A NOTE ON REPRESENTATIONS OF SOME AFFINE
VERTEX ALGEBRAS OF TYPE $D$

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Abstract. In this note we construct a series of singular vectors in
universal affine vertex operator algebras associated to $D^{(1)}_\ell$ of levels $n-\ell+1$,
for $n \in \mathbb{Z}_{>0}$. For $n = 1$, we study the representation theory of the quotient
vertex operator algebra modulo the ideal generated by that singular vector.
In the case $\ell = 4$, we show that the adjoint module is the unique irreducible
ordinary module for simple vertex operator algebra $L_{D_4}(-2,0)$. We also
show that the maximal ideal in associated universal affine vertex algebra
is generated by three singular vectors.

1. Introduction

The classification of irreducible modules for simple vertex operator
algebra $L_\Phi(k,0)$ associated to affine Lie algebra $\Phi$ of level $k$ is still an open
problem for general $k \in \mathbb{C}$ ($k \neq -h^\vee$). This problem is connected with the
description of the maximal ideal in the universal affine vertex algebra $N_\Phi(k,0)$.

One approach to this classification problem is through construction of singular
vectors in $N_\Phi(k,0)$.

The known (non-generic) cases include positive integer levels (cf. [14,
19, 20]) and some special cases of rational admissible levels, in the sense of
Kac and Wakimoto ([17]) (cf. [1, 3, 4, 6, 9, 21, 22]). It turns out that negative
integer levels also have some interesting properties. They appeared in bosonic
realizations in [10], and also recently in the context of conformal embeddings
(cf. [5]).

In this note we study a vertex operator algebra associated to affine Lie
algebra of type $D^{(1)}_\ell$ and negative integer level $-\ell+2$. This level appeared in

2010 Mathematics Subject Classification. 17B69, 17B67, 81R10.

Key words and phrases. Vertex operator algebra, affine Kac-Moody algebra, Zhu’s
algebra.
in the context of conformal embedding of $L_{B_{-1}}(-\ell + 2, 0)$ into $L_{D_1}(-\ell + 2, 0)$. This conformal embedding is in some sense similar to the conformal embedding of $L_{D_1}(-\ell + \frac{3}{2}, 0)$ into $L_{B_{-1}}(-\ell + \frac{3}{2}, 0)$.

We will show that there are also similarities in singular vectors in universal affine vertex algebras $N_{B_{\ell}}(-\ell + \frac{3}{2}, 0)$ (studied in [21]) and $N_{D_{\ell}}(-\ell + 2, 0)$. More generally, we construct a series of singular vectors

$$v_n = \left( \sum_{i=2}^{\ell} e_{\xi_i - \xi_i}(-1)e_{\xi_i + \xi_i}(-1) \right)^n 1$$

in $N_{D_{\ell}}(n - \ell + 1, 0)$, for any $n \in \mathbb{Z}_{>0}$. For $n = 1$, we study the representation theory of the quotient $N_{D_{\ell}}(-\ell + 2, 0)$ modulo the ideal generated by $v_1$. Using the methods from [1, 2, 4, 20], we obtain the classification of irreducible weak modules in the category $\mathcal{O}$ for that vertex algebra. It turns out that there are infinitely many of these modules.

In the special case $\ell = 4$, we obtain the classification of irreducible weak modules from the category $\mathcal{O}$ for simple vertex operator algebra $L_{D_4}(-2, 0)$. This vertex algebra also appeared in [5] in the context of conformal embedding of $L_{G_2}(-2, 0)$ into $L_{D_4}(-2, 0)$. It follows that there are finitely many irreducible weak $L_{D_4}(-2, 0)$–modules from the category $\mathcal{O}$, that the adjoint module is the unique irreducible ordinary $L_{D_4}(-2, 0)$–module, and that every ordinary $L_{D_4}(-2, 0)$–module is completely reducible. We also show that the maximal ideal in $N_{D_4}(-2, 0)$ is generated by three singular vectors.

2. Preliminaries

We assume that the reader is familiar with the notion of vertex operator algebra (cf. [7, 11–14, 16, 18, 19]) and Kac-Moody algebra (cf. [15]).

