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	SUNDAY	MONDAY		TUESDAY		WEDNESDAY		THURSDAY		FRIDAY		
	September 4	September 5		dedicated to the memory of T. Gramchev		September 7		September 8			September 9	
8:00-8:25		*registration*								8:30-8:55	SCARPALEZOS	BOUZAR
8:30-8:40		*opening*			*RODINO*					9:00-9:25	KALAJ	DEBROUWERE
8:40-9:30		TARTAR			TOFT	NICOLA		GUBINELLI		9:30-9:55	DANILOV	KAMINSKI
9:35-10:25		KUNZINGER			RUZHANSKY	NEDELJKOV		VINDAS		10:00-10:25	RAJTER-ĆIRIĆ	MAKSIMOVIĆ
10:30-11:00		coffee break			coffee break	coffee break		coffee break			coffee break	
11:00-11:25		LAZAR	LYSIK		OBERGUGGENBERGER	TODOROV	DEBRUYNE	MINČEVA-K.	BOYADZHIEV		KMIT	
11:30-11:55		KUNŠTEK	BAGLINI		PILIPOVIĆ	HÖRMANN	SELEŠI	TOMIĆ	MICHALIK		VUČKOVIĆ	
12:00-12:25		CRNJAC	VICKERS	12:00-12:50	CORIASCO	PAWILOWSKI	MELNIKOVA	LECKE	HASLER	12:00-12:50	PRANGOSKI	
12:30-12:55		LEVAJKOVIĆ	NIGSCH			SÄMANN	SCHWARZ					
13:00-15:30		LUNCH BREAK		13:00-13:20	POSTER INTRO	LUNCH BREAK				12:50-13:00	*closing*	
				13:20-16:45	LUNCH BREAK							
15:30-15:55		ERCEG	VELINOV			KONJIK	TURUNEN	excursion and conference dinner				
16:00-16:25		MIŠUR	DIMOVSKI			OPARNICA	MARTI					
16:30-16:55		VOJNOVIĆ	TANIGUCHI		sightseeing tour of Dubrovnik	PODHAJECKA	SPREITZER					
17:00-17:25		IVEC	PARK			GARELLO	DELGADO					
17:30-18:00		coffee break				coffee break						
18:00-18:25	registration	MITROVIĆ	MATSUYAMA			General Meeting of the IAGF						
18:30-18:55	reception	BOJANJAC	CHUNG									
19:00-19:25		JANKOV	TARANTO									

SYMPLECTIC STRUCTURES AT INFINITY AND LAGRANGIAN DISTRIBUTIONS ON MANIFOLDS WITH ENDS

SANDRO CORIASCO[1] AND RENÉ SCHULZ[2]

The study of Lagrangian submanifolds is an important branch in symplectic geometry. One of the main motivations for their study is due to the fundamental role they play as carriers of singularities in the global theory of Fourier integral operators on manifolds, see [8, 11, 12, 13, 14]. The fundamental connection is that the kernels of Fourier integral operators are Lagrangian distributions associated with a Lagrangian submanifold (in the simplest case, given by the graph of a canonical relation).

The resulting calculus is especially well-suited for working on compact, boundaryless manifolds, while a global theory of Fourier integral operators on unbounded manifolds, even on $\mathbb{R}^d$, is far from being complete. A natural choice of a class of pseudodifferential operators that such operators should contain are those defined through the so-called SG-symbols, see [2, 18, 19]. There are many contributions to the long-standing problem of introducing a suitable global calculus of SG-Fourier integral operators, see for instance [1, 3, 4, 6]. It is then necessary to understand the suitable class of associated Lagrangian submanifolds that should be considered.

In [15, 16, 17], a geometric approach to the SG-calculus on general asymptotically conic manifolds, the so-called scattering geometry, has been developed. Unbounded geometries are therein viewed as manifolds with boundary and the cotangent bundle is replaced by a rescaled and compactified version, the scattering cotangent bundle. Melrose and Zworski subsequently introduced the so-called Legendrian distributions, see [17], which are smoooth functions with a prescribed singularity at infinity, associated with Legendrian submanifolds "at infinity" (see also [9, 10, 20]). On a vector space, these distributions correspond to Fourier transforms of compactly supported Lagrangian distributions.

In [7] we discussed SG-type tempered oscillatory integrals on $\mathbb{R}^d$, which are Lagrangian distributions with a suitable behaviour at infinity. It turned out that their singularities, encoded by their SG-wave front set (see, e.g., [5]), may be decomposed into two sets: one which admits an interpretation as a Lagrangian submanifold, and one that corresponds to a Legendrian. These sets may thus be used as the starting point of a global theory of SG-Fourier integral operators, and a clear understanding of their geometric properties and local parametrization is then a necessary prerequisite.

We provide the details needed for such analysis. In particular, we introduce a class of pairs of Lagrangian-Legendrian submanifolds and show how they can be parametrized by a class of SG-phase functions. We then review in which sense the resulting objects are suitable to formulate the singularities of SG-Lagrangian distributions. An essential ingredient in our study is the analysis on manifolds with corners.

Key words and phrases. Symplectic structures, Lagrangian submanifolds, Manifolds with ends, Manifolds with corners.

