

MAP OF QUASICOMPONENTS INDUCED BY A SHAPE MORPHISM

NIKITA SHEKUTKOVSKI, TATJANA ATANASOVA-PACHEMSKA AND GJORGJI MARKOSKI

Ss. Cyril and Methodius University and University "Goce Delchev" - Shtip,
Republic of Macedonia

ABSTRACT. Using the intrinsic definition of shape we prove an analogue of well known Borsuks theorem for compact metric spaces.

Suppose X and Y are locally compact metric spaces with compact spaces of quasicomponents QX and QY . For a shape morphism $f : X \rightarrow Y$ there exists a unique continuous map $f_{\#} : QX \rightarrow QY$, such that for a quasicomponent Q from X and W a clopen set containing $f_{\#}(Q)$ the restriction $f : Q \rightarrow W$, is a shape morphism, also.

1. INTRODUCTION

This is the well known theorem of Borsuk about the set of components $\square X$ of a compact metric space X .

THEOREM 1.1 (Borsuk). *Let X and Y be compact metric spaces. Then for any approximative map f from X towards Y , there exists an unique map $f^{\wedge} : \square X \rightarrow \square Y$ such that for any component C_0 of X , the restriction $f|_{C_0}$ is an approximative map from C_0 to $f^{\wedge}(C_0)$.*

Moreover, if f and g define the same shape morphism, then $f^{\wedge} = g^{\wedge}$.

There are several generalizations of the theorem of Borsuk in non compact case. For the case of spaces with compact components see [2–4, 8, 12] and under conditions $\square X$, $\square Y$ to be \mathbb{N} -compact spaces and projections $p : X \rightarrow \square X$, $p : Y \rightarrow \square Y$ to be closed, see [9, Corollary 1.5.].

For non compact spaces, it is expected that we have to use quasicomponents instead of components.

2010 *Mathematics Subject Classification.* 54C56, 55P55, 54C08.

Key words and phrases. Intrinsic definition, continuity up to a covering, proximate sequence, proximate net, quasicomponents.

In [11], it is shown that if $Sh(X) = Sh(Y)$ then \mathbb{N} -compactifications of QX and of QY are homeomorphic. A similar result is given in [10].

Using the intrinsic definition of shape, we give an analogue of the Borsuks theorem in the case of noncompact spaces under condition QX to be compact.

The most known approach for the intrinsic definition of shape of (metric) spaces is by use of functions $f : X \rightarrow Y$ which are near to continuous. The idea of ε -continuity (continuity up to $\varepsilon > 0$) leads to a continuity up to some covering \mathcal{V} of Y , i.e., \mathcal{V} -continuity, and corresponding \mathcal{V} -homotopy.

DEFINITION 1.2. *Let X, Y , be spaces, and \mathcal{V} be a covering of Y . A function $f : X \rightarrow Y$ is \mathcal{V} -continuous, if for any $x \in X$, there exists a neighborhood U of x , such that $f(U) \subseteq V$, for some member $V \in \mathcal{V}$.*

The family of all U , form a covering \mathcal{U} of X . Shortly, we say that $f : X \rightarrow Y$ is \mathcal{V} -continuous, if there exists \mathcal{U} such that $f(\mathcal{U}) \prec \mathcal{V}$.

DEFINITION 1.3. *Two \mathcal{V} -continuous functions $f, g : X \rightarrow Y$ are \mathcal{V} -homotopic, if there exists a function $F : X \times I \rightarrow Y$ such that:*

- 1) $F : X \times I \rightarrow Y$ is $st\mathcal{V}$ -continuous;
- 2) There exists a neighborhood $N = [0, \varepsilon) \cup (1 - \varepsilon, 1]$ of $\{0, 1\}$ in $[0, 1]$ such that $F|_{X \times N}$ is \mathcal{V} -continuous;
- 3) $F(x, 0) = f(x)$, $F(x, 1) = g(x)$.

