New Perspectives in the Theory of Extreme Values
Book of abstracts

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Contents

Regular variation and single big-jump heuristic of Poisson cluster processes

Fabien Baeriswyl

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In this work, we partially extend results due to Dombry, Tillier and Wintenberger (2022) in considering regular variation of three examples of marked Poisson cluster processes with dependent components, and we prove a single big-jump principle when considering convergence on the space of point measures consisting of only one point. We hint at ongoing extensions for hidden regular variation of such processes. We finally use the former results to derive large deviations results of the paths of functionals of the abovementioned processes. This is an ongoing joint work with Valérie Chavez-Demoulin and Olivier Wintenberger.

Limit theory for asymptotically de-correlated spatial random models

Bartlomiej Blaszczyszyn

Inria/ENS Paris

This is an ongoing collaboration with J. Yukich and Yogeshwaran D. The innovation compared to [BJY2019] lies in the introduction of a new mixing property for general marked point processes, as well as the ability to stabilize functionals on them using a double-bounded Lipschitz distance. This advance facilitates the examination of the central limit theorem for the statistics of various specific random geometric models, particularly those involving Gibbs spin systems and interacting diffusions on general spatial random graphs.

Maximal cycles in random geometric complexes

Omer Bobrowski

Technion

Random geometric complexes are simplicial complexes (high-dimensional graphs) whose vertices are generated by a random point process in a metric space. In this talk we will review recent advances in the study of the homology (cycles/holes in various dimensions) of such complexes. We will focus on the size of the largest cycles that can be formed by these processes. In particular, we will argue that the maximal cycles have a universal limit, independent of the original point process distribution.

Maximal degree in random graphs

Gilles Bonnet

University of Groningen

In this talk we will be interested in describing the distribution of the maximal degree of vertices in a large random graph. More specifically, we will be interested in the Poisson-Delaunay and the Poisson-Beta(-prime)-Delaunay graphs. The constructions of these graphs are based on an underlying Poisson point process in the Euclidean space and following specific geometric rules to construct the set of edges. We want to describe the distribution of the maximal degree of all the vertices within a large observation window. We will first present results obtained with Nicolas Chenavier (Bernoulli 2020) showing a concentration on finitely many values in the Poisson-Delaunay model, as the window's size goes to infinity. Then, we will briefly discuss the ongoing work with my PhD student (Joseph Gordon) in the case of the Poisson-Beta-Delaunay and Poisson-Beta-prime-Delaunay models. Joseph will continue this discussion in his talk. Joint work with Nicolas Chenavier and Joseph Gordon.

Facet volume fluctuations of random polytopes

Pierre Calka

Université de Rouen Normandie

We consider the random polytope generated by n independent and uniformly distributed points in a smooth convex body. We show that several functionals of the so-called typical facet of the polytope exhibit limiting distributions. We prove similar explicit convergences of measures associated to the extreme value regime. This leads us in particular to getting limiting extreme value distributions and Poisson approximation in the case of the facet volume. Additionally, we derive a limit distribution for the position of the maximum, as well as a limit shape for the facet which maximizes the volume. This talk is based on joint work with Joe Yukich.

Asymptotic analysis of PCA for extremes

Holger Drees
Universität Hamburg

Consider n iid multivariate regularly varying random vectors. Drees and Sabourin (2021) examined the PCA projection of the angular parts of the k vectors with largest norm. In particular, they derived uniform bounds for the risk of the PCA approximation, that is the expected squared norm of the difference between the angular part and its PCA approximation.

This uniform bound implies a bound on the so-called excess risk of the order $k^{-1/2}$. However, using different techniques it can be shown that the excess risk is of the order k^{-1} if in the limit there is a unique optimal projection. Moreover, in the general case the excess risk is of smaller order than $(k/\log k)^{-1}$. In addition, we determine the limit distribution of the PCA projection and the resulting excess risk in the case that the optimal projection is unique.