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PARACONTROLLED APPROACH TO SINGULAR SPDES

MASSIMILIANO GUBINELLI [1]

Paradifferential calculus can be used to give a meaning and solve a class of singular SPDEs or relevance in mathematical physics, among which one can find the parabolic Anderson model, a generalised Anderson model, the Sardar-Parisi-Zhang equation and the stochastic quantisation equation. In this talk I will introduce the key ideas of the paracontrolled approach to singular SPDEs and discuss merits and limitations.

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GENERALIZED FUNCTIONS AS SET-THEORETICAL MAPS

PAOLO GIORDANO [1], MICHAEL KUNZINGER [2], AND HANS VERNAEVE [3]

Generalized smooth functions (GSF) are a bottom-up approach to nonlinear generalized functions. Contrary to the theory of distributions, generalized functions are viewed as set-theoretical maps defined on, and taking values in, a suitable non-Archimedean ring of scalars, i.e. a ring containing infinitesimal and infinite numbers, namely the ring of Colombeau generalized numbers. GSF are an extension of Colombeau's theory, with a number of improvements. In particular, they can be composed unrestrictedly and they form a concrete category. Moreover, GSF allow for an immediate generalization of many theorems of smooth differential and integral calculus. Differential calculus in this framework is completely intrinsic, when based on the Fermat-Reyes theorem. We report on some recent developments in this field, as well as on applications in mathematical physics.

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Key words and phrases. Generalized smooth functions, Colombeau functions.

A CLASS OF UNBOUNDED SOLUTIONS TO CONSERVATION LAW SYSTEMS

MARKO NEDELJKOV

So called Shadow Wave solution (SDW in the sequel) are introduced to solve some Riemann problems for conservation law systems that do not have classical elementary wave solutions. They resembles shock waves with Dirac delta function sitting on the front.

Such kind of solution is made to be robust enough to check whether they satisfy entropy conditions (using well-known Lax entropy-entropy flux pairs). The main motivation for their construction was the Front Tracking algorithm and one can easily to examine various interactions of elementary waves and waves containing a delta function.

The aim of this talk has two aims. The first one is to demonstrate their usefulness in a number of examples.

The last part of the talk is devoted to some unsolved problems in few situations. The first one is some kind of blow-up situation (Chaplygin gas model), while the second one is a problem of non-uniqueness (generalized Chaplygin gas).

Some of the results are obtained with M. Oberguggenberger, L. Neumann, M. Saho and S. Ružičić.

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HARMONIC ANALYSIS OF THE PATH INTEGRAL

FABIO NICOLA

Path integrals were introduced in 1948 by Richard Feynman to provide a new formulation of Quantum Mechanics and nowadays represent a fundamental tool in most branches of modern Physics. Precisely, a construction of the integral kernel of the Schrödinger propagator was proposed as a suggestive sum-over-histories, defined as a formal integral on the infinite dimensional space of paths joining two points in phase space, similarly to the definition of the Riemann integral, namely by a time slicing approximation procedure. The point is the convergence of such approximations in several function spaces. Whereas the convergence in L^2 (or L^2-based Sobolev spaces) was already addressed in several papers by D. Fujiwara [1, 2] and his school, here we study such issue in the framework of L^p (for smooth potentials) [3]. We use decomposition techniques of functions and operators with respect to suitable wave packets in phase space. We also consider the case of rough potentials in Kato-Sobolev spaces [4].

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Key words and phrases. Path integral, Schrödinger equation, L^p spaces, Kato-Sobolev spaces.

PARAMETRICES AND CONVOLUTION IN QUASIANALYTIC CLASSES OF GELFAND-SHILOV TYPE

STEVAN PILIPOVIC [1], BOJAN PRANGOSKI [2], AND JASSON VINDAS [3]

We construct a special class of ultrapolynomials and use them to construct parametrices in generalised Gelfand-Shilov spaces that have as a special cases the Fourier hyperfunctions and Fourier ultra-hyperfunctions. We apply them in the study of topological and structural properties of several quasianalytic spaces of functions and ultradistributions. As a consequence, we develop a convolution theory for quasianalytic ultradistributions of Gelfand-Shilov type.

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Key words and phrases. Convolution, parametrix method, quasianalytic ultradistributions.

VERY WEAK SOLUTIONS OF WAVE EQUATIONS

MICHAEL RUZHANSKY [1]

In this talk we describe the notion of a very weak solution to hyperbolic (e.g. wave) equations that was introduced in our joint work [1] with Claudia Garetto. This allows one to prove the well-posedness (existence and uniqueness) for the Cauchy problem for the wave equation with distributional coefficients. A difference with Colombeau solutions is that the very weak solution allows one to recover the classical (Gevrey, smooth, distributional or ultradistributional) solutions when they exist. If time permits, we will describe further developments of this notion for the example of the wave equation for the Landau Hamiltonian, allowing one to look at particles moving in a time-dependent electromagnetic field in the presence of electric shocks (e.g. when the electric potential contains delta-functions).

These latter developments are based on joint work with Niyaz Tokmagambetov.

