ON INVERSE LIMITS OF COMPACT SPACES

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In this paper we are concerned with inverse systems \( \{X_a, \pi_{\beta a}\} \) of Hausdorff compact spaces \( X_a \); the systems are taken over arbitrary directed sets \( M = \{a\} \). \( X \) will always denote the inverse limit of the system and \( \pi_a: X \rightarrow X_a \) will be the corresponding natural projections.

We first introduce a Hausdorff paracompact space \( X^* \) associated to every inverse system and consisting of all the spaces \( X_a \) of the system (taken as disjoint sets) and of the limit \( X \). The topology of \( X^* \) is such that the subset \( X \) is actually the limit (in the sense of the directed set \( M \)) of subsets \( X_a \). Several properties of \( X^* \) are given. This generalizes a procedure given by H. Freudenthal ([6], p. 153) in the case of inverse sequences of metrizable compacta.

Next we consider the mapping spaces \( (X, R) \) of all mappings of a Hausdorff compact \( X \) into an ANR and we consider the singular homology group \( H_q((X, R); G) \) (with coefficients in an arbitrary Abelian group \( G \)) as a contravariant functor of \( X \). Using the properties of \( X^* \) we show that \( H_q((X, R); G) \) is continuous with respect to inverse limits (for Hausdorff compacta). This generalizes a previous result of the author ([9], Theorem 13, p. 200) and settles a question raised in the same paper ([9], p. 202).

1. The Space \( X^* \)

Let

\[
X^* = (\bigcup X_a) \cup X, \ a \leq M, \quad (1)
\]

where all \( X_a \) and their limit \( X \) are considered as being disjoint sets. If \( U_a \) is an open set of \( X_a \), let \( U_a^* \subseteq X^* \) be the set defined by

\[
U_a^* = \bigcup_{a \leq \beta} (\pi_{\beta a}^{-1} U_a) \bigcup (\pi_a^{-1} U_a). \quad (2)
\]

Let \( \mathcal{U} \) be the family of subsets of \( X^* \) consisting of all open sets \( U_a \subseteq X_a, \ a \leq M, \) and of all sets \( U_a^*, \ a \leq M \). Since the sets \( \pi_{\alpha}^{-1} U_a \)

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1) Basic definitions and facts concerning inverse systems and their limits can be found in [5] and [8].
form a basis of open sets for \( X \), it follows that \( \mathcal{U} \) is a covering of \( X^* \).
Moreover, the intersection of any two members of \( \mathcal{U} \) is the union of some members of \( \mathcal{U} \). It suffices to prove this statement for the sets \( U_\alpha^* \) and \( U_\alpha'^* \), \( \alpha, \alpha' \in M \). Let \( x \in U_\alpha^* \cap U_\alpha'^* \), if \( x \in X_\beta \), for a \( \beta \leq M \), then \( \alpha \leq \beta, \alpha' \leq \beta \), and \( x \) belongs to the set \( (\pi_{\beta-1} U_\alpha) \cap (\pi_{\beta-1} U_{\alpha'}) \), which is open in \( X_\beta \) and thus belongs to \( \mathcal{U} \). On the other hand, if \( x \notin X \), then \( x \) belongs to the set \( (\pi_{\alpha-1} U_\beta) \cap (\pi_{\alpha-1} U_{\alpha'}) \) which is open in \( X \). Therefore, there is a \( \beta \leq M \) and an open set \( U_\beta \subset X_\beta \) such that \( x \leq \pi_{\beta-1} U_\beta \subset (\pi_{\alpha-1} U_\alpha) \cap (\pi_{\alpha-1} U_{\alpha'}) \). One can also achieve that

\[ U_\alpha^* \subset U_\alpha \cap U_\alpha'^* \]

We now define the topology of \( X^* \) by taking the family \( \mathcal{U} \) for a basis of all open sets. The properties that we established above show that \( \mathcal{U} \) can be given such a role. Notice that \( X_\alpha \) and \( X \) inherit from \( X^* \) their natural topologies as the relative topologies. \( X^* \) is clearly a Hausdorff space if all \( X_\alpha \) are Hausdorff spaces; this enables us to use in \( X^* \) nets and their limits (see [7], Chapter 2).

