ESTIMATES OF KERNELS AND GROUND STATES FOR CLASSICAL SCHRÖDINGER SEMIGROUPS

MIŁOSZ BARANIEWICZ

We consider the Schrödinger operator of the form $H = -\Delta + V$ acting in $L^2(\mathbb{R}^d, dx)$, $d \geq 1$, where the potential $V : \mathbb{R}^d \to [0, \infty)$ is a locally bounded function. The corresponding Schrödinger semigroup $\left\{e^{-tH} : t \geq 0\right\}$ consists of integral operators, i.e.

$$e^{-tH} f(x) = \int_{\mathbb{R}^d} u_t(x, y) f(y) dy, \quad f \in L^2(\mathbb{R}^d, dx), \ t > 0. \tag{1}$$

I will present new estimates for heat kernel of $u_t(x, y)$. Our results show the contribution of the potential is described separately for each spatial variable, and the interplay between the spatial variables is seen only through the Gaussian kernel.

This estimates will be presented on two common classes of potentials. For confining potentials we get two sided estimates and for decaying potentials we get new upper estimate.

Methods we used to estimated kernel of semigroup allow to easily obtain sharp estimates of ground state for slowly varying potentials.

The poster is based on joint work with Kamil Kaleta [1] and my work [2].

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MONTE CARLO ESTIMATION FOR NON-LOCAL PARABOLIC EQUATIONS

DANIEL E. CEDEÑO-GIRON (I. BIOČIĆ AND B. TOALDO)

This work, inspired by research in [1], deals with Monte Carlo approximations for non-local parabolic equations associated with semi-Markov processes, we provide upper bounds on the errors between the exact solution and the Monte Carlo approximation (which could be biased). We also provide confidence intervals estimates using the central limit theorem, and give convergence rates in the central limit theorem using Berry-Esseen-type bounds.

In particular, we consider semi-Markov processes constructed by a time-change of a Feller process, e.g., we look at an isotropic d-dimensional stable process evaluated at a suitable random time. The latter can be the inverse or the undershoot of a general subordinator independent of the stable process. The governing non-local equations for our process of interest are studied in [2, 3]. We note that inverses and undershoots are accurately sampled from their densities using an effective rejection sampling method as in [4, 5], rather than random walk approximations.

Regarding anomalous diffusion, for any subordinator, we have developed an exact method to sample the inverse and the undershoot paths, eliminating the inaccuracies associated with approximating the subordinator paths [6]. Also, bounds of strong and weak error of the Euler-Maruyama approximations are reported.

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NON-LOCAL BOUNDARY VALUE PROBLEMS FOR BROWNIAN MOTIONS

FAUSTO COLANTONI

We explore non-local boundary value problems associated with heat equations, deriving probabilistic representations of the solutions through transformations of Brownian motions involving both additive parts and time changes. We address both the case of the positive half-line, extending to star graphs, and the real case, which generalizes processes like skew Brownian motion. Additionally, we present a connection between non-local boundary conditions in space and stochastic resetting.

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COMPARISON OF RANDOM SETS DISTRIBUTIONS VIA STATISTICAL DEPTHS

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We present several depths for possibly non-convex random sets. The depths are applied to the comparison between two samples of non-convex random sets, using a visual method of DD-plots and statistical tests. The advantage of this approach is to identify sets within the sample that are responsible for rejecting the null hypothesis of equality of the distribution and to provide clues on differences between the distributions. The method is justified on the basis of a simulation study.

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BEURLING–DENY FORMULA FOR SOBOLEV–BREGMAN FORMS

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One way to investigate symmetric Markov processes involves the theory of Dirichlet forms, which corresponds with L^2 space. It is well known that there is a one-to-one correspondence between the class of regular Dirichlet forms $\mathcal{E}$ and the class of symmetric Hunt processes (strong Markovian, quasi-left continuous with càdlàg paths).

