The joint limit distribution of partial maxima and partial sums of a heavy-tailed time series

Thomas Mikosch [1] jointly with Olivier Wintenberger [2]

[1]University of Copenhagen [2]Sorbonne Université

The limits of the ratio of partial maxima and partial sums of an iid real-valued sequence have been studied since the 1959s. The limit theory is well understand since the 1980s; see for example the monograph by Bingham, Goldie, Teugels (1987). This theory is most interesting if the marginal distribution has infinite variance. Then, roughly speaking, the maximum of a given sample and the sum have the same magnitude.

We will consider the infinite variance case and assume stationarity of the underlying sequence. We will exploit regular variation calculus for stationary sequences. It is useful for describing clusters of extremes. These represent the main difference to the iid case. We provide results about the joint convergence of maxima and sums and, in turn, derive limit theory for their ratios. We also touch on related problems on self-normalization of partial sums such as present in sample autovoriances or studentized sums.

Measuring dependence between random vectors via optimal transport

Johan Segers[1] joint work with Gilles Mordant[2]

[1] UCLouvain [2] Georg-August-Universität Göttingen

To quantify the dependence between two random vectors of possibly different dimensions, we propose to rely on the properties of the 2-Wasserstein distance. We first propose two coefficients that are based on the Wasserstein distance between the actual distribution and a reference distribution with independent components. The coefficients are normalized to take values between 0 and 1, where 1 represents the maximal amount of dependence possible given the two multivariate margins. We then make a quasi-Gaussian assumption that yields two additional coefficients rooted in the same ideas as the first two. These different coefficients are more amenable for distributional results and admit attractive formulas in terms of the joint covariance or correlation matrix. Furthermore, maximal dependence is proved to occur at the covariance matrix with minimal von Neumann entropy given the covariance matrices of the two multivariate margins. This result also helps us revisit the RV coefficient by proposing a sharper normalisation. The two coefficients based on the quasi-Gaussian approach can be estimated easily via the empirical covariance matrix. The estimators are asymptotically normal and their asymptotic variances are explicit functions of the covariance matrix, which can thus be estimated consistently too. The results extend to the Gaussian copula case, in which case the estimators are rank-based. The results are illustrated through theoretical examples, Monte Carlo simulations, and a case study involving electroencephalography data.

Branching random walk with infinite progeny mean: a tale of two tails

Philippe Soulier

Université Paris Nanterre

We study the extremes of branching random walks under the assumption that the underlying Galton-Watson tree has infinite progeny mean. It is assumed that the displacements are either regularly varying or they have lighter tails. In the regularly varying case, it is shown that the point process sequence of normalized extremes converges to a Poisson random measure. In the lighter-tailed case, we study the asymptotics of the scaled position of the rightmost particle in the n-th generation and show the existence of a non-trivial constant.

Joint work with Souvik Ray, Rajat Subhra Hazra, and Parthanil Roy.

Extreme positions of regularly varying branching random walk in random environment

Zbigniew Palmowski

Wroclaw University of Science and Technology

In this talk, we consider a Branching Random Walk (BRW) on the real line where the underlying genealogical structure is given through a supercritical branching process in i.i.d. environment and satisfies Kesten-Stigum condition. The displacements coming from the same parent are assumed to have jointly regularly varying tails. Conditioned on the survival of the underlying genealogical tree, we prove that the appropriately normalized (depends on the expected size of the n-th generation given the environment) maximum among positions at the n-th generation converges weakly to a scale-mixture of Frechét random variable. Furthermore, we derive the weak limit of the extremal processes composed of appropriately scaled positions at the n-th generation and show that the limit point process is a member of the randomly scaled scale-decorated Poisson point processes (SScDPPP). Hence, an analog of the predictions by Brunet and Derrida (2011) holds

Limit theory for integrated supOU processes

Danijel Grahovac

University of Osijek, Croatia

Superpositions of Ornstein–Uhlenbeck type (supOU) processes provide a rich class of stationary stochastic processes for which the marginal distribution and the dependence structure may be modeled independently. We investigate the limiting behavior of integrated supOU processes. The limit theorems will be presented both for the fnite and infnite variance case. We next establish the asymptotic behavior of moments by measuring the rate of growth of moments in time. The integrated supOU process may exhibit a phenomenon known as intermittency.