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DEVELOPING OTHER MICROLOCAL TOOLS

LUC TARTAR [1]

Lars Hörmander defined the wave front set WF(S) of a scalar distribution S, outside which S is said to be microlocally regular. He proved that if WF(S) does not intersect WF(T), then the product ST is defined as a distribution. He also proved results of propagation of microlocal regularity if S is solution of a scalar hyperbolic equation with smooth coefficients, but for reasons of propaganda such results are (wrongly) called results of propagation of singularities! A wave front set is a no-man's land, but for understanding a (possibly curved) beam of light, one must measure the amounts of energy and momentum which are transported along it. For doing this, it was natural that I use the H-measures which I had introduced for a question of homogenization. H-measures are a quadratic microlocal tool, and there are a few variants, but for correcting the silly rules of quantum mechanics one should work with semi-linear hyperbolic systems (like the Maxwell-Heaviside equation coupled with the Dirac equation with no mass term) and the (quadratic) nonlinearity creates difficulties. Should one create trilinear microlocal objects? Maybe not, because of the localized waves which one calls "particles", and this requires a better understanding of geometry. Homogenization is a nonlinear microlocal theory, but for questions involving hyperbolic systems and "guesses involving particles", a preliminary step is to develop an existence theory under natural bounds, and compensation effects seem to play an important role.

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MODULATION SPACES, HARMONIC ANALYSIS AND PSEUDO-DIFFERENTIAL OPERATORS

JOACHIM TOFT [1]

In the present talk we present recent results on composition, continuity and Schatten-von Neumann (SvN) properties for operators and pseudo-differential operators (ΨDOs) when acting on modulation spaces. For example we present necessary and sufficient conditions in order for the Weyl product should be continuous on modulation spaces. Such question is strongly connected to questions wether compositions of ΨDOs with symbols in modulation spaces remain as ΨDOs with a symbol in a modulation space.

We also present necessary and sufficient conditions for ΨDOs with symbols in modulation spaces should be SvN operators of certain degree in the interval $(0, \infty]$. Note here that there are so far only few results in the literature on SvN operators with degrees less than one.

Parts of the talk are based on a joint work with Y. Chen, E. Cordero and P. Wahlberg.

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Key words and phrases. Schatten-von Neumann, continuity, composition.

RECENT DEVELOPMENTS ON COMPLEX TAUBERIAN THEOREMS FOR LAPLACE TRANSFORMS

JASSON VINDAS [1]

Complex Tauberian theorems for Laplace transforms have shown to be strikingly useful tools in diverse areas of mathematics such as number theory and spectral theory for differential operators. Many results in the area from the last three decades have been motivated by applications in operator theory and semigroups [1, 4].

In this lecture we shall discuss some recent developments on complex Tauberian theory for Laplace transforms and power series. We will focus on two groups of statements, usually labeled as Ingham-Fatou-Riesz theorems and Wiener-Ikehara theorems. Several classical applications will be discussed in order to explain the nature of these Tauberian theorems.

The results we will present considerably improve earlier Tauberians, on the one hand, by relaxing boundary requirements on Laplace transforms to local pseudofunction boundary behavior, with possible exceptional null sets of boundary singularities, and, on the other hand, by simultaneously considering one-sided Tauberian conditions. Using pseudofunctions allows us to take boundary hypotheses to a minimum, producing "if and only if" type results. In the case of power series, we will extend the Katznelson-Tzafriri theorem, one of the cornerstones in the modern asymptotic theory of operator [3].

The talk is based on collaborative work with Gregory Debruyne [2].

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Key words and phrases. Complex Tauberian theorems; Fatou-Riesz theorem; Wiener-Ikehara theorem; pseudofunctions; pseudomeasures; Laplace transform; power series.

H-DISTRIBUTIONS AND PSEUDO-DIFFERENTIAL OPERATORS

JELENA ALEKSIĆ [1], STEVAN PILIPOVIĆ[2], AND IVANA VOJNOVIĆ [3]

We involve pseudo-differential operators in construction of H-distributions to improve results on H-distributions given in Antonić, N.; Mitrović, D., H-distributions: an extension of H-measures to an $L^p - L^q$ setting, Abstr. Appl. Anal. (2011) and Aleksić, J.; Pilipović, S.; Vojnović, I., H-distributions via Sobolev spaces, Mediterr. J. Math. (2016)

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A FIRST-ORDER APPROXIMATION TO SCALAR SCATTERING FROM THIN, CURVED DIELECTRIC OBJECTS

DARIO BOJANJAC [1], ZVONIMIR ŠIPUŠ [2], AND ANTHONY GRBIC [3]

A first-order asymptotic approximation to scalar scattering from a curved thin dielectric object S_d is presented. In order to solve the scattering problem, a Lippmann-Schwinger integral equation is derived from the governing Helmholtz partial differential equation:

$$u(\mathbf{x}) = u_i(\mathbf{x}) + k_0^2 \int_{S_d} G(\mathbf{x},\mathbf{y})(1-\epsilon_r(\mathbf{y}))u(\mathbf{y})d^3\mathbf{y}.$$

A space distribution of relative permittivity ϵ_r within the integral equation describes the scattering object. Using asymptotic analysis, the initial integral equation over the thin, curved three dimensional object is transformed into an integral equation over a two dimensional object, which approximately describes the thin, curved object. With the described transformation, computational time is significantly reduced since the dimensions of the scattering object are reduced by one. Presented work is an extension of analysis described in paper by D. Ambrose and S. Moskow [1].

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Key words and phrases. scattering theory, electromagnetic waves, reduction of dimension.

MICROLOCAL REGULARITY OF LINEAR PARTIAL DIFFERENTIAL OPERATORS WITH GENERALIZED COEFFICIENTS

CHIKH BOUZAR

The notion of regularity in the algebra $\mathcal{G}(\Omega)$ is based on the subalgebra $\mathcal{G}^{\infty}(\Omega)$ which plays the same role as $\mathcal{C}^{\infty}(\Omega)$ in $\mathcal{D}'(\Omega)$, and it is the basis of the development of local and microlocal analysis within $\mathcal{G}(\Omega)$, see [3], [6] and [7].