**Theorem 1.** Let \( \{X_\alpha, \pi_{\alpha} \} \), \( \alpha \leq M \), be an inverse system of nonempty Hausdorff compacta (over a directed set \( M \)). Choose for every \( \alpha \leq M \) an arbitrary point \( x_\alpha \in X_\alpha \). Then \( \{x_\alpha\}, \alpha \leq M \), is a net in \( X^* \) which has at least one cluster point \( X \in X \subset X^* \).

**Proof.** Let \( M_\alpha \) denote the set of all \( \beta \leq M \) with \( \alpha \leq \beta \). Then \( \{\pi_{\beta} x_\alpha\}, \beta \leq M_\alpha \), is a net of \( X_\alpha \). Let \( A \subset X_\alpha \) be the set of all cluster points of this net. \( A \) is non-empty, because \( X_\alpha \) is compact. Furthermore, \( A \) is closed in \( X_\alpha \). Thus the sets \( B_\beta = \pi_{\beta}\pi_{\beta-1}(A) \subset X_\beta, \beta \leq M_\alpha \), and \( B = \pi_{\alpha-1}(A) \subset X \) are also closed. We shall now prove the following proposition:

(i) The set \( B \subset X \) is not empty.

Take any \( \alpha \leq A \) (\( A \) is non-empty) and any open set \( U_\alpha \subset X_\alpha \) containing \( a \). Since \( a \) is a cluster point of the net \( \{\pi_{\beta} x_\alpha\}, \beta \leq M_\alpha \), for every \( \beta \leq M_\alpha \) there is a \( \gamma \geq \beta \) such that \( \pi_{\gamma} x_\gamma \subset U_\alpha \). On the other hand, \( \pi_{\gamma} x_\gamma = \pi_{\gamma}(\pi_{\beta} x_\alpha) \subset \pi_{\gamma}(X_\beta) \) so that \( (\pi_{\beta}(X_\beta)) \cap U_\alpha \neq 0 \). Consequently, \( a \) is a cluster point of \( \pi_{\beta}(X_\beta) \) and thus \( a \leq \pi_{\gamma}(x_\beta) \), for all \( \beta \leq M_\alpha \). \( \pi_{\beta}(X_\beta) \) is compact. This proves that the sets \( B_\beta = \pi_{\gamma}^{-1}(A) \) are non-empty compact spaces. Since obviously \( \pi_{\beta}', \beta (B_\beta') \subset B_\beta, \beta \leq \beta' \), the sets \( B_\beta \) form an inverse system. The inverse limit of this system is contained in \( B = \pi_{\alpha-1}(A) \subset X \) and is non-empty (see Theorem 3.6, p. 217 of [5]), proving the assertion (i).

Now assume that \( \{x_\alpha\}, \alpha \leq M \), has no cluster points in \( X \). Then for every \( x \leq X \) one can find an open set \( U_\alpha^* \) (given by (2)) and an \( \alpha(x) \leq M \) such that \( U_\alpha^* \) contains no points of \( \{x_\beta\}, \beta \leq M_\alpha \) and \( x \leq U_\alpha^* \). Since \( X \) is compact, there is a finite collection of sets \( U_\alpha(1)^*, \ldots, U_\alpha(n)^* \) covering \( X \) and disjoint with \( \{x_\beta\}, \beta \leq M_\gamma \), where \( \gamma \) is a suitable element of \( M, \gamma \geq \alpha(1), \ldots, \alpha(n) \). Consider now the net \( \{\pi_{\beta} x_\beta\}, \beta \leq M_\gamma \), and the open set \( U_\gamma = \pi_{\gamma -1}(U_\alpha(1)) \cup \ldots \cup \pi_{\gamma -1}(U_\alpha(n)) \) of \( X_\gamma \). Clearly, \( \pi_{\gamma -1}(U_\gamma) \subset X \).

On the other hand, it is readily seen that \( U_\gamma^* \) is contained in the union of the sets \( U_\alpha(1)^*, \ldots, U_\alpha(n)^* \) and therefore contains no points
of \{x_\beta\}, \beta \leq M. Consequently, \{\pi_\beta x_\beta\}, \beta \leq M, is a net entirely contained in the closed set \(X_\gamma \setminus U_\gamma\). Hence, the set \(A\) of its cluster points belongs also to \(X_\gamma \setminus U_\gamma\). According to (i) the set \(B = \pi_\gamma^{-1}(A) \subseteq X\) is not empty and is contained in \(\pi_\gamma^{-1}(U_\gamma)\) by (4). Therefore, \((A \cap U_\gamma) \supset \pi_\gamma B \neq \emptyset\), which is a contradiction to \(A \subseteq X_\gamma \setminus U_\gamma\).

