Elliptic boundary value problems with eigenparameter dependent boundary conditions

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In this talk second order elliptic boundary value problems on bounded domains $\Omega \subset \mathbb{R}^n$ with boundary conditions on $\partial\Omega$ depending nonlinearly on the spectral parameter are investigated in an operator theoretic framework. For a general class of locally meromorphic functions in the boundary condition a solution operator of the boundary value problem is constructed with the help of a linearization procedure. In the special case of rational Nevanlinna or Riesz-Herglotz functions on the boundary the solution operator is obtained in an explicit form in the product Hilbert space $L^2(\Omega) \oplus (L^2(\partial\Omega))^m$, which is a natural generalization of known results on λ-linear elliptic boundary value problems and λ-rational boundary value problems for ordinary second order differential equations.

On conservative realizations of Herglotz-Nevanlinna functions

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In this overview talk we discuss realization problems for linear conservative systems of Livšic type

$$\Theta = \begin{pmatrix} \mathbb{A} & K & J \\ \mathcal{H}_+ \subset \mathcal{H} \subset \mathcal{H}_- & & E \end{pmatrix}, \tag{1}$$

with unbounded state space operator $\mathbb{A}$. The main object of our study is a class of operator-valued Herglotz-Nevanlinna (H-N) functions that can be realized as a linear fractional transformation of the transfer function

$$W_\Theta(z) = I - 2iK^*(\mathbb{A} - zI)^{-1}KJ \tag{2}$$

of a system of the form (1). We will show how direct and inverse problems for such type of systems are solved and provide a complete description of realizable H-N operator-functions in a finite-dimensional Hilbert space E. Various subclasses of the class of realizable H-N functions will be described in connection with specific properties of realizing conservative systems. We will also talk about the general realization problem with non-canonical systems and realizations of generalized Nevanlinna functions that involve spaces with indefinite metric. As an important application, we will discuss the realization of certain scalar H-N functions by systems that are based on a non-self-adjoint Schrödinger operator in $L_2[a, +\infty)$.

This talk is based on joint work with Eduard Tsekanovskiĭ and will survey all the above mentioned developments and connections.

Necessary and sufficient conditions for an indefinite Sturm-Liouville Riesz basis property

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A short survey will be given of some conditions in the literature for the eigenfunctions of a regular Sturm-Liouville problem (with indefinite weight function) to form a Riesz basis in a specific Hilbert space. Some of these conditions look quite different but in certain circumstances they turn out to be equivalent.

Krein spaces applied to Friedrichs' systems

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Friedrichs systems are a class of boundary value problems which allows the study of a wide range of differential equations in a unified framework. They were introduced by K. O. Friedrichs in 1958 in an attempt to treat equations of mixed type (such as the Tricomi equation). The Friedrichs system consists of a first order system of partial differential equations (of a specific type) and an *admissible* boundary condition enforced by a matrix-valued boundary field.

In a recent paper: A. Ern, J.-L. Guermond, G. Caplain: *An Intrinsic Criterion for the Bijectivity Of Hilbert Operators Related to Friedrichs' Systems, Commun. Part. Diff. Equat.* **32** (2007) 317–341 a new approach to the theory of Friedrichs systems has been proposed, rewritting them in terms of Hilbert spaces, and a new way of representing the boundary conditions has been introduced. The admissible boundary conditions have been characterised by two intrinsic geometric conditions in the graph space, thus avoiding the traces at the boundary. Another representation of boundary conditions via boundary operators has been introduced as well, which is equivalent to the intrinsic one (those enforced by two geometric conditions) if two specific operators P and Q on the graph space exist. However, the validity of the last condition was left open. The authors have also shown that their admissible (geometric) conditions imply maximality of the boundary condition.

We note that these two geometric conditions can be naturally written in the terms of an indefinite inner product on the graph space, and by use of simple geometric properties of Krein spaces we show that maximality of boundary condition is equivalent to its admissibility. An aplication of classical results on Krein spaces also allow us to construct a counter–example, which shows that the operators P and Q do not necessarily exist. In the case of one space dimension we give complete classification of admissible boundary conditions (those satisfying the two geometric conditions).