However, the $\mathcal{G}^{\infty}-$regularity does not exhaust the regularity problem inherent to the algebra $\mathcal{G}(\Omega)$.

Given a set of sequences of real numbers $\mathcal{R}$, a sheaf of subalgebras $\mathcal{G}^{\mathcal{R}}(\Omega)$ of $\mathcal{G}(\Omega)$ defines a new notion of local $\mathcal{R}-$regularity for generalized functions of $\mathcal{G}(\Omega)$; the microlocalization of this $\mathcal{R}-$regularity has also been done, see [2] and [1].

The aim of this work is to tackle the problem of $\mathcal{R}-$microlocal regularity of solutions of linear partial differential equations with $\mathcal{R}-$regular functions as coefficients in the spirit of the works of [4] and [5].

Work in collaboration with T. Saidi.

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Key words and phrases. Generalized Functions, Linear Partial Differential Operators, Wave Front Set, Microlocal Regularity.

COMPARISON PRINCIPLE FOR QUASI-LINEAR NON-COOPERATIVE PARABOLIC SYSTEMS

GEORGI BOYADZHIEV[1]

Validity of comparison principle for linear and quasi-linear weakly coupled systems of parabolic PDE is considered. The consept is based on the spectral properties of the sysyem, likewise the approach to the non-cooperative ellyptic systems. Furthermore, comparison principle for the corresponding ellyptic system (when t is fixed) yeilds some local (on t) conditions for comprison principle for the parabolic system.

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Key words and phrases. Comparison principle, parabolic systems, quasi-linear systems of PDE..

NONLINEAR ABSORPTIONS FOR THE EXTINCTIVE SOLUTIONS TO EVOLUTION p-LAPLACIAN EQUATIONS

SOON-YEONG CHUNG [1] AND JEA-HYUN PARK [2]

This work is to study a long time behavior of solutions to the evolution p-Laplace equations with nonlinear absorption as follows: For $p > 1$ and a bounded domain Ω in $\mathbb{R}^N$ $(N \geq 1)$ with smooth boundary $\partial\Omega$,

$$\begin{cases} u_t(x,t) = \Delta_p u(x,t) - f(u(x,t)), & (x,t) \in \Omega \times (0,+\infty), \\ u(x,t) = 0, & (x,t) \in \partial\Omega \times [0,+\infty), \\ u(x,0) = u_0 \geq 0, & x \in \Omega, \end{cases}$$

where $u_0 \in L^\infty(\Omega)$ is non-negative and non-trivial and f is a continuous function on $\mathbb{R}$ satisfying $f(0) = 0$, $f(u) > 0$ for all $u > 0$.

A long time behavior of solutions to the above equation has been studied so far for various types of nonlinear absorption f (see [3], [4], [5], and [6]). Here we give a complete characterization of the nonlinear absorption, via the parameter p and the growth of f near the origin, in order to see when the solution to the equations is extinctive or positive. In addition, we also give upper bounds for extinction times of extinctive solutions. In fact, the main conclusion is summarized by the following table:

	$1 < p < 2$	$2 \leq p$	
$\int_{0+}^{1} \frac{1}{f(s)} ds < \infty$	extinctive	extinctive	
$\int_{0+}^{1} \frac{1}{f(s)} ds = \infty$	extinctive	positive, if $\gamma < \infty$	partially positive, if $\gamma = \infty$

Here, the value γ is given by $\gamma := \limsup_{u \to 0+} \frac{f(u)}{u}$ (see [2] for the details).

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Key words and phrases. evolution p-Laplace equation, nonlinear absorption, extinctive solution, long time behavior.

NEW RESULTS ON NONLINEAR GENERALIZED FUNCTIONS

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Last year I gave a talk on multiplication of distributions in a congress of mathematical physics [1] and in Moscow in a form that shows clearly that L. Schwartz did a conceptual mistake in his famous 1954 result [3]. Since very short I believe it could be interesting to reproduce such argument in introduction of articles. In short when one analyzes objectively the problem one reaches at once to the conclusion that in an hypothetical algebra A of some kind of generalized functions having reasonable properties and containing the distributions the familiar implication $\int F(x)\psi(x)dx = 0$ for all test functions ψ does not imply that $F = 0$ in A. This is not an impossibility but an originality of the new context [2,4] and numerous more recent developments. In turn this originality permits to state coherence with all classical calculations valid inside the distributions without meeting the Schwartz impossibility [3,4].

Then I plan to expose applications that are being developed in Brazil to the different main equations of mathematical physics presently unsolved up to now (theoretical constructive results of global existence of solutions, comparison with known theoretical results and known numerical results, uniqueness) and whose solutions are commonly considered as hopeless. This concerns two very different domains both mathematically and physically: we will give the respective examples of the standard system of ideal gases in multi-D and various other systems and of the scattering operator in QFT.

Now that things are clearly understood with new methods it becomes easy to obtain improvements of the results already obtained and a wealth of related results by following the same ideas and methods. Both existence and uniqueness methods in each domain are completely different from the attempts developed since long time by other approaches and are quite accessible without long prerequisites.

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Key words and phrases. Multiplication of distributions, nonlinear generalized functions, equations of mathematical physics.