**Theorem 2.** Let \(\{X_\alpha, \pi_\alpha\}, \alpha \leq M\), be an inverse system of (non-empty) Hausdorff compacta and let \(U\) be an open set in \(X^*\) such that \(X \subseteq U\). Then there is a \(\gamma \leq M\) such that \(X_\beta \subseteq U\), for all \(\beta \geq \gamma\).

**Proof.** Since the sets (2) form a basis for open sets around points of \(X\) and since \(X\) is compact, it is easy to find an open set \(V\) of \(X^*\) such that \(X \subseteq V \subseteq U\) and that

\[ V = U_{\alpha(1)^*} \cup \ldots \cup U_{\alpha(n)^*}. \]  

(5)

In order to prove Theorem 2, it suffices to find a \(\gamma \leq M\), \(\gamma \geq \alpha(1), \ldots, \alpha(n)\), such that

\[ X_\gamma \subseteq V, \]  

(6)

because (6) will then imply

\[ X_\beta \subseteq V \subseteq U, \text{ for all } \beta \geq \gamma. \]  

(7)

Suppose now that no \(\gamma \leq M\) satisfies (6). Then one could find a point \(x_\gamma \subseteq X_\gamma \setminus V\) for every \(\gamma \leq M\). \(\{x_\gamma\}, \gamma \leq M\), would be a net in \(X^*\), satisfying the conditions of Theorem 1 and contained entirely in \(X^* \setminus V\). Hence, this net could not have cluster points in \(X \subseteq V\), which contradicts Theorem 1.

**Theorem 3.** If \(\{X_\alpha, \pi_\alpha\}\) is an inverse system of (non-empty) Hausdorff compacta, then the space \(X^*\) is Hausdorff and paracompact.

**Proof.** Let \(\{V_\mu\}\) be an open covering of \(X^*\). Since \(X\) is compact, there is a finite subcollection, consisting of sets \(V_\mu(1), \ldots, V_\mu(n)\), which covers \(X\). If \(V\) denotes the union of this subcollection, then there is an \(\alpha \leq M\) such that all \(X_\beta, \beta \leq M_\alpha\), are contained in \(V\) (Theorem 2). Notice that the set

\[ X_\alpha^* = (\bigcup_{\beta \geq \alpha} X_\beta) \cup X \]  

(8)

is an open subset of \(X^*\), because it is of type (2) (with \(U_a = X_a\)).

Now consider the following collection \(\mathcal{V}\) of open sets of \(X^*\): take first the open sets \((X_\alpha^*) \cap V_\mu(1), \ldots, (X_\alpha^*) \cap V_\mu(n)\) for members of \(\mathcal{V}\). Furthermore, for every \(\beta \leq M \setminus M_\alpha\), consider the open covering \(\{X_\beta \cap V_\mu\}\) of \(X_\beta\) and take elements of a finite subcovering as new elements of \(\mathcal{V}\) (recall that \(X_\beta\) is compact and open in \(X^*\)). The family \(\mathcal{V}\) of open sets of \(X^*\), which we just defined, is clearly a star-finite covering of \(X^*\) which refines the covering \(\{V_\mu\}\). \(\mathcal{V}\) is a fortiori a locally finite refinement of \(\{V_\mu\}\).
2. Mappings of $X$ into ANR-s

In this section we are concerned with absolute neighborhood retracts $R$ for metric spaces (abbreviated as ANR). Recall that ANR-s can be characterized as neighborhood retracts of convex subsets $C$ of Banach spaces (see [4], p. 363). We shall also use the following theorem due to R. Arens (Theorem 4.1, p. 18 of [3]; see also [2]):

Let $C$ be a convex subset of a Banach space. Every mapping $f$ of a closed subset of a Hausdorff paracompact space into $C$ admits an extension $f_x$ to the whole space (the values of $f_x$ are in $C$).

The following theorem generalizes a lemma by M. Abe ([1], 2.2, p. 188) and Theorem 11.9, p. 287 of [5].