The relation of homotopy between \mathcal{V} -continuous functions is an equivalence relation.

REMARK 1.4. The conditions 1) and 2), in the previous definition cannot be replaced by a requirement $F : X \times I \rightarrow Y$ to be a \mathcal{V} -continuous function, since in this case the juxtaposition of homotopies doesn't work and the relation of homotopy wouldn't be an equivalence relation. This is shown by [15, Theorem 1.1] and in the example in [1].

We mention that Akaike and Sakai in the paper [1] introduced an intrinsic definition of proper shape for locally compact separable metric spaces, which is shown to be equivalent with the usual proper shape theory [14]. Omitting the word proper in their definitions we obtain a shape theory.

Also, we omit their additional requirement coverings to consist of open sets with compact closures, since they needed this requirement only for proper functions.

We consider the set $covX$ to be the set of all star finite coverings. The requirement is connected with the requirement of $st\mathcal{V}$ -continuity of a homotopy connecting two \mathcal{V} -continuous functions. We mention that in the case of locally compact spaces any covering has a locally finite refinement iff any covering has a star finite refinement. So, for paracompact locally compact spaces any covering has: 1) locally finite refinement 2) star finite refinement

(i.e both locally finite coverings and star finite coverings are cofinite in the set of all coverings) [6].

We repeat the main notions for the intrinsic definition of shape for metric locally compact spaces.

A *proximate net* $(f_{\mathcal{V}}) : X \rightarrow Y$ is a net of functions $f_{\mathcal{V}} : X \rightarrow Y$, $f_{\mathcal{V}}$ is a \mathcal{V} -continuous function, indexed by all coverings of $\text{cov}Y$, such that if $\mathcal{V} \succ \mathcal{W}$ then $f_{\mathcal{V}}$ and $f_{\mathcal{W}}$ are \mathcal{V} -homotopic.

Two proximate nets $(f_{\mathcal{V}}), (f'_{\mathcal{V}}) : X \rightarrow Y$ are homotopic if $f_{\mathcal{V}}$ and $f'_{\mathcal{V}}$ are \mathcal{V} -homotopic, for all $\mathcal{V} \in \text{cov}Y$ which we denote by $(f_{\mathcal{V}}) \sim (f'_{\mathcal{V}})$.

This is an equivalence relation.

If $(f_{\mathcal{V}}) : X \rightarrow Y$ and $(g_{\mathcal{W}}) : Y \rightarrow Z$ are proximate nets, then for a covering $\mathcal{W} \in \text{cov}Z$, there exists a covering $\mathcal{V} \in \text{cov}Y$ such that $g_{\mathcal{W}}(\mathcal{V}) \prec \mathcal{W}$. Then the *composition* of these two proximate nets is a proximate net $(h_{\mathcal{W}})$ defined by $(h_{\mathcal{W}}) = (g_{\mathcal{W}}f_{\mathcal{V}}) : X \rightarrow Z$.

Spaces and homotopy classes of proximate nets form the the category whose isomorphisms induces classification which coincide with shape classification, i.e., isomorphic spaces in this category have the same shape.

2. QUASICOMPONENTS

Usually, the quasicomponent of a point is defined as an intersection of all clopen (=closed and open) sets containing that point. The component of a point x is contained in the quasicomponent of x .

Here we give another definition of quasicomponents using functional separation.

DEFINITION 2.1. *Subsets A and B of X are said to be functionally separated in X if there exists a continuous function $f : X \rightarrow \{0, 1\}$ such that $f(A) = \{0\}$ and $f(B) = \{1\}$.*

DEFINITION 2.2. *Quasicomponent Q of a point x consists of all points y which cannot be functionally separated from x .*

By $Q(X)$ we denote the set of quasicomponents of X . We define a topology of $Q(X)$ by defining a base for this topology to be the family $QF = \{A | A \in Q(X), A \subseteq F\}$, where F is clopen subset of X . The set $Q(X)$ with this topology is the space of quasicomponents of X . The set QF is clopen in $Q(X)$.