VARIANT OF OPTIMALITY CRITERIA METHOD FOR MULTIPLE STATE OPTIMAL DESIGN PROBLEMS

KREŠIMIR BURAZIN [1], IVANA CRNJAC [2], AND MARKO VRDOLJAK [3]

In the multiple state optimal design problems, one is trying to find the best arrangement of given materials, such that the obtained body has some optimal properties regarding different regimes. We consider mixtures of two isotropic materials in context of stationary diffusion equation. The performance of the mixture is measured by an objective function which is an integral functional. It is well known that these problems do not admit classical solution, therefore we use relaxation by homogenization method. We rewrite optimality conditions for relaxed problem in order to apply optimality criteria method to multiple state problems in three dimensions. This problem was considered by Vrdoljak (2010), but optimality criteria method didn't give converging sequence of designs for some energy minimization problems. We present another variant of optimality criteria method that can be applied to those problems as well.

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Key words and phrases. Multiple state optimal design, optimality criteria method, stationary diffusion equation.

WEAK ASYMPTOTICS AND SOLUTIONS OF CAUCHY PROBLEM FOR HYPERBOLIC CONSERVATION LAWS WITH PIECEWISE SMOOTH INITIAL CONDITIONS

VLADIMIR DANILOV [1]

This study is based on the following simple formula for the product of Heaviside functions

$$H(a_1 - x)H(a_2 - x) = B\left(\frac{a_2 - a_1}{\varepsilon}\right)H(a_1 - x) + B\left(\frac{a_1 - a_2}{\varepsilon}\right)F(a_2 - x) + O_{\mathcal{D}'}(\varepsilon),$$

where $O_{\mathcal{D}'}(\varepsilon)$ is a quantity of order $\varepsilon \to_0$ in $\mathcal{D}'$, $B'_z(z) \in \mathbb{S}(\mathbb{R}')$ (Schwartz space), $0 \le B(z) \le 1$, $B(\infty) = 1$, $B(-\infty) = 0$, see [1].

These formulas are generalized to the case of an arbitrary (say, continuous) function [1]

$$f\big(A + BH(a_1 - x) + C(a_2 - x)\big) = \alpha + \beta H(a_1 - x) + \gamma H(a_2 - x) + O_{\mathcal{D}'}(\varepsilon),$$

where α, β, γ are explicitly calculated and have the same structure as above. It is clear that the difference approximation of a differential equation is closely related to the approximation of the solution by step functions. This idea was used in the well-known scheme proposed by J. Glimm [2]. On the other hand, it is clear that the step function can be considered as a linear combination of Heaviside functions. Applying the above-introduced formulas, one can construct a step function which approximates the solution for $t \in [0, T]$ and satisfies the stability conditions. In this way, one can easily obtain the well-known result of S. Dafermos [3] about the structure of singularities of the solution of the hyperbolic conservation law.

The key point in the proposed approach is to calculate a sequence of time instants at which the jumps contained in the step function approximating the problem solution merge.

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Key words and phrases. Weak asymptotics, approximation by step functions, general Couchy problem.

THE COUSIN PROBLEM FOR VECTOR-VALUED QUASIANALYTIC ULTRADIFFERENTIABLE FUNCTIONS

ANDREAS DEBROUWERE

In this talk, the additive Cousin problem for spaces of vector-valued quasianalytic ultradifferentiable functions will be discussed. We study topological properties of the spaces of (vector-valued) quasianalytic ultradifferentiable functions and show how this information leads to a solution of the Cousin problem. As a motivation, we also show how this result can be used to construct Colombeau-type differential algebras in which the space of (infra)hyperfunctions is embedded.

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COMPLEX REMAINDER TAUBERIAN THEOREMS

GREGORY DEBRUYNE

Tauberian theory deals with the question of obtaining asymptotic information on a function S from information of some “average” of the function. To be able to this procedure, one typically needs an extra regularity hypothesis on the function, such as monotonicity or boundedness conditions. These conditions are usually called the Tauberian conditions. The Tauberian theorem we are going to address is the Wiener-Ikehara theorem. In it simplest form, it says that if the Laplace-Stieltjes transform $F(s)$ of a non-decreasing function S with support on the positive half-axis is convergent on $\Re e\ s > 1$ and there exists a such that $F(s) - a/(s-1)$ admits analytic continuation beyond the line $\Re e\, s = 1$, then $S(x) \sim ae^x$. An inherent question of this theorem is how much we can weaken the hypothesis of analytic continuation. It has been established that it is enough to ask that $F(s) - a/(s-1)$ admits local pseudo-function behavior on the line $\Re e\ s = 1$ and one has shown that this is also necessary. If one wishes instead to attain the conclusion $S(x) = ae^x + O(e^x R(x))$ for some remainder function R, one may ask similarly which are the minimal requirements needed on the Laplace-Stieltjes transform. We shall present some results regarding this question. The talk is based on collaborative work with Jasson Vindas.

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Key words and phrases. Wiener-Ikehara theorem, local pseudo-functions, Laplace transforms, remainders.