Theorem 4. Let $\{X_n, \pi_n\}, n \leq M$, be an inverse system of Hausdorff compacta and let $f : X \rightarrow R$ be a mapping of their limit into an ANR. Then there is an $a \leq M$ such that for every $\beta \leq M$, one can define a map $f_\beta : X_\beta \rightarrow R$ with the property that $f_\beta \pi_\beta$ is homotopic to $f$ and $f_\beta \pi_\gamma, \beta \geq \gamma$, for all $\gamma \geq \beta > a$.

Proof. Consider $R$ as a neighborhood retract of a convex set $C$ of a Banach space. Let $V$ be a neighborhood of retraction $R$ in $C$. Consider $f$ as a mapping of $X$ into $C$. Since $X$ is a closed subset of $X^*$ and $X^*$ is Hausdorff and paracompact (Theorem 3), we can apply the theorem of Arens and obtain a mapping $f_x : X^* \rightarrow C$ extending $f$.

Choose now, for every $x \in X$, a convex open set $V(x)$ of $C$ such that $f(x) \subseteq V(x) \subseteq C$ and choose an open set $U_{a(x)}^*$ of type $\beta$ such that $x \subseteq U_{a(x)}^* \subseteq f^{-1}(V(x))$. Notice that $X \cap U_{a(x)}^* = \pi_{a(x)}^{-1} U_{a(x)}$, so that for $a(m) \leq M(x)$, we get

$$\pi_\beta (X \cap U_{a(x)}^*) \subseteq \pi_\beta \pi_a (x)^{-1} (U_{a(x)}) \subseteq U_{a(x)}^* \subseteq f_x^{-1}(V(x)).$$

Thus, for $\beta \leq M(a(x))$,

$$f_\beta \pi_\beta (X \cap U_{a(x)}^*) \subseteq V(x).$$

(10)

The collection $\{U_{a(x)}^*, x \leq X\}$ is an open covering of $X$ and we can choose a finite subcovering consisting of sets $U_{a(1)}^*, \ldots, U_{a(n)}^*$, where $a(i) = a(x_i), x_i \subseteq X$. If we denote the convex set $V(x_i)$ by $V_i$, then (10) goes over into

$$f_{a(i)} \pi_{a(i)} (X \cap U_{a(i)}^*) \subseteq V_i, \quad i = 1, \ldots, n,$$

(11)

and is valid for all $\beta$ greater than $a(1), \ldots, a(n)$.

Now define a homotopy in $C$, connecting $f$ and $f_{a(i)} \pi_{\beta}, \beta \geq a$, by joining points $f(x)$ and $f_{a(i)} \pi_{\beta}(x)$ by a line segment, obviously lying in $C$. We want to show that this segment lies actually in the retraction neighborhood $V$. Given any $x \subseteq X$, there is an $i \leq \{1, \ldots, n\}$ such that $x \subseteq U_{a(i)}^* \subseteq f_{a(i)}^{-1}(V_i)$. Thus, $f(x) = f_{a(i)}(x) \subseteq V_i$. On the other hand, (11) shows that $f_{a(i)} \pi_{\beta}(x) \subseteq V_i$. Since $V_i$ is convex and is lying in $V$, it follows that the segment joining $f(x)$ and $f_{a(i)} \pi_{\beta}(x)$ is contained in $V_i$ and thus in $V$ too. In other words, for $\beta \geq a(1), \ldots, a(n)$,
we have a homotopy in $V$ connecting $f(x)$ and $f_\gamma \pi_\beta(x)$. Choose now an $\alpha \geq a(1), \ldots, a(n)$ such that all $X_\beta, \beta \leq M_\alpha$, lie in $U_{a(1)}^* \cup \ldots \cup U_{a(n)}^* \subset f_\alpha^{-1}(V)$; this is possible due to Theorem 2. Now define

$$f_\beta = \Theta I_{\alpha} | X_\beta, \beta \leq M_\alpha. \quad (12)$$

We have obtained already a homotopy, connecting $f$ and $f_\gamma \pi_\beta$ in $V$, for all $\beta \leq M_\alpha$. Composing this homotopy with the retraction $\Theta$, we now get a homotopy connecting $f$ and $\Theta f_\gamma \pi_\beta = f_\beta \pi_\beta$ in $R$. A similar argument shows that $f_\beta \pi_\gamma$ and $f_\gamma$ are homotopic in $R$, for all $\gamma \geq \beta \geq \alpha$.