Extreme values in the presence of non-stationarity

Lenka Glavaš
University of Belgrade

We discuss two perspectives on this topic. First, we are interested in the situation where a complete sample comes from a strictly stationary random sequence, while an incomplete sample consisting only of the observed members of the original sample corresponds to a random sequence that is non-stationary in the general case. Our goal is to study the joint limiting behavior of a sequence of random vectors of maxima in complete and incomplete samples. We are also able to compare corresponding results for the cases where the underlying time series whose samples we consider are linear processes driven by the noise with different level of tail heaviness. Second, we start with the non-stationary random sequence, in particular, we define a new example of an autoregressive process with periodic dependence structure and derive the extremal features for this time series model.

Properties of typical β- and β'-Voronoi cells

Joseph Gordon
University of Groningen

The β- and β'-Voronoi tessellations are models generalizing classical Poisson-Voronoi tessellation on the Euclidean space introduced recently by Gusakova, Kabluchko, and Thäle in a series of papers. We delve into geometric properties of these models, with consequences for distributions of typical cells. Notably, we derive bounds for tail distributions of a number of facets, and for a fixed number of facets show independence of "volume" and "shape".

Comparison of random sets distributions via statistical depths

Vesna Gotovac Đogaš
University of Split

We present several depths for possibly non-convex random sets. The depths are applied to the comparison between two samples of non-convex random sets, using a visual method of DD-plots and statistical tests. The advantage of this approach is to identify sets within the sample that are responsible for rejecting the null hypothesis of equality of the distribution and to provide clues on differences between the distributions. The method is justified on the basis of a simulation study.

Testing significant dependencies in high dimensions via maxima of U-statistics

Johannes Heiny
Stockholm University

We consider the problem of statistical testing for high-dimensional data. Independence tests, for example, often rely on limit theorems for empirical versions of certain dependence coefficients such as sample covariances, sample correlations, Spearman's rho or Kendall's tau. The latter four dependence coefficients are zero in case of independence. In this talk, we take a different point of view and work under the null hypothesis that the dependence coefficients do not exceed a small fixed threshold. We propose a max-type test that is asymptotically consistent and analyze its behavior under local alternatives. The talk is based on joint work with Patrick Bastian and Holger Dette.

Extreme and central spacings: point processes and local Poisson approximation

Andrii Ilienko
University of Bern

In the simplest setting, we consider the point process generated by uniform spacings, that is, differences between neighboring order statistics constructed by n independent uniform random variables on $[0, 1]$. We show that there is a continuum spectrum of different centerings/normalizations under which the distributional vague convergence to different Poisson point processes holds. Each of these centerings/normalizations allows us to track the asymptotic behavior of the specific part of spacings. The talk is based on the joint project with Yurii Yarosh (Igor Sikorsky Kyiv Polytechnic Institute, Ukraine).

Metric embeddings of tail correlation matrices

Anja Janßen
Universität Magdeburg

The extremal dependence between components of a random vector is im portant for the assessment of risks determined by several factors, for example in the evaluation of risk measures of financial or insurance portfolios. One popular bivariate measure of tail risk is the matrix of tail correlation coefficients. The entries of this ma trix are closely related to a useful distance measure on the space of Fréchet(1)-random variables, named spectral tail distance. We analyze the properties of the related metric and show that it is L^1- and l_1-embeddable, which allows us to better understand its structure. In particular, a given embedding of the spectral tail distance can be related to a certain max-stable random vector realizing a given tail correlation matrix. That way, our approach allows us to conclude about the algorithmic complexity of the decision problem whether a given matrix is a tail correlation matrix or not. Joint work with Sebastian Neblung, Stilian Stoev.

Almost sure properties of Lévy driven supOU processes

Peter Kevei
University of Szeged

Superpositions of Ornstein-Uhlenbeck processes allow a flexible dependence structure, including long range dependence for OU-type processes. These processes were introduced by Barndorff-Nielsen in 2000 to model turbulence. Their complex asymptotic is governed by three effects: the behavior of the Lévy measure both at infinity and at zero, and the behavior at zero of the measure governing the dependence. In the present talk we analyze the a.s. behavior of supOU and integrated supOU process. Our main result is a Marcinkiewicz-Zygmund type SLLN for the integrated process. The talk is based on joint work with Danijel Grahovac.