Extremes of functions of high-dimensional random vectors with applications in statistics

Johannes Heiny

Ruhr-Universität Bochum

High-dimensional data play an important role in many fields of modern sciences. In this talk, we study extremes of functions of high-dimensional random vectors with a particular focus on applications in statistics and geometry. We derive the asymptotic distribution of the largest interpoint distance of high-dimensional random points and deduce a testing procedure for the change-point in their means.

PageRank asymptotics on directed preferential attachment graphs

Mariana Olvera-Cravioto

The University of North Carolina at Chapel Hill

We study a family of evolving directed random graphs that includes the directed preferential model and the directed uniform attachment model. The directed preferential model is of particular interest since it is known to produce scale-free graphs with regularly varying in-degree distribution. We start by describing the local weak limits for our family of random graphs in terms of randomly stopped continuous-time branching processes, and then use these limits to establish the asymptotic behavior of the corresponding PageRank distribution. We show that the limiting PageRank distribution decays as a power-law in both models, which is surprising for the uniform attachment model where the in-degree distribution has exponential tails. And even for the preferential attachment model, where the power-law hypothesis suggests that PageRank should follow a power-law, our result shows that the two tail indexes are different, with the PageRank distribution having a heavier tail than the in-degree distribution.

Palm theory for extremes of time series

Hrvoje Planinić

Department of Mathematics, University of Zagreb

We consider high-level exceedances of stationary time series whose finite-dimensional distributions are regularly varying and whose exceedances occur in clusters. For such a time series, the so-called tail process represents the local distribution of the time series as seen from its typical exceedance over a high threshold. Using the language of the standard Palm theory, we will explain how the tail process can be used to fully describe the asymptotic distribution of the extremal clusters.

Fluctuations of Random Convex Hulls, part I[1] and part II[2]

Joseph Yukich[1] Pierre Calka[2]

[1] Lehigh University [2] Université de Rouen Normandie

Let $X_1, ..., X_n$ be i.i.d. random variables uniformly distributed in the d-dimensional convex body K, assumed to have C^2 boundary. Denote by K_n the convex hull of $X_1, ..., X_n$. The re-scaled *maximal* radial fluctuations of ∂K_n as well as the *maximal* facet areas, are shown to asymptotically follow a Gumbel extreme value distribution as $n \to \infty$. The maximal radial and longitudinal fluctuations include precise logarithmic factors having scaling exponents $1/(d+1)$ and $2/(d+1)$, respectively. The maximal fluctuation results given here resemble those of interfaces in general dynamic and equilibrium systems, including those arising in subcritical random cluster and bond percolation models, where in $d = 2$, the logarithmic correction factors are known to coincide with those found here. Our method is based in particular on the definition of the distribution of a *typical* facet of K_n and the convergence of large deviations at critical thresholds for re-scaled *typical* radial fluctuations of the boundary ∂K_n and *typical* facet area as $n \to \infty$. This is a two-part talk, with the first part concentrating on radial fluctuations and with the second part focussing more on longitudinal and facet area fluctuations.

Extremal lifetimes of persistent cycles

Nicolas Chenavier[1] joint work with Christian Hirsch[2]

[1] Université du Littoral Côte d'Opale [2] University of Groningen

Persistent homology captures the appearances and disappearances of topological features such as loops and cavities when growing disks centered at a Poisson point process. We study extreme values for the lifetimes of features dying in bounded components and with birth resp. death time bounded away from the threshold for continuum percolation and the coexistence region. First, we describe the scaling of the minimal lifetimes for general feature dimensions, and of the maximal lifetimes for cavities in the Čech filtration. Then, we proceed to a more refined analysis and establish Poisson approximation for large lifetimes of cavities and for small lifetimes of loops. Finally, we also study the scaling of minimal lifetimes in the Vietoris-Rips setting and point to a surprising difference to the Čech filtration.