SCHATTEN-VON NEUMANN PROPERTIES ON COMPACT MANIFOLDS

JULIO DELGADO[1] AND MICHAEL RUZHANSKY [2]

In this talk we present some recent results on the study of Schatten-von Neumann properties for operators on compact manifolds. We will explain the point of view of kernels and full symbols. The special case of compact Lie groups is treated separately. We will also discuss about operators on L^p spaces by using the notion of nuclear operator in the sense of Grothendieck and deduce Grothendieck-Lidskii trace formulas in terms of the matrix-symbol . (This a joint work with Michael Ruzhansky)

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ON A CLASS OF TRANSLATION-(MODULATION-)INVARIANT SPACES OF QUASI-ANALYTIC ULTRADISTRIBUTIONS AND CORRELATION WITH NEW MODULATION SPACES

PAVEL DIMOVSKI [1], STEVAN PILIPOVIĆ [2], BOJAN PRANGOSKI [3], AND JASSON VINDAS

A class of translation-invariant Banach spaces of quasi-analytic ultradistributions is introduced and studied. They are Banach modules over a Beurling algebra. Based on this class of Banach spaces, we define corresponding test function spaces $\mathcal{D}^{*}_{E}$ and their strong duals $\mathcal{D}'^{*}_{E'_{*}}$ of quasi-analytic type, and study convolution and multiplicative products on $\mathcal{D}'^{*}_{E'_{*}}$. These new spaces generalize previous works about translation-invariant spaces of tempered (non-quasi-analytic ultra-) distributions; in particular, our new considerations apply to the settings of Fourier hyperfunctions and ultrahyperfunctions. New weighted $\mathcal{D}'^{*}_{L^{p}_{\eta}}$ spaces of quasi-analytic ultradistributions are analyzed. Adding conditions on the modulation we define and study a new class of translation-modulation invariant Banach spaces of quasi-analytic ultradistributions. These new spaces show a certain stability under Fourier transform, duality and tensor product. Multiplication of the Fourier Lebesque spaces $L^{1}_{\hat{\omega}}$ with elements from these spaces, also multiplication of elements from this space with elements from its dual are considered. We associate a new Banach space $\mathcal{M}^{F}$ to translation-modulation invariant Banach space F. These space $\mathcal{M}^{F}$ remains translation-modulation invariant Banach space. The duals of $\mathcal{M}^{F}$ are also considered. The new defined spaces $\mathcal{M}^{F}$ and results concerning them are generalizations of already known Modulation spaces of (ultra)distributions.

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Key words and phrases. Quasi-analytic ultradistributions; Modulation spaces of ultradistributions; Translation-(modulation) invariant Banach spaces of ultradistributions.

ONE-SCALE H-DISTRIBUTIONS

NENAD ANTONIĆ [1] AND MARKO ERCEG [2]

Microlocal defect functionals (H-measures, H-distributions, semiclassical measures etc.) are objects which determine, in some sense, the lack of strong compactness for weakly convergent L^p sequences. In contrast to the semiclassical measures, H-measures are not suitable to treat problems with a characteristic length (e.g. thickness of a plate), while more recant variants, one-scale H-measures [1, 3], have property of being extension of both H-measures and semiclassical measures.

However, H-measures, as well as one-scale H-measures, are adequate only for the L^2 framework. As the generalisation of H-measures to the $L^p - L^{p'}$ setting has already been constructed via H-distributions [2], here we introduce objects which extends the notion of one-scale H-measures, *one-scale H-distributions*, as a counterpart of H-distributions with a characteristic length. Moreover, we address some important features and develop the corresponding localisation principle.

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Key words and phrases. H-measures, H-distribututins, localisation principle, semiclassical measures, characteristic length, Fourier multipliers.

INHOMOGENEOUS MICROLOCAL ANALYSIS IN FOURIER LEBESGUE SPACES

GIANLUCA GARELLO [1] AND ALESSANDRO MORANDO [2]

In the present talk results of microlocal continuity for pseudodifferential operators whose non regular symbols belong to weighted Fourier Lebesgue spaces are given. The focus point is to show that such spaces realize to be algebras with respect to the pointwise multiplication.
Anisotropic local and microlocal propagation of singularities of Fourier Lebesgue type are then studied, with applications to some classes of semilinear equations.

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Key words and phrases. microlocal analysis, pseudodifferential operators, semilinear PDE..

REGULARITY THEORY FOR BLOCH–PERIODIC GENERALIZED FUNCTIONS

MAXIMILIAN HASLER [1]

In this work we develop several aspects of regularity theory known in the framework of other algebras of generalized functions, for the case of asymptotically Bloch-periodic generalized functions. These functions can be written as sum of a function vanishing at infinity and a Bloch-periodic part which must satisfy $f(x+p) = \exp(i\,k\,p)\,f(x)$ for given period and Bloch wave vector $p, k \in \mathbb{R}^n$ and all $x \in D_f \subset \mathbb{R}^n$.

After a short review of the construction of algebras of such generalized function, we elaborate on some sheaf-theoretic properties of these algebras. This is somehow nontrivial in view of the property of periodicity.

Once this framework established, we have in a natural way the notion of ("singular") support of a asymptotically Bloch-periodic generalized function with respect to a given subsheaf of more regular functions.

Following earlier work in the framework of M–extensions [2] and $(\mathcal{C}, \mathcal{E}, \mathcal{P})$–algebras [1], we extend this to the more refined notion of singular spectrum of these generalized functions. The singular spectrum is a generalization of Hörmander's Wave Font Set, allowing microlocal analysis, i.e., to describe not only the points, but also the nature and more refined characterization of the singularities. Finally, we give results about properties these sets and their propagation.

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Key words and phrases. Algebras of generalized functions, Bloch periodic functions, microlocal analysis, regularity theory, singular spectrum.

