrence equations (see Kesten, 1973; Buraczewski, Damek and Mikosch, 2016), and various extensions (Goldie, 1991). In recent years, driven in part by applications in machine learning (see, for example, Hodgkinson and Mahoney, 2021), there has been a growing interest in high- or even infinite-dimensional models. In joint work with prof. Bojan Basrak, we investigate the previously unexplored tail behavior of stationary distributions of infinite-dimensional Markov chains, with particular attention to models arising from point processes. Additionally, we present a tractable framework for deriving tail asymptotics across a broad class of models, including classical finite-dimensional cases, using standard probabilistic tools such as change of measure techniques and the renewal theorem.

Finding trees in a jungle

Nina Kamčev
University of Zagreb

When is it possible to detect a hidden random tree inside a random graph $G(n,p)$? More specifically, given a tree T, we consider a random graph $G_1(T)$ which is a union of a k-vertex tree T, placed randomly into $\{1,\dots,n\}$, and a random graph $G(n,p)$.

Firstly, let T be a 'typical' uniformly random labelled tree. We show that when $p < 1.1/n$ and $k = \Omega(\log n)$, G_1 can be distinguished from $G(n,p)$. This is surprising since at this density, $G(n,p)$ contains random trees of size $\Omega(n^{1/3})$. On the other hand, when $p = C/n$ for a large constant C and $k = o(\sqrt{n})$, G_1 cannot be distinguished. Both bounds on k are optimal up to a constant factor.

We prove similar results for the case when T is an 'unknown' uniformly random tree (i.e., not exposed to the 'detector' prior to sampling G_1), and when T is a d-ary tree.

This line of research presents numerous open problems, including path detection in the critical $G(n,1/n)$, detecting a tree hidden by an adversary or algorithmic detection.

Joint work with Nicolas Broutin, Gábor Lugosi, Bruce Reed and Liana Yepremyan.

Rigidity of random measures

Raphaël Lachièze-Rey
INRIA Paris

Rigidity is an intriguing phenomenon that has been noticed for stationary point processes by Holroyd, Ghosh and Peres in a few seminal papers. It occurs when the (random) number of points in a compact can be exactly and a.s. inferred from the outside configuration, it has been found in many models: determinantal, Coulomb gases, zeros of Gaussian Analytic functions. Sometimes, additional features can be inferred, like the barycenter of the points, i.e. the first moment, or the kth order moments, called k-rigidity. We give a spectral characterisation of k-rigidity, and investigate furthermore the phenomenon of maximal rigidity for a random measure. In this instance of the rigidity phenomenon, the full restriction of the random measure on a subset can be exactly interpolated from the outside. We will show it applies to several processes of interest in physics and mathematics such as stealthy processes, quasicrystals and, surprisingly, some short range Gaussian fields.

Stochastic-block model path encoding and consequences

Vlada Limić
University of Strasbourg

Stochastic-block model (SBM) with two or more blocks is an example of a random graph fundamentally different from the Erdõs-Rényi (E-R) graph. The SBM population is divided into blocks, and the interaction of individuals is heterogenous (block-structure dependent). Since 1997 the connected component sizes of E-R graph have been encoded a number of times via excursions of random walks or alike processes. Size-biased order of connected components is a frequent byproduct of such encodings. The first (and so far the only) "multi-path" encoding of the connected component sizes of SBM was built in a recent joint work with David Clancy and Vitalii Konarovskyi. Size-biasing of components is preserved. The aim of the first half of the talk is to introduce the SBM framework and the random fields featuring in the above encoding. The second half of the talk will be devoted to the notion of a “good function", which is particularly convenient in the scaling limit analysis of SBM (work in progress with David and Vitalii). Good functions may also be interesting to probabilists working in other areas.