The relation between the *classical* representation of admissible boundary conditions (via matrix fields on the boundary), and those given by the boundary operator will be addressed as well: sufficient conditions on the matrix boundary field in order to define boundary operator will be given and tested on examples.

M-matrix inverse problem for Sturm-Liouville equations on graphs.

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We consider an inverse spectral problem for Sturm-Liouville boundary value problems on a graph with formally self-adjoint boundary conditions at the nodes, where the given information is the M-matrix. Based on the results found in S. Currie, B.A. Watson, M-matrix asymptotics for Sturm-Liouville problems on graphs, J. Com. Appl. Math., 218 (2008), pp. 568-578, using the Green's function, we prove that the poles of the M-matrix are at the eigenvalues of the associated boundary value problem and are simple, located on the real axis and that the residue at a pole is a negative semi-definite matrix with rank equal to the multiplicity of the eigenvalue. We define the so called norming constants and relate them to the spectral measure and the M-matrix. This enables us to recover, from the M-matrix, the boundary conditions and the potential, up to a unitary equivalence for co-normal boundary conditions.

First order spectral perturbation theory of square singular analytic matrix functions

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In this talk, we deal with first order eigenvalue perturbation theory of square singular matrices, $A(\lambda)$, depending on a single parameter λ. In particular, we will consider separately the cases ofmatrix pencils, matrix polynomials of arbitrary degree and, finally (if time permits) the more general case of matrices which are analytic in λ. Although this last case includes the previous ones, we treat them separately because in the first two cases we are able to provide additional information in terms of some intrinsic properties of $A(\lambda)$, such as reducing subspaces and bases of certain null spaces. It is well know that eigenvalues of singular matrix polynomials (and, by extension, of singular analytic matrices in λ) are not continuous functions of the entries of the coefficient matrices of the polynomial. Nonetheless, we prove that for most perturbations they are indeed continuous. Given an eigenvalue λ_0 of a matrix polynomial $P(\lambda)$ we prove that, for generic perturbations $M(\lambda)$ with degree less than or equal to the degree of $P(\cdot, \cdot)$, the eigenvalues of $P(\lambda) + \varepsilon M(\lambda)$ admit convergent series expansions near λ_0 and we describe the first order term of these expansions in terms of $M(\lambda_0)$ and certain particular bases of the left and right null spaces of $P(\lambda_0)$. In the important case of λ_0 being a semisimple eigenvalue of $P(\lambda)$ any bases of the left and right null spaces of $P(\lambda_0)$ can be used, and the first order term of the eigenvalue expansions takes a simple form. These results are contained in [1]. In the particular case of $P(\lambda)$ being a matrix pencil the bases of the left and right null spaces are built up from bases of the minimal reducing subspaces of $P(\lambda)$. This case has been treated in [2]. We will finally show how to generalize the previous results to singular analytic matrices $A(\lambda)$, where the perturbation $B(\lambda, \varepsilon)$ is analytic in λ in a neighborhood of the given eigenvalue λ_0 of $A(\lambda)$ and in ε near $\varepsilon = 0$, and satisfying $B(\lambda, 0) \equiv 0$. This will extend to the singular case some results by Langer and Najman [4, 5]. Moreover, once we have "regularized" the problem we will make use of the same techniques as the ones introduced in [5]. When we particularize to semisimple eigenvalues of a regular matrix $A(\lambda)$ we also recover some results from [3].

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Quadratic (weakly) hyperbolic matrix polynomials: Direct and inverse spectral problems

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Let L be a monic quadratic weakly hyperbolic or hyperbolic $n \times n$ matrix polynomial. We discuss the solution of some direct spectral problems: The eigenvalues of a compression of L to an $(n-1)$–dimensional subspace of $\mathbb{C}^n$ block–interlace and the eigenvalues of a one–dimensional perturbation of L $(-,+)$–interlace the eigenvalues of L. A key role in the discussion is played by matrix valued Nevanlinna functions. We also discuss the solution of an inverse spectral problem: Two given block–interlacing sets of real numbers can be viewed as the sets of eigenvalues of L and its compression.

