Workshop on quantitative spectral theory and mathematical physics

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Book of abstracts

A posteriori error estimation for rational eigenvalue problems with applications to band gap computations for photonic crystals

Luka Grubišić

University of Zagreb, Croatia

luka@math.hr

Using the perturbation theory for block operator matrices we develop a posteriori approximation error estimation techniques for rational eigenvalue problems. We consider applications of the theory to band gap computations for photonic crystals. We use the Lorenz permitivity model for the description of material properties.

This is a joint work with Christian Engström, Vadim Kostrykin, and Christiane Tretter.

An indefinite operator on finite metric graphs

Amru Hussein

University of Mainz, Germany

hussein@mathematik.uni-mainz.de

Differential expressions of the form $-D_x \operatorname{sgn}(x) D_x$ on the real line are considered. Self-adjoint realisations of this expression and their spectral properties are considered. Generalizations to metric graphs are discussed as well. This is motivated by a model of a cloaking phenomenon in $\mathbb{R}^2$.

Faster solution of large, overdetermined, dense linear systems

Željko Jeričević

University of Rijeka, Croatia

zeljko.jericevic@riteh.hr

The solution of linear least squares system requires the solution of overdetermined system of equations. For large dense systems that requires prohibitive number of operations. We developed a novel numerical approach for approximate solution of large linear systems of a dense type. The method is based on Fourier transform although any unitary, orthogonal transform which concentrates power in a small number of coefficients can be used. This is the strategy borrowed from digital signal processing where pruning off redundant information from spectra or filtering of selected information in frequency domain is the usual practice. For the least squares problem the procedure is to transform the linear system along the column to the frequency domain, generating a transformed system. The least significant portions in the transformed system are deleted as the whole rows, yielding a smaller pruned system. The pruned system is solved in transform domain, generating the transform of approximate solution. Inverting the transform of approximate solution yields the approximate solution of original system. The quality of solution is compared against exact solution and is on the level of numerical noise up to reduction to square system. Numerical experiments illustrating feasibility of the method and quality of the approximation and operation count are presented.

Joint work with Ivica Kožar.

On 2×2 symmetric matrices

Vadim Kostrykin

University of Mainz, Germany

kostrykin@mathematik.uni-mainz.de

Derivation of a Model of the Prestressed Elastic String

Maroje Marohnić

University of Zagreb, Croatia

maroje.marohnic@math.hr

We derive an one-dimensional model for displacement and torsion of the elastic string starting from the cylindrical three-dimensional prestressed elastic body with small thickness ε. We assume that the stress in the body is due to a prior elastic deformation φ of isotropic, homogenous, elastic body for which constitutive equations are known. By formal asymptotic expansion we deduce the scaling of forces. Then we prove that the solutions of three-dimensional problems converge to the solutions of the string equations via singular perturbation techniques. Coefficients of the string model depend on three-dimensional elasticity coefficients and tension produced by deformation φ.

Joint work with J. Tambača.

On the correspondence between spectra of the pencil $A - \lambda B$ and of the operator $B^{-1}A$

Ivica Nakić

University of Zagreb, Croatia

nakic@math.hr

In this talk er are concerned with the reduction of the spectral problem for symmetric linear operator pencils to a spectral problem for the single operator. Also, a Rayleigh–Ritz–like bounds on eigenvalues of linear operator pencils are obtained.

Spectral properties of elliptic differential operators and Dirichlet–to–Neumann maps

Jonathan Rohleder

TU Graz, Austria

rohleder@math.tugraz.at

We consider a formally symmetric second order elliptic differential expression of the form

$$\mathcal{L}u = -\sum_{j,k=1}^{n} (\partial_j a_{jk})\partial_k u + au$$

with variable coeffcients on an interior or exterior domain $\Omega \in \mathbb{R}^n$ and a class of selfadjoint realizations of $\mathcal{L}$ in $L^2(\Omega)$ with certain nonlocal boundary conditions. The main objective of this talk is to show how the eigenvalues and absolutely continuous spectra of these operators can be characterized with the help of a Dirichlet–to–Neumann map associated with $\mathcal{L}$ on the boundary of Ω.

The talk is based on a joint work with J. Behrndt.

Structure and Spectral properties of Inhomogeneous Random Graphs

Reza Samavat

TU Chemnitz, Germany

reza.samavat@mathematik.tu-chemnitz.de

We concern an inhomogeneous random graph in a sense that two vertices are connected not only by the random parameter p but also by the type of vertices or colors. It means that for two vertices of different types i and j the probability that an edge between these two vertices exists, is p_{ij}. The degree distribution of the model is not like random graphs or small world graphs. Here the exponential decay of degree distribution in last two models changes to power-law decay. We show that integrated density of states exhibits a Lifshitz tail at the lower spectral edge.