Asymptotic fluctuations of smooth linear statistics of independently perturbed lattices

Gabriel Mastrilli
ENSAI

Perturbed lattices have been extensively studied, as they form a central and flexible class capable of replicating complex behaviors of point processes exhibiting stronger spatial correlations than the Poisson process, known as hyperuniform processes. In this talk, we present results on the fluctuations of smooth linear statistics of i.i.d. perturbed lattices. More precisely, we highlight three distinct classes of limiting behavior, depending on the dimension and on the tails of the perturbations. For dimensions larger than two, Gaussian central limit theorems hold under mild assumptions. In contrast, dimension one exhibits a transition from Gaussian to non-Gaussian stable limits, with an intermediate regime showing non-Gaussian non-stable laws.

Nagaev-type large deviations for heavy-tailed time series

Thomas Mikosch
University of Copenhagen

In 1979, Sergey Nagaev published a paper (in AoP) on large deviations for sums of iid random variables with regularly varying tails. The results of the paper describe the very narrow large deviation zone of normal approximation and the very wide large deviation zone due to a single very big value in the sample. We will talk about related results for time series with regularly tails and apply them to central limit, renewal and ruin theory.

Transversals in quasirandom latin squares

Rudi Mrazović
University of Zagreb

A transversal in a $n \times n$ latin square is a set of entries not repeating any row, column, or symbol. A famous conjecture of Brualdi, Ryser, and Stein predicts that every latin square has at least one transversal provided n is odd. We will discuss an approach motivated by the circle method from the analytic number theory which enables us to count transversals in latin squares which are quasirandom in an appropriate sense.

Homogeneous substructures in random ordered hyper-matchings

Andrzej Ruciński

Adam Mickiewicz University

An ordered r-uniform matching of size n is a collection of n pairwise disjoint r-subsets of a linearly ordered set of rn vertices. For $n = 2$, such a matching is called an *r-pattern*, as it represents one of $\frac{1}{2}\binom{2r}{r}$ ways two disjoint edges may intertwine. Given a set $\mathcal{P}$ of r-patterns, a *$\mathcal{P}$-clique* is a matching with all pairs of edges order-isomorphic to a member of $\mathcal{P}$.

We are interested in the size of a largest $\mathcal{P}$-clique in a *random* ordered r-uniform matching selected uniformly from all such matchings on a fixed vertex set $\{1, 2, ..., rn\}$. We determine this size (up to multiplicative constants) for several sets $\mathcal{P}$, including all sets of size $|\mathcal{P}| \leqslant 2$, the set $\mathcal{R}^{(r)}$ of all r-partite patterns, as well as sets $\mathcal{P}$ enjoying a Boolean-like, symmetric structure.

This is joint work with A. Dudek, J. Grytczuk, J. Przybyło.

Non-Gaussian limits for diameter and perimeter of convex hulls of multiple random walks

Stjepan Šebek

University of Zagreb

We prove large-time L^2 and distributional limit theorems for perimeter and diameter of the convex hull of N trajectories of planar random walks whose increments have finite second moments. Earlier work considered $N \in \{1, 2\}$ and showed that, for generic configurations of the mean drifts of the walks, limits are Gaussian. For perimeter, we complete the picture for $N = 2$ by showing that the exceptional cases are all non-Gaussian, with limits involving an Itô integral (two walks with the same non-zero drift) or a geometric functional of Brownian motion (one walk with zero drift and one with non-zero drift), and establish Gaussian limits for generic configurations when $N \geqslant 3$. For the diameter we obtain a complete picture for $N \geqslant 2$, with limits (Gaussian or non-Gaussian) described explicitly in terms of the drift configuration.

On the optimal prediction of extreme events

Stilian Stoev
University of Michigan

The prediction of the extreme values of Y in terms of a vector of covariates X is a fundamental problem. Doing so optimally is particularly important in a regime of extremes where the events $\{Y > y_0\}$ are rare and the data are scarce. We start with a Neyman-Pearson type perspective to optimality, characterize the optimal predictors in terms of density ratios and discuss several examples. This leads us to the natural notion of *optimal extremal predictors* $h(X)$ as ones that maximize the tail-dependence coefficient

$$\lambda(Y, h(X)) := \lim_{p \to 1} \mathbf{P}[Y > F_Y^{\leftarrow}(p) \mid h(X) > F_{h(X)}^{\leftarrow}(p)],$$

over a class of functions $\mathcal{H} \ni h$.