Let $V$ be a vertex operator algebra. Denote by $A(V)$ the associative algebra introduced in [23], called the Zhu’s algebra of $V$. As a vector space, $A(V)$ is a quotient of $V$, and we denote by $[a]$ the image of $a \in V$ under the projection of $V$ onto $A(V)$. We recall the following fundamental result from [23]:

**Proposition 2.1.** The equivalence classes of the irreducible $A(V)$–modules and the equivalence classes of the irreducible $\mathbb{Z}_+$–graded weak $V$–modules are in one-to-one correspondence.

Let $\mathfrak{g}$ be a simple Lie algebra with a triangular decomposition $\mathfrak{g} = \mathfrak{n}_- \oplus \mathfrak{h} \oplus \mathfrak{n}_+$, and $\hat{\mathfrak{g}}$ the (untwisted) affine Lie algebra associated to $\mathfrak{g}$. Denote by $V(\mu)$ the irreducible highest weight $\mathfrak{g}$–module with highest weight $\mu$, and by $L(k, \mu)$ the irreducible highest weight $\hat{\mathfrak{g}}$–module with highest weight $k\Lambda_0 + \mu$.

Furthermore, denote by $N(k, 0)$ (or $N_{\hat{\mathfrak{g}}}(k, 0)$) the universal affine vertex algebra associated to $\mathfrak{g}$ of level $k \in \mathbb{C}$. For $k \neq -\hbar^\vee$, $N(k, 0)$ is a vertex operator algebra with Segal-Sugawara conformal vector, and $L(k, 0)$ is a
simple vertex operator algebra. The Zhu’s algebra of $N(k,0)$ was determined in [14]:

Proposition 2.2. The associative algebra $A(N(k,0))$ is canonically isomorphic to $U(\mathfrak{g})$. The isomorphism is given by $F: A(N(k,0)) \rightarrow U(\mathfrak{g})$

$$F([x_1(-n_1-1)\cdots x_m(-n_m-1)1]) = (-1)^{n_1+\cdots+n_m}x_m\cdots x_1,$$

for any $x_1, \ldots, x_m \in \mathfrak{g}$ and any $n_1, \ldots, n_m \in \mathbb{Z}^+.$

We have:

Proposition 2.3. Assume that a $\hat{\mathfrak{g}}$–submodule $J$ of $N(k,0)$ is generated by $m$ singular vectors $(m \in \mathbb{Z}_{>0})$, i.e. $J = U(\hat{\mathfrak{g}})v^{(1)} + \cdots + U(\hat{\mathfrak{g}})v^{(m)}$. Then

$$A(N(k,0)/J) \cong U(\mathfrak{g})/I,$$

where $I$ is the two-sided ideal of $U(\mathfrak{g})$ generated by $u^{(1)} = F([v^{(1)}]), \ldots, u^{(m)} = F([v^{(m)}]).$

Let $J = U(\hat{\mathfrak{g}})v^{(1)} + \cdots + U(\hat{\mathfrak{g}})v^{(m)}$ be a $\hat{\mathfrak{g}}$–submodule of $N(k,0)$ generated by singular vectors $v^{(1)}, \ldots, v^{(m)}$. Now we recall the method from [1,2,4,20] for the classification of irreducible $A(N(k,0)/J)$–modules from the category $\mathcal{O}$ by solving certain systems of polynomial equations.

Denote by $L$ the adjoint action of $U(\mathfrak{g})$ on $U(\mathfrak{g})$ defined by $X_Lf = [X,f]$ for $X \in \mathfrak{g}$ and $f \in U(\mathfrak{g})$. Let $R^{(j)}$ be a $U(\mathfrak{g})$–submodule of $U(\mathfrak{g})$ generated by the vector $u^{(j)} = F([v^{(j)}])$ under the adjoint action, for $j = 1, \ldots, m$. Clearly, $R^{(j)}$ is an irreducible highest weight $U(\mathfrak{g})$–module. Let $R_0^{(j)}$ be the zero-weight subspace of $R^{(j)}$.

The next proposition follows from [1,4,20]:

Proposition 2.4. Let $V(\mu)$ be an irreducible highest weight $U(\mathfrak{g})$–module with the highest weight vector $v_\mu$, for $\mu \in \mathfrak{h}^*$. The following statements are equivalent:

1. $V(\mu)$ is an $A(N(k,0)/J)$–module,
2. $R^{(j)} V(\mu) = 0$, for every $j = 1, \ldots, m,$
3. $R_0^{(j)} v_\mu = 0$, for every $j = 1, \ldots, m$.