On indefinite Differential Operators of second order

Stephan Schmitz

University of Mainz, Germany

schmist@uni-mainz.de

Let $\Omega \subset \mathbb{R}^n$ be a bounded open set with C^2 boundary. Let $A(\cdot)$ be a symmetric, not necessarily definite $n \times n$ matrix with entries in $L^\infty(\Omega)$, such that the inverse $A(\cdot)^{-1}$ has entries in $L^\infty(\Omega)$ again and $\int_\Omega A(x)^{-1} dx$ is invertible.

We show, that under these assumptions, there is a unique self-adjoint operator associated with the indefinite quadratic form

$$\langle \operatorname{grad} u, A(\cdot) \operatorname{grad} u \rangle_{L^2(\Omega)^n}$$

with domain lying in the domain of the gradient.

This operator has a compact resolvent and its spectral properties are studied as well.

The talk is based on a joint work with A. Hussein, D. Krejcirik and V. Kostrykin.

Lifshitz Asymptotics for Hamiltonians with some Monotonicity in the Randomness

Christoph Schumacher

TU Chemnitz, Germany

christoph.schumacher@mathematik.tu-chemnitz.de

The integrated density of states of a random Schrödinger operator is in many instances known to exhibit Lifshitz tails at fluctuation boundaries. Usually the randomness enters the operator in a monotone fashion, for example in alloy type models or breather models with repulsion. By using an inequality due to Thirring instead of Temple's inequality we can relax the monotonicity condition to a positivity condition. This includes all known previous examples and additionally enables us to deal with a new class of breather type potentials.

Joint work with Ivan Veselić.

A Banach space–valued ergodic theorem and applications

Fabian Schwarzenberger

TU Chemnitz, Germany

`fschwarzenberger@web.de`

In this talk we consider Cayley graphs over amenable groups. We present an ergodic theorem for Banach space–valued, almost additive functions defined on the set of finite subsets of the group. We furthermore show applications of this theorem in the spectral theory of selfadjoint operators, where we get convergence of the normalized eigenvalue counting functions in supremum norm to a function called integrated density of states. An other application of the ergodic theorem comes from percolation theory, where we are able to prove the uniform existence of certain density functions.

Mathematical modeling of vascular stents

Josip Tambača

University of Zagreb, Croatia

tambaca@math.hr

The stents are elastic structures made of struts. They are inserted into the blood vessels in order to reduce the effects of stenosis or to bridge aneurysms. Their design is very complex and thus computationally very expensive to treat as three-dimensional elastic objects. In this talk I will present mathematical modeling of elastic stents based on a one-dimensional model for stent struts.

Wegner estimate for alloy-type models with sign-changing exponentially decaying single-site potentials

Martin Tautenhahn

TU Chemnitz, Germany

martin.tautenhahn@mathematik.tu-chemnitz.de

We study Schroedinger operators on $\ell^2(\mathbb{Z}^d)$ of the form $H_\omega = -\Delta + V_\omega$ where Δ denotes the discrete Laplacian and V_ω is a random multiplication operator by the function $V_\omega(x) = \sum_{k\in\mathbb{Z}^d} \omega_k u(x-k)$. We assume that $u : \mathbb{Z}^d \to \mathbb{R}$ is exponentially decaying and $\{\omega_k\}_{k\in\mathbb{Z}^d}$ is a sequence of independent identically distributed random variables. The main result is a Wegner estimate for this model which is polynomial in the volume of the box and linear in the size of the energy interval. In particulae, it can be used for a localization proof vie the multiscale analysis. The methods also apply to a certain class of non-monotone alloy-type models on $L^2(\mathbb{R}^d)$.

Joint work with Norbert Peyerimhoff and Ivan Veselić.

Eigenvalue flows for linear pencils of operators

Ivan Veselić

TU Chemnitz, Germany

`ivan.veselic@mathematik.tu-chemnitz.de`

In perturbation theory one often considers operator families with an affine linear structure. One-parameter families are a typical example, but multi-parameter families play an important role as well, especially in the modelling of disordered systems. An important question concerns the movement of eigenvalues, or the flow of eigenvalues under the variation of the parameters. We discuss several approaches on how the flow may be studied and finally specialise to the case of finite matrices with block structure.

List of the participants

Participant	e-mail address
Luka Grubišić	luka.grubisic@math.hr
Amru Hussein	hussein@mathematik.uni-mainz.de
Željko Jeričević	zeljko.jericevic@riteh.hr
Vadim Kostrykin	kostrykin@mathematik.uni-mainz.de
Maroje Marohnić	maroje.marohnic@math.hr
Ivica Nakić	nakic@math.hr
Jonathan Rohleder	rohleder@math.tugraz.at
Reza Samavat	reza.samavat@mathematik.tu-chemnitz.de
Stephan Schmitz	schmist@uni-mainz.de
Christoph Schumacher	christoph.schumacher@mathematik.tu-chemnitz.de
Fabian Schwarzenberger	fschwarzenberger@web.de
Josip Tambača	tambaca@math.hr
Martin Tautenhahn	martin.tautenhahn@mathematik.tu-chemnitz.de
Ivan Veselić	ivan.veselic@mathematik.tu-chemnitz.de