**Theorem 5.** Let $\{X_\alpha, \pi_\beta\}$ and $\{Y_\alpha, \sigma_\beta\}$, $\alpha \leq M$, be two inverse systems of Hausdorff compacta and let $X_\alpha \subset Y_\alpha, \sigma_\beta | X_\beta = \pi_\beta \sigma_\beta$; let $X \subset Y$ be the corresponding limits. Let $R$ be an ANR and let, for a fixed $\alpha \leq M$, $f_\alpha : X_\alpha \rightarrow R$ be a given mapping such that $f_\alpha \pi_\alpha : X \rightarrow R$ is extendible to $Y$. Then there is a $\beta \leq M_\alpha$ such that $f_\alpha \pi_\beta : X_\beta \rightarrow R$ is extendible to $Y_\beta$.

This theorem generalizes Lemma 8, p. 199 of [9]. Disposing of Theorem 2 and other properties of the spaces $X^*$ and $Y^*$ it is easy to carry on the necessary modifications in the proof given in [9] in order to obtain a proof of Theorem 5. Notice in particular that the space $X^*_\alpha$, defined in (8), is a closed subset of the corresponding space $Y^*_\alpha$. Furthermore, let $\pi_\alpha^* : X^*_\alpha \rightarrow X_\alpha$ be a mapping coinciding with $\pi_\alpha$ on $X$, and coinciding with $\pi_\alpha$ on $X$. The fact that the sets (2) are open in $X^*$ insures the continuity of $\pi_\alpha^*$.

### 3. Continuity Theorem for Homology of Function Spaces

Let $X$ be a Hausdorff compact and $Y$ a metrizable space. We denote by $\langle X, Y \rangle$ the space of all continuous mappings $f : X \rightarrow Y$; $\langle X, Y \rangle$ is given the compact-open topology (e.g., see [7], p. 221). If $X'$ is another Hausdorff compact and $g : X' \rightarrow X$ is a mapping, then the transformation $G : \langle X, Y \rangle \rightarrow \langle X', Y \rangle$ defined by

$$G(f) = fg, \quad (13)$$

i.e. by composing $f$ and $g$. If $C'$ is a closed subset of $X'$, then $C'$ and $g(C')$ are compact. If $U$ is an open set of $Y$, then

$$G^{-1} \{ f' | f' \subset \langle X', Y \rangle, f'(C') \subset U \} = \{ f | f \subset \langle X, Y \rangle, f g(C') \subset U \}. \quad (14)$$

This shows that $G$ is continuous.

Now consider an inverse system of Hausdorff compact spaces $\{X_\alpha, \pi_\beta\}$, $\alpha \leq M$, and a metrizable space $Y$. Let $\Pi_{a, b} : \langle X_\alpha, Y \rangle \rightarrow \langle X_\beta, Y \rangle$ be the induced mappings. Let $H_q(\langle X_\alpha, Y \rangle, G)$ denote the $q$-th singular homology group of $\langle X_\alpha, Y \rangle$ with coefficients in the group $G$ and let $\Pi_{a, b}$ be the homomorphism induced by $\Pi_{a, b}$. Then $\{H_q(\langle X_\alpha, Y \rangle, G), \Pi_{a, b}\}$, $a \leq M$, is a direct system of groups. Furthermore, if $X$ is the limit of $X_\alpha$ then the mappings $\pi_\alpha : X \rightarrow X_\alpha$...
induce mappings \( \Pi_a : \langle X_a, Y \rangle \to \langle X, Y \rangle \) and we have homomorphisms 
\[ \Pi_a \ast : X_a(\langle X_a, Y \rangle, G) \to H_0(\langle X, Y \rangle, G), \]
which induce a natural homomorphism \( \pi \) of the direct limit of \( H_0(\langle X_a, Y \rangle, G) \) into \( H_0(\langle X, Y \rangle, G) \).

**Theorem 6.** Let \( \{ X_a, \pi_{\theta a} \}, \alpha \leq \mathcal{M}, \) be an inverse system of Hausdorff compacta and let \( R \) be an absolute neighborhood retract. Then \( \pi \) establishes a natural isomorphism between the direct limit of \( H_0(\langle X_a, R \rangle, G) \) and the group \( H_0(\langle X, R \rangle, G) \), where \( X \) is the inverse limit of \( \{ X_a, \pi_{\theta a} \} \) and \( G \) is any group of coefficients (the homology is taken in the sense of singular theory).