Let $r \in R_0^{(j)}$. Clearly there exists the unique polynomial $p_r \in S(\mathfrak{h})$ such that

$$r v_\mu = p_r(\mu) v_\mu.$$

Set $T^{(j)}_0 = \{ p_r \mid r \in R_0^{(j)} \}$, for $j = 1, \ldots, m$. We have:

Corollary 2.5. There is one-to-one correspondence between

1. irreducible $A(N(k,0)/J)$–modules from the category $\mathcal{O}$,
2. weights $\mu \in \mathfrak{h}^*$ such that $p(\mu) = 0$ for all $p \in T^{(j)}_0$, for every $j = 1, \ldots, m$. 

In the case $m = 1$, we use the notation $R$, $R_0$ and $P_0$ for $R^{(1)}$, $R_0^{(1)}$ and $P_0^{(1)}$, respectively.

3. Vertex operator algebra associated to $D^{(1)}_\ell$ of level $-\ell + 2$

In this section we study the representation theory of the quotient of universal affine vertex operator algebra associated to $D^{(1)}_\ell$ of level $-\ell + 2$, modulo the ideal generated by a singular vector of conformal weight two.

Denote by $g$ the simple Lie algebra of type $D_\ell$. We fix the root vectors for $g$ as in [8, 10]. We have:

**Theorem 3.1.** Vector

$$v_n = \left( \sum_{i=2}^\ell e_{\epsilon_1 - \epsilon_i}(-1)e_{\epsilon_1 + \epsilon_i}(-1) \right)^n 1$$

is a singular vector in $N_{D_\ell}(n - \ell + 1, 0)$, for any $n \in \mathbb{Z}_{>0}$.

**Proof.** Direct verification of relations $e_{t_{k-\ell+1}}(0)v_n = 0$, for $k = 1, \ldots, \ell - 1$, $e_{t_{\ell-1}+\epsilon_i}(0)v_n = 0$ and $f_{\epsilon_1+\epsilon_2}(1)v_n = 0$.

In the case $n = 1$, we obtain the singular vector

$$v = \sum_{i=2}^\ell e_{\epsilon_1 - \epsilon_i}(-1)e_{\epsilon_1 + \epsilon_i}(-1)1$$

in $N_{D_\ell}(-\ell + 2, 0)$.

**Remark 3.2.** Vector $v$ from relation (3.1) has a similar formula as singular vector

$$\frac{1}{4} e_{\epsilon_1}(-1)^2 1 + \sum_{i=2}^\ell e_{\epsilon_1 - \epsilon_i}(-1)e_{\epsilon_1 + \epsilon_i}(-1)1$$

for $B^{(1)}_\ell$ in $N_{D_\ell}(-\ell + \frac{3}{2}, 0)$. The representation theory of the quotient of $N_{B_\ell}(-\ell + \frac{3}{2}, 0)$ modulo the ideal generated by that vector was studied in [21].

We will consider representations of the vertex operator algebra

$$V_{D_\ell}(-\ell + 2, 0) = \frac{N_{D_\ell}(-\ell + 2, 0)}{U(g)v}.$$ 

Proposition 2.3 gives:

**Proposition 3.3.** The associative algebra $A(V_{D_\ell}(-\ell + 2, 0))$ is isomorphic to the algebra $U(g)/I$, where $I$ is the two-sided ideal of $U(g)$ generated by

$$u = \sum_{i=2}^\ell e_{\epsilon_1 - \epsilon_i}e_{\epsilon_1 + \epsilon_i}.$$
We have the following classification:

**Theorem 3.4.** For any subset $S = \{i_1, \ldots, i_k\} \subseteq \{1, 2, \ldots, \ell - 2\}$, $i_1 < \cdots < i_k$, and $t \in \mathbb{C}$, we define weights

$$\mu_{S,t} = \sum_{j=1}^{k} \left( i_j + 2 \sum_{s=j+1}^{k} (-1)^{s-j} i_s + (-1)^{k-j+1} (t + \ell - 1) \right) \omega_{ij} + t \omega_{t-1},$$

$$\mu'_{S,t} = \sum_{j=1}^{k} \left( i_j + 2 \sum_{s=j+1}^{k} (-1)^{s-j} i_s + (-1)^{k-j+1} (t + \ell - 1) \right) \omega_{ij} + t \omega_{t},$$

where $\omega_1, \ldots, \omega_t$ are fundamental weights for $g$. Then the set

$$\{L_{D_i}(-\ell + 2, \mu_{S,t}), \ L_{D_i}(-\ell + 2, \mu'_{S,t}) \mid S \subseteq \{1, 2, \ldots, \ell - 2\}, t \in \mathbb{C} \}$$

provides the complete list of irreducible weak $V_{D_i}(-\ell + 2, 0)$-modules from the category $O$.