The Beurling–Deny formula provides the following unique decomposition of a regular Dirichlet form $\mathcal{E}$, defined on an appropriate domain $\mathcal{D}(\mathcal{E})$ in L^2 into an (inexplicit) strongly local part, the jumping part, and the killing part:

$$\begin{aligned}\mathcal{E}(u,v) &= \mathcal{E}^{(c)}(u,v)\\ &\quad+\frac{1}{2}\iint\limits_{(E\times E)\setminus\mathrm{diag}}(u(y)-u(x))(v(y)-v(x))J(dx,dy)+\int\limits_E u(x)v(x)k(dx),\end{aligned}$$

where J is the *jumping measure*, a k is the *killing measure*. Here u, v are continuous (or, more generally, quasi-continuous versions of) functions from the domain $\mathcal{D}(\mathcal{E})$. The three parts describe the local, jumping and killing behavior of the corresponding Hunt process.

The Sobolev–Bregman form $\mathcal{E}_p$ is an extension to L^p of the Dirichlet form $\mathcal{E}$. The applications of this notion can be found in [2, 3, 4, 5, 6, 7], although the name *Sobolev–Bregman* was introduced only recently in [5].

Our goal is to derive the following Beurling–Deny formula for $\mathcal{E}_p$:

$$\begin{aligned}\mathcal{E}_p[u] &= \frac{4(p-1)}{p^2}\mathcal{E}^{(c)}(u^{\langle p/2\rangle},u^{\langle p/2\rangle})\\ &\quad+\frac{1}{p}\iint\limits_{(E\times E)\setminus\mathrm{diag}}F_p(u(x),u(y))J(dx,dy)+\int\limits_E|u(x)|^p k(dx),\end{aligned}$$

where the function $F_p(a,b) := |b|^p - |a|^p - pa^{\langle p-1\rangle}(b-a)$ is so-called *Bregman divergence* and $a^{\langle\kappa\rangle} := |a|^\kappa \operatorname{sgn} a$.

The presentation is based on the joint work [1] with Mateusz Kwaśnicki (Wrocław University of Science and Technology).

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ON THE CONVEX HULL OF TWO PLANAR RANDOM WALKS

DANIELA IVANKOVIĆ (TOMISLAV KRALJ, NIKOLA SANDRIĆ, STJEPAN ŠEBEK)

In this research, we study the limiting behavior of the perimeter and diameter functionals of the convex hull spanned by the first n steps of two independent planar random walks with drift. The geometric properties of the smallest convex set containing points of our random walks can provide useful information about their dispersion. The techniques and ideas used to obtain our results are, in part, developed in [2, 3] where similar results are obtained for a single planar random walk.

First, we prove the strong law of large numbers for the convex hull of $m \geq 1$ independent random walks with scaling n^{-1} and identify the limit as convex hull of the set of their drift vectors including the null vector. Using the appropriate variant of the continuous mapping theorem, the strong law is extended to all intrinsic volumes, with adequate scaling, and consequently to the perimeter and diameter of the convex hull.

Afterwards, we introduce the necessary assumptions on the drift vectors of the two independent random walks and give the L^2 approximations of the perimeter and diameter processes which, along with determining appropriate variances of the limiting normal law, enables us to state and prove the central limit theorem for perimeter and diameter of these random sets. Key elements that we use here are martingale difference sequence and the Cauchy formula for perimeter and diameter of convex compact sets.

In the end, we comment on possible extensions of our results to the case of more than two independent planar random walks with drifts. Finally, we run a simulation study to test our conjecture about non–Gaussian distributional limit in the case when any of our assumptions on the drift vectors of random walks are not satisfied.

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TOPOLOGICAL DATA ANALYSIS FOR RANDOM SETS AND ITS APPLICATION IN DETECTING OUTLIERS AND GOODNESS OF FIT TESTING

MARCELA MANDARIĆ (VESNA GOTOVAC ĐOGAŠ)

We present the methodology for detecting outliers and testing the goodness-of-fit of random sets using topological data analysis. We construct the filtration from level sets of the signed distance function and consider various summary functions of the persistence diagram derived from the obtained persistence homology such as accumulative persistence function and support function of lift zonoid. The outliers are detected using functional depths for the summary functions. Global envelope tests using the summary statistics as test statistics were used to construct the goodness-of-fit test. The procedures were justified by a simulation study using germ-grain random set models.