THEOREM 2.3. *If \mathcal{W} is a covering of Y consisting of disjoint open sets (shortly disjoint covering) and $f : X \rightarrow Y$ is a \mathcal{W} -continuous function, then for each quasicomponent Q of X , there exists $W_Q \in \mathcal{W}$ such that $f_{\mathcal{W}}(Q) \subseteq W_Q$*

PROOF. Let $x \in X$, Q is quasicomponent of X and $W_Q \in \mathcal{W}$ such that $f(x) \in W_Q$. We define a continuous map $h : Y \rightarrow \{0, 1\}$ by $h(W_Q) = \{0\}$ and $h(Y \setminus W_Q) = \{1\}$.

We will prove that $hf : X \rightarrow \{0, 1\}$ is continuous.

For an arbitrary point $z \in X$ there exists a neighborhood U_z and there exists $W \in \mathcal{W}$ such that $f(U_z) \subseteq W$.

Now, if $W = W_Q$, then $hf(U_z) \subseteq h(W_Q) \subseteq \{0\}$, while if $W \neq W_Q$, $hf(U_z) \subseteq h(W) \subseteq \{1\}$. It follows that the composition $hf : X \rightarrow \{0, 1\}$ is continuous.

If we suppose that there is a point $y \in Q$ such that $f(y) \notin W_Q$, then x and y will be functionally separated by the map $hf : X \rightarrow \{0, 1\}$. We conclude that $f(Q) \subseteq W_Q$. \square

COROLLARY 2.4. *If \mathcal{W} is a covering consisting of disjoint open sets and $f : X \rightarrow Y$ is a \mathcal{W} -continuous function, then for each component $Q \in \square X$, there exists $W_Q \in \mathcal{W}$ such that $f_{\mathcal{W}}(Q) \subseteq W_Q$.*

THEOREM 2.5. *If \mathcal{W} is a covering consisting of disjoint open sets, and $H : X \times I \rightarrow Y$ is a \mathcal{W} -homotopy connecting \mathcal{W} -continuous functions $f : X \rightarrow Y$ and $g : X \rightarrow Y$, then for each quasicomponent Q of X , there exists $W_Q \in \mathcal{W}$ such that $f(Q) \subseteq W_Q$ and $g(Q) \subseteq W_Q$.*

PROOF. Since $H : X \times I \rightarrow Y$ is $\text{st}\mathcal{W}$ continuous and $\mathcal{W} = \text{st}\mathcal{W}$, it follows that $H : X \times I \rightarrow Y$ is \mathcal{W} -continuous. Similarly as in the previous theorem, there exists $W_Q \in \mathcal{W}$ such that $H(Q \times I) \subseteq W_Q$. Then $f(Q) = H(Q \times \{0\}) \subseteq W_Q$ and $g(Q) = H(Q \times \{1\}) \subseteq W_Q$. \square

COROLLARY 2.6. *If \mathcal{W} is a covering consisting of disjoint open sets, and $H : X \times I \rightarrow Y$ is a \mathcal{W} -homotopy connecting \mathcal{W} -continuous functions $f : X \rightarrow Y$ and $g : X \rightarrow Y$, then for each component $Q \in \square X$, there exists $W_Q \in \mathcal{W}$ such that $f(Q) \subseteq W_Q$ and $g(Q) \subseteq W_Q$.*

THEOREM 2.7. *Let $(f_{\mathcal{V}}) : X \rightarrow Y$ be a proximate net, and \mathcal{W} be a covering consisting of disjoint open sets. Then, for each quasicomponent Q of X , there exists $W_Q \in \mathcal{W}$ such that for every $\mathcal{V} \prec \mathcal{W}$, $f_{\mathcal{V}}(Q) \subseteq W_Q$ (and specially $f_{\mathcal{W}}(Q) \subseteq W_Q$).*