The proof is carried on first by interpreting singular homology of the mapping spaces \( \langle X, R \rangle \) as \( X \)-homology of \( R \), in the sense of [9] (see I. 4, p. 190). Obvious modifications of the arguments on p. 200–202 of [9] give a proof of Theorem 6. Notice that the Lemma of Abe and Lemma 8 of [9] have to be replaced by the above Theorems 4 and 5.

**REFERENCES:**


O INVERZNIM LIMESIMA KOMPAKTNIH PROSTORA

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**Sadržaj**

U ovom članku se promatraju inverzni1) sistemi \( \{ X_a, \pi_{\theta a} \} \) Hausdorffovih kompaktnih prostora \( X_a \) i to nad proizvoljnim usmjerenim skupovima \( M = \{ \alpha \} \). Relacijom (1) se uvodi u razmatranje skup sastavljen od svih članova sistema (koje smatramo disjunktnima)

1) Osnovne definicije i svojstva inverznih sistema izloženi su na pr. u [5] i [8].
i od graničnog skupa $X$. U skup $X^*$ se uvodi topologija time, što se definira jedna baza otvorenih skupova $\mathcal{U}$ na ovaj način. $\mathcal{U}$ se sastoji iz svih skupova $U_e \subseteq X_e$, koji su otvoreni u $X_e$, $e \in M$, te iz svih skupova oblika (2); pri tome je $\pi_e : X \to X_e$ prirodno preslikavanje, koje pripada promatranom sistemu.

Pokazuje se više svojstava prostora $X^*$. Napose se pokazuje da svaki otvoren skup $U$ iz $X^*$, koji sadrži $X$, sadrži i sve $X_\beta$, počevši od nekog dovoljno velikog $\alpha \leq M$ (Theorem 2.). Kao posljedica dobiva se da je $X^*$ Hausdorffov i parakompaktan. Ove činjenice omogućuju da se primijeni jedan teorem R. Arensa o proširivanju neprekidnih preslikavanja, koja su definirana na nekom zatvorenom dijelu nekog parakompaktog prostora, a vrijednosti tim leže u nekom konveksnom dijelu nekog Banachovog prostora. Služeći se tim teoremom dokazuje se na primjer ovo (Theorem 4):

Neka je $\{X_e, \pi_\beta\}$ jedan inverzni sistem Hausdorffovih kompakata, neka je $R$ jedan apsolutni okolinski retrakt (za metričke prostore) i neka je dano neprekidno preslikavanje $f : X \to R$. Tada postoji $\alpha \leq M$ sa svojstvom da je, za svaki $\beta \geq \alpha$, moguće definirati jedno neprekidno preslikavanje $f_\beta : X_\beta \to R$ i to na takav način, da je preslikavanje $f_\beta \pi_\beta$ homotopno sa $f$, dok je $f_\beta \pi_\beta$ homotopno sa $f_\alpha$ za sve $\gamma \geq \beta \geq \alpha$.

Služeći se ovim i još jednim sličnim rezultatom (Theorem 5) dokazuje se glavni rezultat radnje:

Neka je $R$ jedan apsolutni okolinski retrakt a $\langle X_e, R \rangle$ i $\langle X, R \rangle$ neka su prostori svih neprekidnih preslikavanja od $X_e$ u $R$, odnosno od $X$ u $R$. Neka je $G$ neka Abelova grupa, a $H_q(Y, G)$ neka označuje q-dimenzionalnu singularnu grupu homologije prostora $Y$ s koeficijentima u $G$. Tada inverznom sistemu $\{X_e, \pi_\beta\}$ pripada direktni sistem grupa $\{H_q(\langle X_e, R \rangle, G)\}$. Direktni limes ovog sistema je grupa izomorfnog grupe $H_q(\langle X, R \rangle, G)$.

Ovaj teorem, dakle, pokazuje da je funktor homologije funkcionalnog prostora $\langle X_e, R \rangle$ neprekidan s obzirom na prijelaz varijable $X_e$ na inverznu granicu. Time je dobiveno poopćenje jednog teorema iz autorove disertacije (vidi Theorem 13, str. 200 u [9]) i riješen je Problem 1, koji se tamo navodi ([9], str. 202).

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