The lecture is based on joint work with Tomas Azizov, Karl-Heinz Foerster, and Peter Jonas.

A variational expression for generlized relative entropy

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We give a variational expression for one-parameter extended relative entropy. In addition, we derive a one-parameter extended thermodynamic inequality and a generalized Peierls-Bogoliubov inequality. Finally we give a one-parameter extended Golden-Thompson type inequality, for positive operators.

Structured operator equations in spectral analysis

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We prove a $\sin\Theta$ theorem for quasi-definite operators in a Hilbert space. A typical example of a quasi-definite operator is an operator associated with Stokes system of partial differential equations. We obtain estimates for all unitary invariant norms. The estimates involve a constant which measures stability of a quasi-definite form representation theorem.

We compare these results with a $\sin\Theta$ theorem which is obtained by the use of weak operator Riccati equation. This technique yield estimates of the angle operator in a spectral norm only but without the use of additional stability constants. An infinite dimensional setting is used to give a new information on the ingredients of the known finite dimensional estimates.

This is a joint work with V. Kostrykin, K. Makarov, N. Truhar and K. Veselić.

The hierarchical matrices technique

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Matrices of large size arise in particular from elliptic partial differential equations and integral equations. In the former case one make use of the sparsity, in the latter case a standard treatment of the matrices leads already to storage problems. The technique of hierarchical matrices allows to organise the storage as well as all matrix operations (including inversion) with almost linear complexity. The hierarchical matrix operations yield only approximations, but the arising error can be made at least as small as the discretisation error. The lecture explains the matrix representation, the organisation of the operations and underlines the black-box character of the method. Furthermore, applications are described which are usually considered to be impossible for large scale matrices: computation of functions of matrices and solution of matrix equations (Lyapunov etc.).

Spectral properties of unbounded Jacobi matrices versus properties of bounded ones

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The spectral theory of bounded, hermitian Jacobi matrices is well developed (see the two volume treatise of B.Simon). In the talk some results on the spectral properties of unbounded Jacobi matrices obtained in the last 15 years will be given. Next a few comments on similarities and differences between bounded and unbounded cases will discussed.

Spectral theory of Sturm–Liouville operators with point interactions

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We investigate spectral properties (self-adjointness, lower semiboundedness, and discreteness of the spectrum) of Sturm–Liouville operators with local interactions on a discrete set $X = \{x_n\}_{n=1}^{\infty} \subset \mathbb{R}_+$. In the case when the distance between the interactions sites tends to zero, $d_* := \inf_{n \neq k} |x_n - x_k| = 0$, we show that spectral properties of Sturm–Liouville operators are closely connected with the spectral properties of some unbounded Jacobi matrices. We exploit this connection to study spectral properties of Sturm–Liouville operators with δ- and δ'-interactions when $d_* = 0$.

On indefinite quadratic forms

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We develop a perturbation theory for indefinite (that is, not necessarily semibounded) quadratic forms. For such forms we prove operator representation theorems. Some applications will be also discussed. Results have been obtained jointly with L. Grubišić, K. Makarov, and K. Veselić.

Structued condition numbers for eigenvalue problems

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The condition number of an eigenvalue provides a first-order measure for the sensitivity of this eigenvalue under perturbations. Restricting the set of perturbations to preserve certain structures gives rise to the concept of structured eigenvalue condition numbers. A question of practical importance is to estimate the extent to which structured condition numbers can be below unstructured ones. The first part of this talk will discuss this question for multiple eigenvalues of matrices, building on the work of Langer and Najman. The second part of this talk will be concerned with matrix polynomials. In particular, it will be shown how to select a structured linearization of a given polynomial such that the structured eigenvalue condition numbers do not deteriorate.