**Proof.** We use the method for classification of irreducible $A(V_{D_i}(-\ell + 2, 0))$-modules in the category $O$ from Corollary 2.5. In this case $R \cong V_{D_i}(2\omega_1)$, and similarly as in [21, Lemma 28] one obtains that

$$\dim R_0 = \ell - 1.$$

Furthermore, one obtains by direct calculation that

$$(f_{e_{i-\ell}} f_{e_{i+\ell}})_{L} u \in p_i(h) + U(g)n_+, \quad (f_{e_{i-\ell+1}} f_{e_{i+\ell+1}} - f_{e_{i-\ell}} f_{e_{i+\ell}})_{L} u \in p_i(h) + U(g)n_+, \ i = 2, \ldots, \ell - 1,$$

where

$$p_i(h) = h_i(h_{e_{i+\ell+1}}+\ell - i - 1), \quad \text{for} \ i = 1, \ldots, \ell - 1$$

are linearly independent polynomials in $p_0$. Here $h_i$ $(i = 1, \ldots, \ell)$ denote the simple coroots for $g$ and

$$h_{e_{i+\ell+1}} = h_i + 2h_{i+1} + \ldots + 2h_{\ell-2} + h_{\ell-1} + h_{\ell}, \quad \text{for} \ i < \ell - 1.$$

Corollary 2.5 now implies that the highest weights of irreducible $A(V_{D_i}(-\ell + 2, 0))$-modules from the category $O$ are given as solutions of polynomial equations

$$p_i(h) = 0, \ i = 1, \ldots, \ell - 1.$$

First we note that for $i = \ell - 1$, we obtain the equation

$$h_{\ell-1} h_{\ell} = 0.$$

Thus, either $h_{\ell-1} = 0$ or $h_{\ell} = 0$. Assume first that $h_{\ell-1} = 0$, and let $S = \{i_1, \ldots, i_k\}$, $i_1 < \ldots < i_k$ be the subset of $\{1, 2, \ldots, \ell - 2\}$ such that
We conclude that the set provides the complete list of irreducible ordinary modules from the category $\mathcal{O}$. The claim of theorem now follows from Zhu’s theory.

**Example 3.5.** For $\ell = 4$, we have subsets $S = \emptyset, \{1\}, \{2\}, \{1, 2\}$ of the set $\{1, 2\}$, so we obtain that the set

\[
\{L_D(\ell + 2, t\omega_1), L_D(\ell + 2, t\omega_2), L_D(\ell + 2, -(2 - t)\omega_1 + t\omega_3),
L_D(\ell + 2, -(2 - t)\omega_1 + t\omega_3), L_D(\ell + 2, (-1 - t)\omega_2 + t\omega_3),
L_D(\ell + 2, (-1 - t)\omega_2 + t\omega_3)\mid t \in \mathbb{C}\}
\]

(3.5)

provides the complete list of irreducible weak $V_D(\ell - 2, 0)$-modules from the category $\mathcal{O}$.

Recall that a module for vertex operator algebra is called ordinary if $L(0)$ acts semisimply with finite-dimensional weight spaces. We have:

**Corollary 3.6.** The set

\[
\{L_D(\ell + 2, t\omega_{r-1}), L_D(\ell + 2, t\omega_r)\mid t \in \mathbb{Z}_{\geq 0}\}
\]

provides the complete list of irreducible ordinary $V_D(\ell - 2, 0)$-modules.