REGULARIZATIONS AND GENERALIZED FUNCTION SOLUTIONS FOR SCHRÖDINGER-TYPE EQUATIONS

GÜNTHER HÖRMANN [1]

We review results on distributional and generalized solutions to Schrödinger-type equations with non-smooth principal part and discuss particular aspects in examples from global seismology and Bohmian quantum mechanics.

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Key words and phrases. Schrödinger equation, regularizations, Colombeau generalized functions.

ON CONTINUITY OF LINEAR OPERATORS ON MIXED-NORM LEBESGUE SPACES

NENAD ANTONIĆ [1] AND IVAN IVEC [2]

Pretty extensive study of the continuity of pseudo-differential operators on Lebesgue and Sobolev spaces has been done in the last few decades, resulting in well-rounded theory. However, a little has been said about behaviour on spaces with mixed norm.

The first goal of this work was to study and to find the most general conditions that insure the continuity of linear operators on Lebesgue spaces with mixed norm. Then continuity of pseudo-differential operators with symbols in class $S^0_{1,\delta}$, $\delta \in [0,1)$ are investigated.

Techniques involved in the proof are Calderón-Zygmund decomposition of a summable function, and Marcinkiewicz interpolation theorem.

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Key words and phrases. pseudodifferential operators, mixed-norm Lebesgue spaces.

HOMOGENISATION OF ELASTIC PLATE EQUATION

KREŠIMIR BURAZIN [1], JELENA JANKOV [2], AND MARKO VRDOLJAK [3]

We consider a homogeneous Dirichlet boundary value problem for $\mathrm{divdiv}(M\nabla\nabla u) = f$ which describes an elastic symmetric plate clamped at the boundary. We are interested in homogenisation of this equation. The physical idea of homogenisation is to average heterogeneous media in order to derive effective properties. Homogenisation theory is well developed for a second order elliptic equation where a key role plays H-convergence, which was introduced by Spagnolo under the name of G-convergence (1968), and further generalised by Tartar (1975) and Murat and Tartar (1978) as H-convergence.

The theory can be well adapted to general elliptic equations and systems. We shall demonstrate this approach for elastic plate equation which is a fourth order elliptic equation and prove properties of H-convergence, such as locality, irrelevance of the boundary conditions, corrector results, ...

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Key words and phrases. elastic plate, H-convergence, homogenisation.

ENERGY MINIMAL DIFFEOMORPHISMS BETWEEN ANNULI ON SURFACES

DAVID KALAJ [1]

Let M and N be doubly connected Riemann surfaces with boundaries and with non-vanishing conformal metrics σ and ρ respectively, and assume that ρ is a smooth metric with bounded Gauss curvature $\mathcal{K}$ and finite area. The paper establishes the existence of homeomorphisms between M and N that minimize the Dirichlet energy.

Among all homeomorphisms $f\colon M \to N$ *between doubly connected Riemann surfaces such that* $\operatorname{Mod} M \leq \operatorname{Mod} N$ *there exists, unique up to conformal authomorphisms of* M*, an energy-minimal diffeomorphism which is a harmonic diffeomorphism.*

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THE CONVOLUTION AND PRODUCT OF ULTRADISTRIBUTORS IN THE CONTEXT OF RÉNYI'S THEORY OF PROBABLILITY

ANDRZEJ KAMIŃSKI

Alfred Rényi created in [3, 4] a generalization of the classical probability theory of Kolmogorov based on axioms expressed in terms of conditional probablility. The theory admits unbounded probability distributions and leads to interesting problems concerning the convolution and product of Schwartz distributions and Beurling-Roumieu ultradistributions considered in the sense of the respective quotient spaces, called the spaces of distributors and ultradistributors. We present the results which are extensions of those obtained in [1] and [2].

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Key words and phrases. Rényi's probablility spaces, (tempered) ultradistributors, convolution of (tempered) ultradistributors, product of (tempered) ultradistributors.

SOLUTION REGULARITY AND SMOOTH DEPENDENCE FOR ABSTRACT EQUATIONS WITH APPLICATIONS TO HYPERBOLIC PDES

IRINA KMIT [1] AND LUTZ RECKE [2]

First we present a generalized implicit function theorem for abstract equations of the type $F(\lambda, u) = 0$. We suppose that $F(\lambda, \cdot)$ is smooth for all λ. It should be stressed that we do not suppose that $F(\cdot, u)$ is smooth for all u. Let $F(0, u_0) = 0$. We state conditions under which for all $\lambda \approx 0$ there exists exactly one solution $u \approx u_0$, this solution u is smooth in a certain abstract sense, and the data-to-solution map $\lambda \mapsto u$ is smooth. Then we apply this to time-periodic solutions of first-order hyperbolic systems

$$\partial_t u_j + a_j(x, \lambda)\partial_x u_j + b_j(t, x, \lambda, u) = 0$$

and second-order hyperbolic equations

$$\partial_t^2 u - a(x, \lambda)^2 \partial_x^2 u + b(t, x, \lambda, u, \partial_t u, \partial_x u) = 0.$$

Here we impose some conditions that prevent small divisors from coming up and ensure smooth dependence of b_j and b on t (which will yield smooth dependence of the solution on λ).

The talk is based on the results obtained in [1].

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Key words and phrases. generalized implicit function theorem, nonlinear first-order and second-order hyperbolic PDEs, boundary value problems, time-periodic solutions.