Deformed commutations of operators and their relations with braided algebras and convolutions

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The q-normality of operators, a q-deformation of the notion of normality ($q > 0$), was introduced and studied by S. Ôta. Several classes: q-normal, q-quasinormal, q-hyponormal and q-subnormal operators were investigated. It turns out that many basic properties of these q-deformed operators are different from that of the corresponding undeformed operators. We will focus on the fact that q-normality gives a q-analogue of positive definiteness, which in the classical case ($q = 1$) is related to moment problems. The integral representation corresponding to the q-positive definiteness will be presented, giving the notion of q-moment sequences. Moreover, we observe that they are in correspondence with states on unital $*$-algebras generated by q-normal elements.

Next, we study unital $*$-algebras generated by families of elements, which are q-normal and satisfy some additional relations (called (p, q)-commutation relations). A realization of such elements can be found in the $*$-algebra of operators on a pre-Hilbert space or in a $*$-braided algebra. The distribution of sum of such operators, with respect to a given state, gives rise to a formula which can be interpreted as a new convolution of measures. We investigate its properties and also present a corresponding Fourier transform.

A part of the results is a joint work with Éric Ricard.

Quadratic eigenvalue problems - old and new.

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I will review the spectral theory of quadratic eigenvalue problems and describe some recent attacks on inverse spectral problems.

Spectral Theory of the Klein Gordon Equation

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With the Klein-Gordon equation operators in Pontryagin and Krein spaces are associated and their spectra are studied. (joint work with Branko Najman and Christiane Tretter)

Spectra of a Class of non-selfadjoint Random Jacobi Matrices

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We study spectra and pseudospectra of two classes of (one- or twosided) infinite matrices. The first type is supported on the main and superdiagonal and the second type is supported its sub- and superdiagonal. All these diagonals are made of samples from a random variable.

The study of these matrices and their spectra was motivated by work on so-called non-selfadjoint quantum mechanics by J. Feinberg and A. Zee.

On the similarity of indefinite Sturm-Liouville operator to a selfadjoint operator

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Sufficient conditions for the similarity of indefinite Sturm-Liouville operator to a selfadjoint operator will be discussed. This topic has its origin in the works of B. Ćurgus, B. Najman, H. Langer and A. Fleige (see [1] and references therin). In the case of decaying potential we present some sharp conditions for the similarity to a self-adjoint operator. The counterexamples demonstrated the sharpness of the obtained sufficient conditions will be discussed too.

These results complements the previous results by B. Ćurgus, B. Najman, M. Faddeev, R. Shterenberg and others.

The talk is based on the recent paper [2].

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Convexity of ranges and connectedness of level sets of quadratic forms

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This is a joint work with I. Feldman and N. Krupnik.

Let H be a complex inner product space and S be its unit sphere. By the classical theorems of F. Hausdorff and O. Toeplitz the level sets of an Hermitian form on S are connected and the range of a quadratic form on S is convex. We consider the following question: which subsets of H besides S have these properties? Between the sets which we examine are some subsets of S and also spheres, balls, exteriors of balls, annuli in various norms. The answer on the question depends both on the norm and on the dimension of H (the main difference appears between $\dim H = 2$ and $\dim H > 2$).

Spectral Gaps of Indefinite Singular Sturm-Liouville-Operators

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In this talk we consider the differential expression

$$\operatorname{sgn}(\cdot)\,(-f'' + qf)$$

on the real line, where q is a locally summable, real-valued function. Assume that this differential expression is in the limit point case at $\pm\infty$. Then the differential expression gives rise to a self-adjoint operator in a Krein space $(L^2(\mathbb{R}), [\cdot,\cdot]))$, where the inner product is generated by the indefinite weight function $[\cdot,\cdot] = (\operatorname{sgn}\cdot,\cdot)$.

We give sufficient conditions for the existence of a real spectral gap of A in terms of the potential q. In general, eigenvalues of A within the gap may accumulate to the boundary of the essential spectrum. We give some conditions for q which guarantee accumulation and non-accumulation, respectively. Moreover, we derive an estimate for the maximal number of eigenvalues in the gap.

On singular two-parameter eigenvalue problems

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In the 1960s Atkinson introduced an abstract algebraic setting for multiparameter eigenvalue problems. He showed that a nonsingular multiparameter eigenvalue problem is equivalent to the associated system of generalized eigenvalue problems. Many theoretical results and numerical methods for nonsingular multiparameter eigenvalue problems are based on this relation.