**Proof.** If $L_D(\ell + 2, \mu)$ is an ordinary $V_D(\ell + 2, 0)$-module, then $\mu$ is a dominant integral weight. Then $\mu(h_{\epsilon, \tau_{i+1}}) \in \mathbb{Z}_{\geq 0}$, for $i = 1, \ldots, \ell - 1$. Relations (3.2) and (3.3) then give that

\[
\mu(h_i) = 0, \quad \text{for } i = 1, \ldots, \ell - 2.
\]
and \( \mu(h_{t-1}) = 0 \) or \( \mu(h_t) = 0 \). Thus, \( \mu = t\omega_{t-1} \) or \( \mu = t\omega_t \), and \( t \in \mathbb{Z}_{\geq 0} \) since \( \mu \) is a dominant integral weight.

It follows that:

**Corollary 3.7.** The set of irreducible ordinary \( L_{D_4}(-\ell + 2, 0) \)-modules is a subset of the set

\[
\{ L_{D_4}(-\ell + 2, t\omega_{t-1}), L_{D_4}(-\ell + 2, t\omega_t) \mid t \in \mathbb{Z}_{\geq 0} \}.
\]

4. Case \( \ell = 4 \)

In this section we study the case \( \ell = 4 \). We determine the classification of irreducible weak \( L_{D_4}(-2, 0) \)-modules from the category \( \mathcal{O} \). It turns out that there are finitely many of these modules and that the adjoint module is the unique irreducible ordinary \( L_{D_4}(-2, 0) \)-module. We also show that the maximal ideal in \( N_{D_4}(-2, 0) \) is generated by three singular vectors.

Denote by \( \theta \) the automorphism of \( N_{D_4}(-2, 0) \) induced by the automorphism of the Dynkin diagram of \( D_4 \) of order three such that

\[
\theta(e_1 - e_2) = e_3 - e_4, \quad \theta(e_2 - e_3) = e_2 - e_3, \\
\theta(e_3 - e_4) = e_3 + e_4, \quad \theta(e_3 + e_4) = e_1 - e_2.
\]

Relation (3.1) implies that

\[
v = (e_{e_1-e_2}(-1)e_{e_1+e_2}(-1) + e_{e_1-e_3}(-1)e_{e_1+e_3}(-1) + e_{e_1-e_4}(-1)e_{e_1+e_4}(-1))1
\]

is a singular vector in \( N_{D_4}(-2, 0) \). Furthermore,

\[
\theta(v) = (e_{e_3-e_4}(-1)e_{e_3+e_4}(-1) - e_{e_2-e_4}(-1)e_{e_1+e_3}(-1) \\
+ e_{e_2+e_3}(-1)e_{e_1-e_4}(-1))1,
\]

and

\[
\theta^2(v) = (e_{e_3+e_4}(-1)e_{e_3+e_2}(-1) - e_{e_2+e_4}(-1)e_{e_1+e_3}(-1) \\
+ e_{e_1+e_4}(-1)e_{e_2+e_3}(-1))1
\]

are also singular vectors in \( N_{D_4}(-2, 0) \). We consider the vertex operator algebra

\[
\tilde{L}_{D_4}(-2, 0) = \frac{N_{D_4}(-2, 0)}{J},
\]

where \( J \) is the ideal in \( N_{D_4}(-2, 0) \) generated by vectors \( v, \theta(v) \) and \( \theta^2(v) \).

Proposition 2.3 gives that the associative algebra \( A(\tilde{L}_{D_4}(-2, 0)) \) is isomorphic to the algebra \( U(\mathfrak{g})/I \), where \( I \) is the two-sided ideal of \( U(\mathfrak{g}) \) generated by \( u, \theta(u) \) and \( \theta^2(u) \), and

\[
u = e_{e_1-e_2}e_{e_1+e_2} + e_{e_1-e_3}e_{e_1+e_3} + e_{e_1-e_4}e_{e_1+e_4}.
\]

**Proposition 4.1.** We have:
The set
\[ \{ L_{D_4}(-2,0), L_{D_4}(-2,-2\omega_1), L_{D_4}(-2,-2\omega_3), L_{D_4}(-2,-2\omega_4), L_{D_4}(-2,-\omega_2) \} \]
provides a complete list of irreducible weak \( \tilde{L}_{D_4}(-2,0) \)-modules from the category \( \mathcal{O} \).

(ii) \( L_{D_4}(-2,0) \) is the unique irreducible ordinary module for \( \tilde{L}_{D_4}(-2,0) \).