Lah distributions and convex hulls of high-dimensional random walks

Alexander Marynych

Taras Shevchenko National University of Kyiv

Motivated by a problem arising in the analysis of convex hulls of high-dimensional random walks, we introduce a new class of discrete probability distributions, which we call Lah distributions, and which involve Stirling numbers of both kinds. We provide a combinatorial interpretation of the Lah distributions in terms of random compositions and records and prove various limit theorems for them. This talk is based on a recent joint work with Zakhar Kabluchko (Münster, Germany).

Couplings and Poisson approximation of determinantal point processes

Moritz Otto

Otto-von-Guericke-Universität Magdeburg

We consider functionals of determinantal point processes with a fast decay of correlations. Under a stabilization and a monotonicity assumption, we show a new Poisson process approximation result for the functional. Our proof is based on Stein's method and uses a coupling of the original process and its Palm version. We present an application of our findings in the theory of random geometric graphs.

On extremes of random clusters and marked renewal cluster processes

Nikolina Milinčević [1]
joint work with Bojan Basrak[1] and Petra Žugec [2]

[1]Department of Mathematics, University of Zagreb
[2] Faculty of Organization and Informatics, University of Zagreb

We describe the limiting distribution of the extremes of observations that arrive in clusters. We study the tail behaviour of $H = \max\{X_j : j = 0, \dots, K\}$, where $(X_j)_{j\geq 0}$ denotes an i.i.d. sequence of observations belonging to the maximum domain of attraction of some extreme value distribution, and where K denotes an integer-valued random variable, possibly dependent on the observations X_j. We show that whenever K has a finite mean and represents a stopping time with respect to a suitable filtration, then H belongs to the same maximum domain of attraction as X_j's. Using this result, we determine the limiting distribution of $\max\{X_j : j = 0, \dots, K(t)\}$, where $K(t)$ is the number of observations (X_j) arriving up to the time t according to a general marked renewal cluster process.

Regular Variation on Quotient Spaces

Ilya Molchanov

University of Bern

If a Polish space is equipped with an equivalence relationship, then the corresponding quotient space is not necessarily metrisable. Still, under some assumptions on the equivalence, it is Hausdorff and so becomes a Souslin space. We develop the theory of regular variation in Souslin spaces and then apply the obtained results to the setting of quotient spaces.
This is joint work in progress with Bojan Basrak and Nikolina Milinčević.

A new definition of random set

Vesna Gotovac Đogaš[1], Kateřina Helisova [2], Lev B. Klebanov [3], Jakub Staněk [4], Irina V. Volchenkova

[1] Faculty of Science, University of Split
vgotovac@pmfst.hr
[2] Czech Technical University in Prague
[3] Department of Probability and Mathematical Statistics, Charles University, Prague, Czech Republic
[4] Department of Mathematics Education, Charles University, Prague, Czech Republic

In this talk, a new definition of random sets is proposed. Motivation for introducing the new definition is that the classical approach deals with Hausdorff distance between realisations of the random sets, which is not satisfactory for statistical analysis in many cases. We place the realisations of the random sets in a complete Boolean algebra (B.A.) endowed with a positive finite measure intended to capture important characteristics of the realisations. A distance on B.A. is introduced as a square root of measure of symmetric difference between its two elements. The distance is then used to define a class of Borel subsets of B.A. Consequently, random sets are defined as measurable mappings taking values in the B.A. This approach enables us to use a more general family of distances between realisations of random sets. It additionally provides us with some statistical tests concerning equality of some characteristics of random set distributions.