Central Limit Theorem for the perimeter of convex hulls spanned by two independent random walks in a two-dimensional plane

Tomislav Kralj
University of Zagreb

In this research, we focus on the Euclidean plane $\mathbb{R}^2$ and explore the perimeter of convex hulls generated by two independent random walks, with linearly independent drift vector. We denote this perimeter up to time n as L_n. Our primary finding is the convergence in distribution of the normalized and centered variable $1/\sqrt{n}\,(L_n - \mathbb{E}[L_n])$ to a Gaussian distribution $N(0, \sigma^2)$ as $n \to \infty$. The limiting variance $\sigma^2 > 0$ is explicitly computable.

Limit theorems for unbounded cluster functionals

Rafal Kulik

University of Ottawa

A blocks method is used to define clusters of extreme values in stationary time series. The cluster starts at the first large value in the block and ends at the last one. The block cluster measure (the point measure at clusters) encodes different aspects of extremal properties. Its limiting behaviour is handled by vague convergence, hence the set of test functions consists of bounded, shift-invariant functionals that vanish around zero. If unbounded or non shift-invariant functionals are considered, we may obtain convergence at a different rate, depending on the type of the functional and the block size (small vs. large blocks). There are two prominent examples of such functionals: the locations of large jumps and the cluster length. We obtain a comprehensive characterization of the limiting behaviour of the block cluster measure evaluated at such functionals for stationary, regularly varying time series. Once the convergence of the block cluster measure is established, we can proceed with consistency of the empirical cluster measure. Consistency holds in the small and moderate blocks scenario, while fails in the large blocks situation. Next, we continue with weak convergence of the empirical cluster processes. The starting point is the seminal paper by Drees and Rootzen (2010). Under the appropriate uniform integrability condition (related to small blocks) the results in the latter paper are still valid. In the moderate and large blocks scenario, the Drees and Rootzen empirical cluster process diverges, but converges weakly when re-normalized properly. We will also provide some extensions to continuous time processes and extremal problems in stochastic geometry.

Isoperimetry and Extremal Processes

Ilya Molchanov

University of Bern

The talk deals with asymptotic results for distributions of homogeneous functionals of min-stable or max-stable processes and for their conditional distributions given a homogeneous functional. This is a joint work with Daniel Hug and Kirstin Strokorb.

Cumulant method for weighted random connection models

Moritz Otto

Aarhus University

Recently, the class of weighted random connection models (WRCM) has appeared as a promising approach for accurately describing phenomena in large complex networks. Instead of a fixed connection threshold, the WRCM allows to quantify the connectivity via a profile function and a weight distribution. However, due to the complexity of these models, the analysis of their asymptotic behavior is still in its infancy. In this talk, I will address the question of asymptotic normality of suitable functionals such as subgraph counts and power-weighted edge lengths on the WRCM. One of the main tools in the analysis of asymptotic normality is the method of cumulants. As underlying random point patterns, we will discuss Poisson and determinantal processes. These classes of point processes are particularly well-suited for cumulant-based methods since their cumulant density can be expressed in closed form in terms of the kernel function. The talk is based on an ongoing joint project with Nils Heerten (Bochum) and Christian Hirsch (Aarhus).

Limit theorems for high-dimensional Betti numbers in the multiparameter random simplicial complexes

Takashi Owada

Purdue University

We consider the multiparameter random simplicial complex on a vertex set $\{1, \ldots, n\}$, which is parameterized by multiple connectivity probabilities. Our key results concern the topology of this complex of dimensions higher than the critical dimension. We show that the higher-dimensional Betti numbers satisfy strong laws of large numbers and central limit theorems. Moreover, lower tail large deviations for these Betti numbers are also discussed. Some of our results indicate an occurrence of phase transitions in terms of the scaling constants of the central limit theorem, and the exponentially decaying rate of convergence of lower tail large deviation probabilities. This is joint work with Gennady Samorodnitsky.

