KOREAN - CROATIAN PROBABILITY SUMMER CAMP
ŠIBENIK, 26. - 28.6.2024.

MINI-COURSES

There will be three 3x45 min mini-courses given by the following experts in the field:

Soobin Cho (University of Illinois at Urbana-Champaign):

Advances in analyzing non-local Dirichlet forms on metric measure spaces

Abstract: In this presentation, I will discuss a comprehensive overview of recent progress in the analysis of non-local regular Dirichlet forms on metric measure spaces. Adopting both analytic and probabilistic approaches, I will focus on equivalent relations among key aspects such as heat kernel estimates, regularity estimates for non-local operators, and various function inequalities, including localized Poincaré, cutoff Sobolev, and Faber-Krahn inequalities. The presentation highlights the inherent stability structures of these relations. Furthermore, the talk revisits findings on subordinate Markov processes, emphasising their crucial role in stability results. Recent advancements in the analysis of these processes are explored, revealing robust function inequalities and regularity outcomes for harmonic functions in metric measure spaces. The final section presents recent results for non-local Dirichlet forms with blow-up jump measures.

Mateusz Kwaśnicki (Wrocław University of Science and Technology):

Harmonic extension technique - probabilistic and analytic perspective

Abstract: Consider a path of the reflected Brownian motion in the half-plane $\{y \geq 0\}$, and erase its part contained in the interior $\{y > 0\}$. What is left is, in an appropriate sense, a path of a jump-type stochastic process on the line $\{y = 0\}$ - the boundary trace of the reflected Brownian motion. It is well-known that this process is, in fact, a 1-stable Lévy process, also known as the Cauchy process.

The PDE interpretation of the above fact is the following. Consider a bounded harmonic function u in the half-plane $\{y > 0\}$, with sufficiently smooth boundary values f. Let g denote the normal derivative of u at the boundary. The mapping $f \mapsto g$ is known as the Dirichlet-to-Neumann operator, and it is again well-known that this operator coincides with the square root of the 1-dimensional Laplace operator $-\Delta$. Thus, the Dirichlet-to-Neumann operator coincides with the generator of the boundary trace process.

Molchanov and Ostrovski proved that isotropic stable Lévy processes are boundary traces of appropriate diffusions in half-spaces. Caffarelli and Silvestre gave a PDE counterpart of this result: the fractional Laplace operator is the Dirichlet-to-Neumann operator for an appropriate second-order elliptic operator in a half-space. Again, the Dirichlet-to-Neumann operator turns out to be the generator of the boundary trace process.

During my talk, I will discuss boundary trace processes and Dirichlet-to-Neumann operators in a more general context. My main goal will be to explain the connections between probabilistic and analytical results. Along the way, I will introduce the necessary machinery: Brownian local times and additive functionals, Krein's spectral theory of strings, and Fourier transform methods.

Stjepan Šebek (University of Zagreb):

Geometric functionals of the Brownian convex hull

Abstract: In this mini-course, we study the convex hull of the standard planar Brownian motion. This object has been well-studied from Lévy (1948) onwards. We focus exclusively on geometric functionals of the Brownian convex hull. In the first part, we deal with the expected values of various geometric quantities that describe the size of the convex hull spanned by a path of the standard planar Brownian motion (namely, perimeter, area, diameter, inradius, and circumradius). The second part will be focused on finding bounds on the related inverse processes (that correspond to the perimeter, area, diameter, circumradius, and inradius of the convex hull), which provide us with some information on the speed of growth of the size of the Brownian convex hull.