PROOF. By Theorem 2.3, there exists a $W_Q \in \mathcal{W}$ such that $f_{\mathcal{W}}(Q) \subseteq W_Q$. If $\mathcal{V} \prec \mathcal{W}$, then there exists a $\text{st}\mathcal{W}$ -continuous function, $H_{\mathcal{W}\mathcal{V}} : X \times I \rightarrow Y$. Since $\text{st}\mathcal{W} = \mathcal{W}$, $H_{\mathcal{W}\mathcal{V}} : X \times I \rightarrow Y$ is also a \mathcal{W} -continuous function. Since $H_{\mathcal{W}\mathcal{V}}(Q \times \{0\}) = f_{\mathcal{W}}(Q) \subseteq W_Q$, it follows that $H_{\mathcal{W}\mathcal{V}}(Q \times I) \subseteq W_Q$, and that $f_{\mathcal{V}}(Q) = H_{\mathcal{W}\mathcal{V}}(Q \times \{1\}) \subseteq W_Q$. \square

REMARK 2.8. Let \mathcal{V} be a disjoint covering. For every proximate net $(f_{\mathcal{U}}) : X \rightarrow Y$ we can define a proximate net $(f'_{\mathcal{U}}) : X \rightarrow Y$ by putting

$$f'_{\mathcal{U}} = \begin{cases} f_{\mathcal{U}}, & \text{if } \mathcal{U} \prec \mathcal{V} \\ f_{\mathcal{U} \cap \mathcal{V}}, & \text{if not } \mathcal{U} \prec \mathcal{V}. \end{cases}$$

Since $f_{\mathcal{U} \cap \mathcal{V}}$ is $\mathcal{U} \cap \mathcal{V}$ continuous, it follows that it is \mathcal{U} continuous. If $\mathcal{U}' \prec \mathcal{U}$, then $\mathcal{U}' \cap \mathcal{V} \prec \mathcal{U} \cap \mathcal{V}$, and since $f_{\mathcal{U}} = f_{\mathcal{U} \cap \mathcal{V}}$ and $f_{\mathcal{U}'} = f_{\mathcal{U}' \cap \mathcal{V}}$ are $\mathcal{U} \cap \mathcal{V}$ -homotopic, it follows that they are \mathcal{U} -homotopic.

Then, $(f'_{\mathcal{U}})$ is a proximate net, and $(f_{\mathcal{U}})$ and $(f'_{\mathcal{U}})$ are homotopic.

Since restriction of \mathcal{U} -continuous function is \mathcal{U} -continuous, and $f_{\mathcal{U}}(Q) \subseteq W_Q$ for all coverings \mathcal{U} , it follows that the restriction $(f'_{\mathcal{U}}) : Q \rightarrow W_Q$ is a proximate net.

COROLLARY 2.9. *Let $(f_{\mathcal{V}}) : X \rightarrow Y$ be a proximate net, and \mathcal{W} be a covering consisting of disjoint open sets. Then, for each component $Q \in \square X$, there exists $W_Q \in \mathcal{W}$ such that for every $\mathcal{V} \prec \mathcal{W}$, $f_{\mathcal{V}}(Q) \subseteq W_Q$ (and specially $f_{\mathcal{W}}(Q) \subseteq W_Q$).*

3. COMPACT METRIC CASE

In the case of compact metric spaces it is enough to consider proximate sequences $(f_n) : X \rightarrow Y$, indexed by the set of natural numbers, [13].

A sequence of finite coverings, $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \dots$ of a space such that for any covering \mathcal{V} , there exists $n \in \mathbb{N}$, such that $\mathcal{V} \succ \mathcal{V}_n$, we call a cofinal sequence of finite coverings.