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ERGODICITY AND ULTRACONTRACTIVITY OF DISCRETE FEYNMAN–KAC SEMIGROUPS AND RELATED OPERATORS

MATEUSZ ŚLIWIŃSKI (JOINT WORK WITH WOJCIECH CYGAN, KAMIL KALETA AND RENÉ SCHILLING)

We present results of our investigation of a particular discrete-time counterpart of the Feynman–Kac semigroup with a confining potential in a countably infinite space. These findings are a continuation of our work, described in detail in the paper [1]. We focus on Markov chains with the direct step property, which is satisfied by a wide range of typically considered kernels. In our joint work with Wojciech Cygan, René Schilling and Kamil Kaleta [3], we introduce the concept of progressive intrinsic ultracontractivity (pIUC) and investigate links between pIUC of Feynman–Kac semigroups, their uniform quasi-ergodicity and uniform ergodicity of their intrinsic semigroups. We conclude with a simple observation which allows us to linearly interpolate certain related operators on ℓ^p spaces, $1 \leqslant p \leqslant \infty$.

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INTRINSIC ULTRACONTRACTIVITY OF FEYNMAN-KAC SEMIGROUPS FOR CYLINDRICAL STABLE PROCESSES

KINGA SZTONYK

The following Schrödinger operator

$$K = K_0 + V,$$

where

$$K_0 = \sqrt{-\frac{\partial^2}{\partial x_1^2}} + \sqrt{-\frac{\partial^2}{\partial x_2^2}}$$

is an example of a nonlocal, anisotropic, singular Lévy operator. We consider potentials $V : \mathbb{R}^2 \to \mathbb{R}$ such that $V(x)$ goes to infinity as $|x| \to \infty$. The operator $-K_0$ is a generator of a process $X_t = (X_t^{(1)}, X_t^{(2)})$, sometimes called cylindrical, such that $X_1^{(1)}$, $X_2^{(2)}$ are independent symmetric Cauchy processes in $\mathbb{R}$.

We define the Feynman-Kac semigroup

$$T_t f(x) = E^x \left(\exp\left(-\int_0^t V(X_s)\, ds \right) f(X_t) \right).$$

Operators T_t are compact for every $t > 0$. There exists an orthonormal basis $\{\phi_n\}_{n=1}^{\infty}$ in $L^2(\mathbb{R}^2)$ and a corresponding sequence of eigenvalues $\{\lambda_n\}_{n=1}^{\infty}$, $0 < \lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \dots$, $\lim_{n\to\infty} \lambda_n = \infty$ such that $T_t \phi_n = e^{-\lambda_n t} \phi_n$. We can assume that ϕ_1 is positive and continuous on $\mathbb{R}^2$. The main result I would like to present concerns estimates for ϕ_1 and intrinsic ultracontractivity of the semigroup T_t under certain conditions on the potential V.

This poster is based on my master's thesis [1].

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EXCURSION THEORY FOR THE WRIGHT–FISHER DIFFUSION

IVANA VALENTIĆ (PAUL A. JENKINS, JERE KOSKELA, JAROMIR SANT, DARIO SPANÒ)

In this work, we develop excursion theory for the Wright–Fisher diffusion with recurrent mutation. Our construction is intermediate between the classical excursion theory where all excursions begin and end at a single point and the more general approach considering excursions of processes from general sets. Since the Wright–Fisher diffusion has two boundary points, it is natural to construct excursions which start from a specified boundary point, and end at one of two boundary points which determine the next starting point. In order to do this we study the killed Wright–Fisher diffusion, which is sent to a cemetery state whenever it hits either endpoint. We then construct a marked Poisson process of such killed paths which, when concatenated, produce a pathwise construction of the Wright–Fisher diffusion.

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