We extend the above relation to singular two-parameter eigenvalue problems and show that under very mild conditions the finite regular eigenvalues of both problems agree. This enables one to solve a singular two-parameter eigenvalue problem by computing the common regular eigenvalues of two singular generalized eigenvalue problems.

We consider *the algebraic two-parameter eigenvalue problem*

$$\begin{aligned} W_1(\lambda,\mu)x_1 := (A_1+\lambda B_1+\mu C_1)x_1 &= 0, \\ W_2(\lambda,\mu)x_2 := (A_2+\lambda B_2+\mu C_2)x_2 &= 0, \end{aligned} \tag{3}$$

where A_i, B_i, and C_i are $n_i \times n_i$ matrices over $\mathbb{C}$, $\lambda,\mu\in\mathbb{C}$, and $x_i\in\mathbb{C}^{n_i}$. A pair (λ,μ) is an *eigenvalue* if it satisfies (3) for nonzero vectors x_1, x_2.

A two-parameter eigenvalue problem can be expressed as two coupled generalized eigenvalue problems. On the tensor product space $S := \mathbb{C}^{n_1}\otimes\mathbb{C}^{n_2}$ of the dimension $N := n_1n_2$ we define *operator determinants*

$$\begin{aligned} \Delta_0 &= B_1\otimes C_2 - C_1\otimes B_2, \\ \Delta_1 &= C_1\otimes A_2 - A_1\otimes C_2, \\ \Delta_2 &= A_1\otimes B_2 - B_1\otimes A_2. \end{aligned} \tag{4}$$

The problem (3) is then related to a coupled pair of generalized eigenvalue problems

$$\begin{aligned} \Delta_1 z &= \lambda\Delta_0 z, \\ \Delta_2 z &= \mu\Delta_0 z, \end{aligned} \tag{5}$$

for decomposable tensors $z\in S$, $z = x\otimes y$. In the nonsingular case, when Δ_0 is nonsingular, the eigenvalues of (3) agree with the eigenvalues of the associated pair of generalized eigenvalue problems (5).

Our main result is that under very mild conditions we can similarly relate the finite regular eigenvalues of (3) to the finite regular eigenvalues of (5). This enables one to compute the eigenvalues of a general singular two-parameter eigenvalue problem (3) from the corresponding pair of singular generalized eigenvalue problems. This relation also opens many new possibilities in the study of singular two-parameter eigenvalue problems.

This is a joint work with Bor Plestenjak.

Minimization of the energy of a vibrational system

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In this talk we will study vibrational systems of the form

$$M\ddot{x} + C\dot{x} + Kx = 0,$$

(here M, C, K are symmetric nonnegative matrices representing mass, damping and stiffness of the system), in terms of the minimization of the energy of the system given by $E(t) = 1/2((M\dot{x},\dot{x}) + (Kx,x))$.

A part of the results is a joint work with S.J. Cox, A. Rittmann and K. Veselić.

The behavior of functions of operators under perturbations

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I am going to speak about recent joint results with A.B. Aleksandrov. It is well known that a Lipschitz function does not have to be operator Lipschitz. In other words, The inequality $|f(x) - f(y)| \leq \text{const}\,|x - y|$ does not imply that $\|f(A) - f(B)\| \leq \text{const}\,\|A - B\|$ for self-adjoint operators A and B. It turned out that the situation dramatically changes if we consider functions in Hölder–Zygmund classes. We prove that if $0 < \alpha < 1$ and f is in the Hölder class $\Lambda_\alpha(\mathbb{R})$, then for arbitrary self-adjoint operators A and B with bounded $A-B$, the operator $f(A)-f(B)$ is bounded and $\|f(A) - f(B)\| \leq \text{const}\,\|A - B\|^\alpha$. We prove a similar result for functions f of the Zygmund class $\Lambda_1(\mathbb{R})$: $\|f(A+K) - 2f(A) + f(A-K)\| \leq \text{const}\|K\|$, where A and K are self-adjoint operators. Similar results also hold for all Hölder-Zygmund classes $\Lambda_\alpha(\mathbb{R})$, $\alpha > 0$. We also study properties of the operators $f(A) - f(B)$ for $f \in \Lambda_\alpha(\mathbb{R})$ and self-adjoint operators A and B such that $A - B$ belongs to the Schatten–von Neumann class S_p. We consider the same problem for higher order differences. Similar results also hold for unitary operators and for contractions.