DEFINITION 3.1. *The sequence (f_n) of functions $f_n : X \rightarrow Y$ is a proximate sequence from X to Y , if there exists a cofinal sequence of finite coverings, $\mathcal{V}_1 \succ \mathcal{V}_2 \succ \dots$ of Y , such that for each natural number n the function $f_n : X \rightarrow Y$ is a \mathcal{V}_n -continuous function, and if $m \geq n$ then f_n and f_m are homotopic as \mathcal{V}_n -continuous functions. In this case we say that (f_n) is a proximate sequence over (\mathcal{V}_n) .*

Two proximate sequences (f_n) and $(f'_n) : X \rightarrow Y$ are homotopic if f_n and f'_n are \mathcal{V}_n -homotopic, for all $n \in \mathbb{N}$.

This is an equivalence relation.

Let $(f_n) : X \rightarrow Y$ be a proximate sequence over (\mathcal{V}_n) and $(g_k) : Y \rightarrow Z$ be a proximate sequence over (\mathcal{W}_k) . For a covering \mathcal{W}_k of Z , there exists a covering \mathcal{V}_{n_k} of Y such that $g_k(\mathcal{V}_{n_k}) \prec \mathcal{W}_k$. Then, the composition is a proximate sequence $(h_k) = (g_k f_{n_k}) : X \rightarrow Z$.

Spaces and homotopy classes of proximate sequences form the category whose isomorphisms induces classification of compact metric spaces which coincide with shape classification, i.e., isomorphic spaces in this category have the same shape.

THEOREM 3.2. *If X and Y are compact metric spaces, then any proximate sequence $(f_n) : X \rightarrow Y$ induces a function*

$$(f_n)_{\#} : \square X \rightarrow \square Y.$$

PROOF. Let $(f_n) : X \rightarrow Y$ be a proximate sequences over covering (\mathcal{V}_n) and let C be a component of connectedness of the point $x \in X$. There exists

a point of accumulation $y \in Y$, of the sequence $(f_n(x))$, i.e. there exists a subsequence (f_{n_i}) such that $f_{n_i}(x) \rightarrow y$.

Suppose $y \in D$, where D is the component of connectedness of Y .

We will show that D does not depend on the choice of the accumulation point.

Suppose the contrary, there exists another point of accumulation $z \in E \neq D$, E a component of Y , such that $f_{n_j}(x) \rightarrow z$.

Then there exist an open disjoint sets V and W such that $D \subseteq V$, $E \subseteq W$, and $\mathcal{W} = \{V, W\}$ is a covering of Y .

There exists n_0 such that f_{n_0} is \mathcal{W} -continuous functions and for $n \geq n_0$, f_n and f_{n_0} are homotopic as \mathcal{V}_{n_0} -continuous functions, and it follows as \mathcal{W} -continuous functions. Then if $f_{n_0}(x) \in V$ it follows that $f_n(x) \in V$ for all $n \geq n_0$. And this is a contradiction. (the same contradiction is obtained if we suppose $f_{n_0}(x) \in W$).

We will show that D does not depend on the choice of $x \in C$.

If $x' \in C$ and the component E of Y is chosen, in the same way like the component D is chosen for x , then there exist two open disjoint sets V and W such that $V \cup W = Y$, and $D \subseteq V, E \subseteq W$. Put $\mathcal{W} = \{V, W\}$. By Theorem 2.3, there exists n such that $f_n(C) \subseteq V$ and $f_n(C) \subseteq W$ which is a contradiction.

It follows that the function $(f_n)_\# : \square X \rightarrow \square Y$ defined by $(f_n)_\#(C) = D$ is well defined. □

We omit the proof of the following theorem since is similar to the corresponding theorem of the next section.

THEOREM 3.3. *Let $(f_n), (g_n) : X \rightarrow Y$ be two proximate sequences. If $(f_n) \sim (g_n)$, then the induced functions are equal, i.e.,*

$$(f_n)_\# = (g_n)_\# : \square X \rightarrow \square Y$$

4. NON COMPACT CASE

Let X and Y be locally compact metric spaces with compact space of quasicomponents.