2RV+HRV and Testing for Strong VS Full Dependence

Sidney Resnick
Cornell University

Preferential attachment models of network growth are bivariate heavy tailed models for in- and out-degree with limit measures which either concentrate on a ray of positive slope from the origin or on all of the positive quadrant depending on whether the model includes reciprocity or not. Concentration on the ray is called full dependence. If there were a reliable way to distinguish full dependence from not-full, we would have guidance about which model to choose. This motivates inventigating tests that distinguish between (i) full dependence; (ii) strong dependence (limit measure concentrates on a proper subcone of the positive quadrant; (iii) concentration on positive quadrant. We give two test statistics and discuss their asymptotically normal behavior under full and not-full dependence.

Clustering in extremes and in large deviations

Gennady Samorodnitsky
Cornell University

We describe the cluster of large deviations events that arise when one such large deviations event occurs. We work in the framework of an infinite moving average process with a noise that has finite exponential moments. The combined effect of conditioning and stationarity makes, somewhat unexpectedly, the limiting cluster of large deviations share certain properties with the tail process arising in extreme value theory.

Simulating Random Samples from X-Vine Multivariate Pareto Distributions

Johan Segers
Université catholique de Louvain

Regular vines provide a way to organize the components of a random vector along a sequence of interlocked trees. The first tree corresponds to a Markov random field whereas the next trees capture higher-order effects. For multivariate extremes, we show how arbitrary bivariate exponent measure densities and bivariate copula densities can be combined along a regular vine into a multivariate exponent measure density which we call an X-vine. The multivariate Pareto distribution associated to this exponent measure can be used to model multivariate excesses over high thresholds. Exploiting recursive properties of regular vines, we construct an algorithm to simulate random samples from such multivariate Pareto distributions. Joint work with Anna Kiriliouk, Jeongjin Lee.

Graphical models for infinite measures with applications to extremes and Lévy processes

Kirstin Strokorb

Cardiff University

Conditional independence and graphical models are well studied for probability distributions on product spaces. We propose a new notion of conditional independence for any measure Λ on the punctured Euclidean space $\mathbb{R}^d \setminus \{0\}$ that explodes at the origin. The importance of such measures stems from their connection to infinitely divisible and max-infinitely divisible distributions, where they appear as Lévy measures and exponent measures, respectively. We characterize independence and conditional independence for Λ in various ways through kernels and factorization of a modified density, including a Hammersley-Clifford type theorem for undirected graphical models. As opposed to the classical conditional independence, our notion is intimately connected to the support of the measure Λ. Our general theory unifies and extends recent approaches to graphical modeling in the fields of extreme value analysis and Lévy processes. Our results for the corresponding undirected and directed graphical models lay the foundation for new statistical methodology in these areas.

Intersection of randomly translated elements of the extended convex ring

Tommaso Visona

University of Bern

Let $\Xi_n = \{\xi_1, \dots, \xi_n\}$ be a sample of n independent points distributed on a regular closed element K of the extended convex ring in $\mathbb{R}^d$ according to a probability measure $\mathbf{P}$ on K, admitting a density function. We consider random sets generated from the intersection of the translations of K by elements of Ξ_n, as

$$X_n = \bigcap_{i=1}^{n} (K - \xi_i)\,.$$

The aim of this work is to obtain limit theorems for properly scaled X_n as $n \to \infty$. These limit results rely on the behaviour of the density of $\mathbf{P}$ near the boundary of K. The properly scaled sequence of random sets converges in distribution to the zero cell of a tessellation of hyperplanes generated by a Poisson Point Process.

Radial Fluctuations of Random Polytopes

Joseph Yukich
Lehigh University

Consider the random polytope K_n obtained as the convex hull of n i.i.d. points, uniformly distributed inside a d-dimensional smooth convex body K. We show that the maximal radial fluctuations of the convex hull, as given by the Hausdorff distance between K_n and K, converges to a Gumbel distribution as n tends to infinity. The approach depends upon showing that the Hausdorff distance may be approximated by a supremum of 'facet distances' satisfying a spatial mixing condition analogous to classical mixing conditions. This leads to an explicit formula for the extremal index of facet distances. The talk is based on joint work with Pierre Calka.