On a Model in Poro-Elasticity

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A modification of the material law associated with the well-known Biot system as suggested by M. A. Murad and J. H. Cushman in 1996 and first rigorously investigated by R. E. Showalter in 2000 is re-considered in the light of a new approach to a comprehensive class of evolutionary problems and extended to large class of anisotropic inhomogeneous media.

The paper is joint work with Des McGhee (Glasgow, U.K.).

Preservers of generalized orthogonality on finite dimensional vector and projective spaces

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Let V be a finite dimensional vector space over a field (or more generally division ring) F. If A and B are invertible linear transformations on V, then a map $f : V \to V$ is said to be (A, B)-orthogonality preserving if $Bf(x)$ is orthogonal to $f(y)$ as soon as Ax is orthogonal to y; here, the orthogonality is understood in the sense of a fixed sesquilinear form. Analogously, (A, B)-orthogonality preserving maps on the corresponding projective space are defined. Recent results, open problems, and counterexamples concerning characterizations of (A, B)-orthogonality preserving maps will be presented. (Joint work with Peter Šemrl.)

Chernoff's theorem for backward propagators

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The classical Chernoff theorem is a statement about discrete-time approximations of semigroups, where the approximations are constructed as products of time-dependent contraction operators strongly differentiable at zero. We generalize a version of Chernoff's theorem for semigroups proved in a paper by Smolyanov, Weizsaecker, and Wittich, and obtain a theorem about discrete-time approximations of backward propagators $U_{s,t}$. The discrete-time approximations are constructed as products of two-parameter contraction operators $Q_{s,t}$ whose derivatives at $s = t$ coincide with the generators of $U_{s,t}$. We consider the situation when the backward propagator is represented by a transition density of a time-inhomogeneous diffusion process, and the contraction operators are represented by integral operators with probabilistic kernels. The latter situation is realized in a number of applications. For example, when the backward propagator is associated with a time-inhomogeneous diffusion on a manifold, and the discrete-time approximations are distributions of diffusion processes in the surrounding Euclidean space. We then obtain the approximation of the distribution on the manifold by distributions in the Euclidean space.

Matrices with normal defect one

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A $n \times n$ matrix A has normal defect one if it is not normal, however can be embedded as a north-western block into a normal matrix of size $(n+1) \times (n+1)$. The latter is called a minimal normal completion of A. A construction of all matrices with normal defect one is given. Also, a simple procedure is presented which allows one to check whether a given matrix has normal defect one, and if this is the case — to construct all its minimal normal completions. A characterization of the generic case for each n under the assumption that self-commutator of A has rank 2 (which is necessary for A to have normal defect one) is obtained. Both the complex and the real cases are considered. It is pointed out how these results can be used to solve the minimal commuting completion problem in the classes of pairs of $n \times n$ Hermitian (resp., symmetric, or symmetric/antisymmetric) matrices when the completed matrices are sought of size $(n+1) \times (n+1)$. An application to the $2 \times n$ separability problem in quantum computing is described.

This is joint work with D. Kaliuzhnyi-Verbovetskyi and H. Woerdeman.

On a commutative WJ^*-algebra of $D^+_\varkappa$-class and its bicommutant

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WJ^*-algebra is a weakly closed symmetric (according to the inner product) algebra of operators in a Krein space that contains the identity. An operator family belongs to the $D^+_\varkappa$-class if it has at least one common invariant subspace that is a maximal non-negative subspace and can be presented as a direct sum of its $\varkappa$-dimensional isotropic part and a uniformly positive subspace, $\varkappa < \infty$. Let us note that every commutative operator family of self-adjoint operators in Pontryagin spaces belongs to $D^+_\varkappa$-class for some $\varkappa$. Finally, the bicommutant of an operator family is the algebra of operators that commute with every operator which commutes with all operators of the given family. We'll discuss the relation between a function representation for a commutative WJ^*-algebra of $D^+_\varkappa$-class and its bicommutant.