THEOREM 4.1. *If X and Y are topological spaces with compact space of quasicomponents QY , then any proximate net $(f_\nu) : X \rightarrow Y$ induces a continuous function*

$$(f_\nu)_\# : QX \rightarrow QY.$$

PROOF. If $(f_\nu) : X \rightarrow Y$ is a proximate net, and Q is a quasicomponent of X , then for a disjoint covering \mathcal{W} of Y , there exists $W_Q \in \mathcal{W}$ such that $f_\mathcal{W}(Q) \subseteq W_Q$. Moreover if \mathcal{W}' is another disjoint covering of Y such that $\mathcal{W} \succ \mathcal{W}'$, and if $f_{\mathcal{W}'}(Q) \subseteq W'_Q$ then $W'_Q \subseteq W_Q$.

To prove the last statement we denote by $F_{\mathcal{W}\mathcal{W}'}$ the homotopy connecting $f_{\mathcal{W}}$ and $f_{\mathcal{W}'}$. Since $F_{\mathcal{W}\mathcal{W}'}$ is $\text{st}\mathcal{W}$ -continuous there exists $W_1 \in \mathcal{W}$ such that $F_{\mathcal{W}\mathcal{W}'}(Q \times I) \subseteq \text{st}W_1$.

Since \mathcal{W} is disjoint $\text{st}\mathcal{W}_1 = \mathcal{W}_1$, and since $F_{\mathcal{W}\mathcal{W}'}(Q \times \{0\}) = f_{\mathcal{W}}(Q) \subseteq W_Q$ we obtain $W_1 = W_Q$. Then from $f_{\mathcal{W}'}(Q) = F_{\mathcal{W}\mathcal{W}'}(Q \times \{1\}) \subseteq W'_Q$ it follows that $W'_Q \subseteq W_Q$.

Notice that there always exists a covering which is finer than a finite number of coverings. From the previous statement it follows that for a fixed Q , the intersections of a finite number of sets $QW_Q = \{A \mid A \in QY, A \subseteq W_Q\}$ is not empty.

The set $QW_Q = \{A \mid A \in QY, A \subseteq W_Q\}$ is closed in QY . Since QY is compact it follows that the intersection of all QW_Q , where W_Q is taken from a disjoint covering \mathcal{W} , is not empty and consists of one quasicomponent T . We put $(f_{\mathcal{W}})_{\#}(Q) = T$.

To prove the continuity of $f_{\#}$, let

$$f_{\#}(S) = T, \quad (f_{\mathcal{V}}) : X \rightarrow Y$$

be a proximate net and $G, G \supseteq T$, be a clopen subset of Y . Then $QG = \{A \mid A \in QY, A \subseteq G\}$ is a basis neighborhood of T in QY . There exists a $\mathcal{V} \in \text{cov}Y$ that refines the covering $\{G, Y \setminus G\}$ of Y . Since the function $f_{\mathcal{V}}$ is \mathcal{V} -continuous, it is $\{G, Y \setminus G\}$ -continuous, also.

Now, we will show that the set

$$F = \{x \mid x \in X, f_{\mathcal{V}}(x) \in G\}$$

is clopen in X . Suppose $x \in F$. Since the function $f_{\mathcal{V}}$ is $\{G, Y \setminus G\}$ -continuous and $f_{\mathcal{V}}(x) \in G$, it follows that there exists a neighborhood U of x such that $f_{\mathcal{V}}(U) \subseteq G$. We obtained that $U \subseteq F$, so F is open. Similarly, $X \setminus F = \{x \mid x \in X, f_{\mathcal{V}}(x) \in Y \setminus G\}$ is open, i.e., F is closed.