Proof. (i) We use the method for classification from Corollary 2.5. In this case \( R^{(1)} \cong V_{D_4}(2\omega_1), R^{(2)} \cong V_{D_4}(2\omega_3), R^{(3)} \cong V_{D_4}(2\omega_4) \) and
\[ \dim R^{(1)}_0 = \dim R^{(2)}_0 = \dim R^{(3)}_0 = 3. \]
Using polynomials from relation (3.2) and automorphisms \( \theta \) and \( \theta^2 \), one obtains that the highest weights \( \mu \) of \( \tilde{L}_{D_4}(-2,0) \)-modules \( V_{D_4}(\mu) \) are obtained as solutions of these 9 polynomial equations:
\[
\begin{align*}
&h\epsilon_1 - \epsilon_2 (h\epsilon_1 + \epsilon_2 + 2) = 0, \\
&h\epsilon_2 - \epsilon_3 (h\epsilon_2 + \epsilon_3 + 1) = 0, \\
&h\epsilon_3 - \epsilon_4 h\epsilon_3 + \epsilon_4 = 0, \\
&h\epsilon_3 - \epsilon_4 (h\epsilon_1 + \epsilon_2 + 2) = 0, \\
&h\epsilon_2 - \epsilon_3 (h\epsilon_1 + \epsilon_4 + 1) = 0, \\
&h\epsilon_3 + \epsilon_4 h\epsilon_1 - \epsilon_2 = 0, \\
&h\epsilon_3 + \epsilon_4 (h\epsilon_1 + \epsilon_2 + 2) = 0, \\
&h\epsilon_2 - \epsilon_3 (h\epsilon_1 - \epsilon_4 + 1) = 0, \\
&h\epsilon_1 - \epsilon_2 h\epsilon_3 - \epsilon_4 = 0.
\end{align*}
\]
This easily gives that \( \mu = 0, -2\omega_1, -2\omega_3, -2\omega_4 \) or \( -\omega_2 \), and the claim follows from Zhu’s theory.

Claim (ii) follows from the fact that \( \mu = 0 \) is the only dominant integral weight such that \( L_{D_4}(-2,\mu) \) is in the set given in the claim (i).

We have:

**Theorem 4.2.** Vertex operator algebra \( \tilde{L}_{D_4}(-2,0) \) is simple, i.e.,
\[ L_{D_4}(-2,0) = \frac{N_{D_4}(-2,0)}{U(\hat{g}).v + U(\hat{g}).\theta(v) + U(\hat{g}).\theta^2(v)}. \]

Proof. Let \( w \) be a singular vector for \( \hat{g} \) in \( \tilde{L}_{D_4}(-2,0) \). The classification result from Proposition 4.1 (ii) implies that \( U(\hat{g}).w \) is a highest weight \( \hat{g} \)-module with highest weight \( -2\Lambda_0 \) and that \( w \) is proportional to \( 1 \). The claim follows.

We conclude:
Theorem 4.3. \( (i) \) The set
\[
\{ L_{D_4}(-2,0), L_{D_4}(-2,-2\omega_1), L_{D_4}(-2,-2\omega_3), L_{D_4}(-2,-2\omega_4), L_{D_4}(-2,-\omega_2) \}
\]
provides a complete list of irreducible weak \( L_{D_4}(-2,0) \)-modules from the category \( \mathcal{O} \).

\( (ii) \) \( L_{D_4}(-2,0) \) is the unique irreducible ordinary module for \( L_{D_4}(-2,0) \).

\( (iii) \) Every ordinary \( L_{D_4}(-2,0) \)-module is completely reducible.

**Proof.** Proposition 4.1 and Theorem 4.2 imply claims (i) and (ii).

(iii) Let \( M \) be an ordinary \( L_{D_4}(-2,0) \)-module, and let \( w \) be a singular vector for \( \hat{\mathfrak{g}} \) in \( M \). The classification result from (ii) implies that \( U(\hat{\mathfrak{g}}).w \) is a highest weight \( \hat{\mathfrak{g}} \)-module with highest weight \( -2\Lambda_0 \). Claim (ii) also implies that any singular vector in \( U(\hat{\mathfrak{g}}).w \) has highest weight \( -2\Lambda_0 \) and it is proportional to \( w \). Thus, \( U(\hat{\mathfrak{g}}).w \) is an irreducible \( \hat{\mathfrak{g}} \)-module and the claim follows.

Acknowledgements.
The author thanks Dražen Adamović for his helpful advice and valuable suggestions.

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Received: 15.5.2012.