Dilations and the reproducing kernel property

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I intend to scrutinise diverse dilation theorems from the point of view of the reproducing kernel property. This property is the main tool in construction while for the environment Hilbert C*-modules are going to serve.

Bounds on the solution of Sylvester equation

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The solution to a Sylvester equation $AX - XB = GF^*$ with a low rank right-hand side is analyzed quantitatively through Low-rank Alternating-Directional-Implicit method (LR-ADI) with exact shifts. New bounds for $\|X\|$ are obtained. Also perturbation bounds for $\|\delta X\|$, for perturbed Sylvester equation:
$(A + \delta A)(X + \delta X) - (X + \delta X)(B + \delta B) = (G + \delta G)(F + \delta F)^*$, are obtained. A distinguished feature of these bounds is that they reflect the interplay between the eigenvalue decompositions of A and B and the right-hand side factors G and F. Numerical examples suggest that because of this inclusion of details, new perturbation bounds are much sharper than the existing ones.

Dirichlet forms on graphs

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We describe diffusion of particles on a finite directed graph Γ, consisting of a set V of vertices and of a set E of edges connecting vertices. The edges $e \in E$ are assumed to be modelled by the interval $[0, 1]$, equipped with a finite measure μ_e, and a weight μ_v is assigned to each vertex $v \in V$. The particles are described by densities $f \in L_2(\Gamma, \mu) = \bigoplus_{e\in E} L_2([0, 1], \mu_e) \oplus \ell_2(V, (\mu_v)_{v\in V})$. The self-adjoint operator H governing the diffusion by the evolution equation

$$u'(t) = -Hu(t)$$

is associated with the sesquilinear form τ in $L_2(\Gamma, \mu)$,

$$\tau(f, g) := \sum_{e\in E} \gamma_e \int_0^1 f_e'(x)\overline{g_e'(x)}\, dx + \sum_{v\in V} \gamma_v f_v \overline{g_v},$$

where $\gamma_e, \gamma_v \geq 0$ are certain weights. The domain of τ is a suitable subspace of $L_2(\Gamma, \mu)$, in particular incorporating the boundary conditions (glueing conditions) at the vertices. The main objects of the investigation are

- the closedness of the form τ,
- positivity of the associated C_0-semigroup (i.e., validity of the first Beurling-Deny criterion),
- the second Beurling-Deny ctiterion.

The talk is a report on joint work with U. Kant, T. Klaußand M. Weber.

An inverse nodal problem for two-parameter Sturm-Liouville problems

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We consider uniqueness and reconstruction of the potentials q_j and (separated) boundary conditions from a dense sequence of nodal points for a two-parameter system

$$y_j'' + (\lambda a_j + \mu b_j + q_j)y_j = 0, \quad j = 1, 2,$$

of Sturm-Liouville equations linked by the eigenvalue pairs (λ, μ).

Hecke algebras on homogeneous trees and relation with Hankel and Toeplitz matrices

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On a homogeneous tree X we define a Hecke algebra $H(X)$, which consists of distant dependent kernels $F(x, y) = f(\text{dist}(x, y))$. It is considered as a commutative subalgebra of some bigger non-commutative algebras, including all bounded operators on $L_2(X)$. We study when the Hecke algebra is maximal abelian subalgebra, and show that it is the case if $\deg(X) > 2$ (degree of homogeneity). For $\deg(X) = 2$, the case of $X = Z$ (integers) we show that the commutant of $H(Z)$ is a direct sum of Hankel and Toeplitz operators.

On operators with single spectrum

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We intend to discuss a number of recent results and questions concerning analytic and geometric properties of linear operators with the single spectrum $\{1\}$.

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