Next, we focus on the special case where (Y, X) are (jointly) multivariate regularly varying and characterize the optimal predictors in the class of non-negative continuous homogeneous functions $h : \mathbf{R}^d \to \mathbf{R}_+$. This amounts to solving a calculus of variations problem for an integral functional of the spectral (i.e., angular) measure, which we solve in the general, spectrally continuous case. In the special, spectrally discrete case, we encounter an interesting *perfect asymptotic precision* phenomenon, where *for some models* $\lambda^{(\mathrm{opt})}(Y, X) := \lambda(Y, h^{(\mathrm{opt})}(X)) = 1$. Notably, in many linear models that share the same sets of heavy-tailed factors, somewhat contrary to intuition, the extreme events can be predicted with an asymptotically perfect precision.

The two characterizations lead to two types of statistical estimators of the optimal predictors, which we briefly illustrate with simulations and apply to the prediction of extreme solar flares.

Mass-Stationarity and Shift-Coupling

Hermann Thorisson
University of Iceland

Mass-stationarity is a formalization of the intuitive idea that the origin of a random measure ξ° is at a **typical location in its mass**. It extends the concept of point-stationarity of a simple point-process, which means that its origin is at a **typical point**. These concepts characterize the Palm version of a stationary random measure ξ.

Actually, there are **two** types of Palm versions. For the less known type, the stationary ξ and the mass-stationary ξ° can be **shift-coupled**, that is, ξ and ξ° can be represented as a single random measure up to a random shift of the origin.

The isoperimetric problem for convex hulls and large deviations rate functionals of random walks

Vladislav Vysotskiy
University of Sussex

We study the asymptotic shape of the most likely trajectories of a planar random walk that result in large deviations of the area of the convex hull of the first n steps of the walk, as $n \to \infty$. If the increments of the walk have finite Laplace transform, such a scaled limit trajectory h solves the inhomogeneous anisotropic isoperimetric problem for the convex hull, where the usual length of h is replaced by the large deviations rate functional $\int_0^1 I(h'(t))dt$ with I being the rate function of the increments. Assuming that the distribution of increments is not contained in a half-plane, we show that the optimal trajectories are smooth, convex, and satisfy the Euler–Lagrange equation, which we solve explicitly for every I. Our solution resembles that of the isoperimetric problem in the Minkowski plane found by Busemann (1947).

Invariant transports of random measures and hyperuniformity

D. Yogeshwaran
Indian Statistical Institute, Bangalore

The talk will focus on asymptotic variances of invariant Euclidean random measures under transports. We will show natural mixing criteria for transport that preserves asymptotic variance under the transport map. We pay special attention to the case of a vanishing asymptotic variance, known as hyperuniformity. By constructing suitable transports from a source point process, we can establish or refute hyperuniformity for many point processes and random measures. In particular, we will see the example of hyperuniformerer that transforms any ergodic point process with finite intensity into a hyperuniform process by randomizing each point within its cell of a fair partition.

Normal approximation in stochastic geometry

Joseph Yukich
Lehigh University

In recent years there has been considerable progress in establishing qualitative and quantitative central limit theorems for functionals of random spatial models. This includes functionals of random spatial graphs, statistics of particle systems on random point sets, as well as classical statistics of Boolean models. The first part of the talk reviews some of the notable central limit theorems in the subject. The second part introduces new ways to prove asymptotic normality; in this way we significantly broaden the scope of applications and, in the case of functionals of Poisson input, we may often sharpen the existing quantitative bounds to the normal.