Since $S \subseteq F$, QF is a neighborhood of S in QX . If $S' \in QF$, then $f_{\{G, Y \setminus G\}}(S') \subseteq G$, and from the construction of $f_{\#}$ it follows that $f_{\#}(S') \in QG$. We obtained $f_{\#}(QF) \subseteq QG$, i.e., $f_{\#}$ is continuous. \square

THEOREM 4.2. *Let $(f_{\mathcal{V}}), (g_{\mathcal{V}}) : X \rightarrow Y$ are homotopic proximate nets $(f_{\mathcal{V}}) \sim (g_{\mathcal{V}})$. Then, for the induced maps holds:*

$$(f_{\mathcal{V}})_{\#} = (g_{\mathcal{V}})_{\#}.$$

PROOF. Let $(H_{\mathcal{V}}) : X \times I \rightarrow Y$ be a proximate net, i.e., $H_{\mathcal{V}} : X \times I \rightarrow Y$ is a homotopy connecting \mathcal{V} -continuous functions $f_{\mathcal{V}}, g_{\mathcal{V}} : X \rightarrow Y$.

Let Q be a quasicomponent of X such that $(f_{\mathcal{V}})_{\#}(Q) = T, (g_{\mathcal{V}})_{\#}(Q) = R$ and $T \neq R$. Then there exist an open disjoint sets V and W such that $T \subseteq V, R \subseteq W$, and $\mathcal{W} = \{V, W\}$ is a covering of Y . $H_{\mathcal{W}} : X \times I \rightarrow Y$ is a $\text{st}\mathcal{W}$ -continuous, and since $\text{st}\mathcal{W} = \mathcal{W}$, $(H_{\mathcal{W}}) : X \times I \rightarrow Y$ is \mathcal{W} -continuous.

Let x be an arbitrary point from the quasicomponent Q . Then $H_{\mathcal{W}}((x, 0)) = f_{\mathcal{W}}(x) \in V$, while $H_{\mathcal{W}}((x, 1)) = g_{\mathcal{W}}(x) \in W$. This is a

contradiction, since by Theorem 2.3, $H_{\mathcal{W}}(Q \times I) \subseteq V$ or $H_{\mathcal{W}}(Q \times I) \subseteq W$. So, $T = R$, i.e., for the induced maps holds: $(f_{\mathcal{V}})_{\#} = (g_{\mathcal{V}})_{\#}$. \square

The last theorem is the main result, an analogue of Borsuks theorem, in the case of non compact spaces. It is a consequence of Theorems 4.1, 4.2 and 2.7.

THEOREM 4.3. *Suppose X and Y are locally compact metric spaces with compact spaces of quasicomponents QX and QY .*

If a shape morphism $f : X \rightarrow Y$ is presented by a proximate net $(f_{\mathcal{V}}) : X \rightarrow Y$, then there exists a unique mapping $(f_{\mathcal{V}})_{\#} : QX \rightarrow QY$, such that, if $(f_{\mathcal{V}}), (g_{\mathcal{V}}) : X \rightarrow Y$ are homotopic proximate nets such that $(f_{\mathcal{V}}) \sim (g_{\mathcal{V}})$, i.e., they define the same shape morphism, then $(f_{\mathcal{V}})_{\#} = (g_{\mathcal{V}})_{\#}$.

Moreover, the restriction of f to any clopen set W containing $(f_{\mathcal{V}})_{\#}(Q)$, presented by the restriction of proximate net $(f_{\mathcal{V}}) : Q \rightarrow W$ is also a shape morphism.

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N. Shekutkovski
Institute of Mathematics
Faculty of Natural Sciences and Mathematics
Sts. Cyril and Methodius University
1000 Skopje
Republic of Macedonia
E-mail: `nikita@pmf.ukim.mk`

T. Atanasova-Pachemska
University "Goce Delchev" - Shtip
Faculty of Informatics
2000 Shtip
Republic of Macedonia
E-mail: `tatjana.pacemska@ugd.edu.mk`

G. Markoski
Institute of Mathematics
Faculty of Natural Sciences and Mathematics
Sts. Cyril and Methodius University
1000 Skopje
Republic of Macedonia
E-mail: `gorgim@pmf.ukim.mk`

Received: 26.4.2011.

Revised: 9.9.2011.