ELEMENTARNA MATEMATIKA 1 – jednadžbe

U sljedećim zadacima traže se sva realna rješenja dane jednadžbe, nejednadžbe odnosno sustava. Nepoznanice su označene s x i y, a parametri a, b. Ukoliko se u jednadžbi javljaju parametri, treba odrediti njena rješenja u ovisnosti o vrijednosti (realnih) parametara.

1. $1 - \dfrac{x}{1 + \frac{x}{1-x}} = x^2$
2. $\dfrac{\frac{a}{x} + \frac{b}{a}}{\frac{b}{x} - \frac{a}{b}} = \dfrac{b}{a}$
3. $\frac{(4x^2-4x+1)(2-x-x^2)(x-4)}{(x^2-4)(x+3)} \geq 0$
4. $\frac{(x^2-3x-4)(2x+5)}{-x^2+x} \geq 0$
5. $\left|\dfrac{x+4}{3x+2}\right| > \dfrac{1}{x}$
6. $\left|x + \left|\dfrac{1}{2}x + \left|\dfrac{1}{3}x - 1\right|\right|\right| = 1$
7. $\sqrt{x+1} + \sqrt{x-1} = 1$
8. $\sqrt{2x^2+3x-5} \geq x+1$
9. $\sqrt{x+3} < \sqrt{2-x} - \sqrt{x}$
10. $\sqrt[3]{15-x} + \sqrt[3]{20+x} = 5$
11. $\sqrt[3]{x+1} + \sqrt[3]{x+2} + \sqrt[3]{x+3} = 0$
12. $2 \cdot 9^{x-1} - 3^{2(x-2)} = \left(3^{x+2} - \sqrt[3]{27^{x-1}}\right) \cdot 9^{-2} + \frac{25}{243}$
13. $4^{x+\sqrt{x^2-2}} - 5 \cdot 2^{x-1+\sqrt{x^2-2}} = 6$
14. $8^x + 1 = 4^{x+0.5}$
15. $3^{x-3} = 5^{x^2-7x+12}$
16. $\dfrac{9^x - 5 \cdot 15^x + 4 \cdot 25^x}{-9^x + 8 \cdot 15^x - 15 \cdot 25^x} < 0$
17. $\left|\log_{\sqrt{2}} x - 2\right| - |2 - \log_2 x| > 1$
18. $\dfrac{1}{\log_{x-3}(x+1)} + \dfrac{1}{\log_x(x^2+2x+1)} < 1$
19. $25^{2x-x^2+1} + 9^{2x-x^2+1} = 34 \cdot 15^{2x-x^2}$
20. $\log_{1-2x}(6x^2-5x+1) - \log_{1-3x}(4x^2-4x+1) = 2$
21. $\log_2\left(\sqrt{x^2-4x}+3\right) > \log_{\frac{1}{2}}\left(\frac{2}{\sqrt{x^2-4x}+\sqrt{x+1}+1}\right) + 1$
22. $\begin{cases} \left(\frac{3}{2}\right)^{x-y} - \left(\frac{2}{3}\right)^{x-y} = \frac{65}{36} \\ xy - x + y = 118 \end{cases}$
23. $\begin{cases} x^{\log y} + y^{\log x} = 200 \\ xy = 1000 \end{cases}$
24. $\begin{cases} \log_2 x - \log_4 y = \log_4(4-x) \\ \log_3(x+y) = \log_{\frac{1}{3}} \frac{y}{x} \end{cases}$
25. $3\sin x + 4\cos x = 5$
26. $\dfrac{1-\cos x}{\sin x} + \mathrm{tg}^2\dfrac{x}{2} = 0$
27. $4\cos x \cdot \cos 2x \cdot \cos 5x + 1 = 0$
28. $4^{\sin^2(\pi x)} + 3 \cdot 4^{\cos^2(\pi x)} \leq 8$
29. $\begin{cases} \sin^2 2x + 1 = \sin^2 3y \\ x + y = \frac{\pi}{6} \end{cases}$
30. Odredite sve $a \in \mathbb{R}$ za koje jednadžba $\sin x \cos x - \sin x - \cos x + a = 0$ ima bar jedno rješenje.

Rješenja

[1.] Nema rješenja. [2.] Za $a = 0$, $b = 0$ ili $a = b$ nema rješenja. Za $a \neq 0$, $b \neq 0$ i $a \neq \pm b$ rješenje je $x = b - a$. Za $a \neq 0$, $b \neq 0$ i $a = -b$ rješenja su $x \in \mathbb{R} \setminus \{0, a\}$. [3.] $x \in \langle -3, -2\rangle \cup \{\frac{1}{2}\} \cup [1, 2\rangle \cup \langle 2, 4]$. [4.] $x \in \langle -\infty, -\frac{5}{2}] \cup [-1, 0\rangle \cup \langle 1, 4]$. [5.] $x \in \langle -\infty, -\frac{2}{3}\rangle \cup \langle -\frac{2}{3}, 0\rangle \cup \langle 1, +\infty\rangle$. [6.] $x_1 = 0, x_2 = -\frac{12}{7}$. [7.] Nema rješenja. [8.] $x \in \langle -\infty, -\frac{5}{2}] \cup [2, +\infty\rangle$. [9.] Nema rješenja. [10.] $x \in \{7, -12\}$. [11.] $x = -2$. [12.] $x = 0$. [13.] $x = \frac{3}{2}$. [14.] $x = 0$, $x = \log_2 \frac{1+\sqrt{5}}{2}$. [15.] $x = 3$, $x = 4 + \log_5 3$. [16.] $x \in \langle -\infty, 0] \cup \langle 1, \log_3 4] \cup \langle \log_3 5, +\infty\rangle$. [17.] $x \in \langle 0, \frac{1}{2}\rangle \cup \langle 2^{\frac{5}{3}}, \infty\rangle$. [18.] $x \in \langle 3, 3+2\sqrt{2}\rangle \setminus \{4\}$. [19.] $x \in \{0, 2, 1+\sqrt{3}, 1-\sqrt{3}\}$. [20.] $x \in \emptyset$. [21.] $x \in [-1, 0]$. [22.] $(x, y) \in \{(12, 10), (-10, -12)\}$. [23.] $(x, y) \in \{(10, 100), (100, 10)\}$. [24.] $(x, y) = (\frac{4}{3}, \frac{2}{3})$. [27.] $x = \frac{\pi}{8} + k\frac{\pi}{2}$, $x = \pm\frac{1}{2}\arccos\frac{-1-\sqrt{5}}{4} + k\pi$, $x = \pm\frac{1}{2}\arccos\frac{-1+\sqrt{5}}{4} + k\pi$, za $k \in \mathbb{Z}$. [28.] $x \in \bigcup_{k\in\mathbb{Z}}\left[\frac{1}{4}+k, \frac{3}{4}+k\right]$. [29.] $(x, y) = (m\pi, \frac{\pi}{6} - m\pi), m \in \mathbb{Z}$. [30.] $a \in \left[-\frac{2\sqrt{2}+1}{2}, 1\right]$.