References

[1] Gotovac V. (2019) *Similarity between random sets consisting of many components* Image Anal Stereol. Vol. 38: 185–99

[2] Gotovac Đogaš V., Helisova K., Klebanov L.B., Staněk J., Volchenkova I.V (2021+) *A new definition of random set*

Disagreement coupling and Poisson approximation for some Gibbs processes

Günter Last

Karlsruher Institut für Technologie

We discuss a thinning and an embedding procedure to construct finite Gibbs processes with a given Papangelou intensity. Extending some earlier work of Hofer-Temmel (2019) and of Hofer-Temmel and Houdebert (2019) we will use this to couple two finite Gibbs processes with different boundary conditions. We then combine this coupling with a classical result by Barbour and Brown (1992) to establish Poisson approximation of point processes derived from certain subcritial Gibbs processes via dependent thinning.

This is joint work with Moritz Otto.

Extremes for marked point processes

Bojan Basrak

Department of Mathematics, University of Zagreb

We consider stationary configurations of points in R^d which are marked by real-valued scores. In our setup, the scores are allowed to depend on location of points and outside sources of randomness. We show that in a neighbourhood of a point with an extreme score one can often rescale positions and scores of nearby points to obtain a limiting point process we call tail configuration. Under some assumptions on dependence between scores, based on this local limit result we can derive global asymptotics for extreme scores in increasing sections of R^d.

This is joint work in progress with Ilya Molchanov and Hrvoje Planinić

Heavy-tailed phase-type distributions: a unified approach

Jorge Yslas

University of Bern

Heavy-tailed phase-type random variables have mathematically tractable distributions and are conceptually attractive to model physical phenomena due to their interpretation in terms of a hidden Markov structure. Three recent extensions of regular phase-type distributions give rise to models which allow for heavy tails: discrete- or continuous-scaling; fractional-time semi-Markov extensions; and inhomogeneous time-change of the underlying Markov process. In this talk, we present a unifying theory for heavy-tailed phase-type distributions for which all three approaches are particular cases. Our main objective is to provide useful models for modeling heavy tails, but our specification also captures other tail behaviors. Furthermore, we provide two multivariate extensions inspired by the univariate construction, which can be considered a matrix version of a frailty model. Finally, we present fully explicit EM algorithms for the estimation of all models and illustrate their use.

Strong renewal theorem and local limit theorem in the absence of regular variation

Péter Kevei

University of Szeged

We obtain a strong renewal theorem with infinite mean beyond regular variation, when the underlying distribution belongs to the domain of geometric partial attraction of a semistable law with index $\alpha \in (1/2, 1]$. In the process we obtain local limit theorems for both finite and infinite mean, that is for the whole range $\alpha \in (0, 2)$. We also derive the asymptotics of the renewal function for $\alpha \in (0, 1]$. The talk is based on recent joint work with Dalia Terhesiu (University of Leiden).

An analysis of the proportional tail model from extreme value theory with the coupling method

Clément Dombry

Université de Franche-Comté

The proportional tail model has recently been considered in Extreme Value Theory (EVT) as an interesting framework for the analysis of extreme quantile regression. The model assumes that the tail distribution of the response variable Y conditionally on the covariate $X = x$ is asymptotically proportional to the unconditional tail. The proportionality factor is the so-called skedasis function that quanti fies the impact of X on the extremes of Y. In this paper, we revisit the analysis of the proportional tail model thanks to the coupling method recently developed in EVT (Bobbia et al., Bernoulli 2021). Under some regular variation condition, we prove that the point measure of exceedances over high threshold jointly considered with the concomitant covariates can be approximated by a simple limit model and we provide quantitative estimates in terms of Wasserstein distance. This allows us to extend earlier results on the estimation of the extreme value index and the integrated skedasis function to lower threshold where a limit bias appear. Besides, we develop new results for the model validation and propose a test based on the bootstrap together with its analysis thanks to the coupling method. A short simulation study illustrates the good properties of the test procedure in terms of level and power.

Renewal theorems for the i.i.d. cluster point processes

Marina Dajaković

Faculty of Science, University of Split

In this talk we consider i.i.d. cluster point processes on and give, under certain conditions, variants of the classical renewal theorems (Blackwell's theorem and the key renewal theorem). Furthermore, we prove the extension of the renewal theorem, namely that shifted i.i.d. cluster point process converges in distribution to the stationary i.i.d. cluster process, with respect to the vague topology.