WAVES AND FRACTIONAL DERIVATIVES OF COMPLEX ORDER

SANJA KONJIK [1]

Wave propagation in viscoelastic media can be accurately described by the use of fractional derivatives, i.e., derivatives of noninteger order. So far mostly fractional derivatives of real order have been employed for that purpose. We present a new approach in modelling wave phenomena via complex order fractional derivatives. We shall discuss various topics that include well-posedness of the problem, physical and mathematical constraints for the corresponding constitutive equation, existence and uniqueness of solutions, advantages over the real order fractional models, numerical verifications, etc.

This talk is based on joint work with T. M. Atanacković, M. Janev, S. Pilipović and D. Zorica from University of Novi Sad, and relies on [1, 2, 3].

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Key words and phrases. wave equation, fractional derivative, thermodynamical restrictions, fundamental solution.

OPTIMAL DESIGN PROBLEM ON AN ANNULUS FOR A TWO-COMPOSITE MATERIAL MAXIMIZING THE ENERGY

PETAR KUNŠTEK [1] AND MARKO VRDOLJAK [2]

We optimize a distribution of two isotropic materials that occupy an annulus in two or three dimensions, heated by a uniform heat source, aiming to maximize the total energy. In elasticity, the problem models the maximization of the torsional rigidity of a cylindrical rod with annular cross section made of two homogeneously distributed isotropic elastic materials.

Commonly, optimal design problems do not have solutions (such solutions are called *classical*), so one considers proper relaxation of the original problem. Relaxation by the homogenization method consists in introducing generalized materials, which are mixtures of original materials on the micro-scale.

However, by analysing the optimality conditions, we are able to show that the solution is unique, classical and radial. Depending on the amounts of given materials, we find two possible optimal configurations. The precise solution can be determined by solving a system of nonlinear equations, which can be done only numerically.

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Key words and phrases. stationary diffusion, optimal design, homogenization, optimality conditions.

EXPLORING LIMIT BEHAVIOUR OF NON-QUADRATIC TERMS VIA H-MEASURES. APPLICATION TO SMALL AMPLITUDE HOMOGENISATION.

MARTIN LAZAR [1]

Original H-measures explore a quadratic limit behaviour of bounded L^2 sequences. We investigate possibilities of handling a general $\mathrm{L}^p, p > 2$ sequences and describing, roughly speaking, a microlocal limit of $\int |u_n|^p$ via (original) H-measures.

The method is applied to the small amplitude homogenisation problem for a stationary diffusion equation, in which coefficients are assumed to be analytic perturbations of a constant, enabling formulae for higher order correction terms. Explicit expressions in terms of Fourier coefficients are obtained under periodicity assumption. The method allows of its generalisation and application to the corresponding non-stationary equation, as well as to some other small amplitude homogenisation problems.

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Key words and phrases. H-measures, small amplitude homogenisation, non-quadratic terms.

INTRODUCTION TO THE CALCULUS OF VARIATIONS IN GENERALIZED SMOOTH FUNCTIONS

ALEXANDER LECKE, LORENZO LUPERI BAGLINI, AND PAOLO GIORDANO

The aim of this talk is to introduce the calculus of variations into the theory of generalized smooth functions (GSF) [1, 2, 3]. GSF are smooth set-theoretical functions defined on a non-Archimedean extension of the real field. They embed Schwartz distributions but are freely close with respect to composition. This feature facilitates the transposition of classical results into this generalized setting. In order to do this, we begin with a brief introduction to the theory of generalized smooth functions. After this, we give some interesting results like the fundamental lemma of calculus of variations or the Legendre – Hadamard condition in the GSF setting. We conclude the talk with examples from low regular Riemannian geometry such as that (with some assumptions) the standard part of the minimal length in GSF exists and is equal to the minimal lenght in the "standard world".

This is a joint work with Lorenzo Luperi Baglini and Paolo Giordano (University of Vienna).

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Key words and phrases. Generalized Smooth Functions. Calculus of Variations.

THE STOCHASTIC LINEAR QUADRATIC OPTIMAL CONTROL PROBLEM

TIJANA LEVAJKOVIĆ [1]

We consider an infinite dimensional stochastic linear quadratic control problem with the state equation

$$dy(t) = (\mathbf{A}y(t) + \mathbf{B}u(t))\,dt + \mathbf{C}y(t)\,dW(t), \qquad y(0) = y^0, \quad t \in [0,T],$$

defined on Hilbert state space $\mathcal{H}$ and the quadratic cost functional

$$\mathbf{J}(u) = \mathbb{E}\left[\int_0^T \left(\|\mathbf{R}y\|^2_{\mathcal{H}} + \|u\|^2_{\mathcal{U}}\right)\,dt \;+\; \|\mathbf{G}y_T\|^2_{\mathcal{H}}\right].$$

The objective is to minimize the functional over all possible controls u and subject to the condition that y satisfies the state equation. The operators $\mathbf{A}$ and $\mathbf{C}$ are operators on $\mathcal{H}$ and $\mathbf{B}$ acts from the control space $\mathcal{U}$ to the state space $\mathcal{H}$, the process $W(t)$ is a $\mathcal{H}$-valued Brownian motion, while the operators $\mathbf{R}$ and $\mathbf{G}$ are bounded observation operators taking values in $\mathcal{H}$ and $y_T = y(T)$. In order to preserve mean dynamics, we represent the random perturbation as a stochastic convolution and obtain the Wick-version of the state equation. Using the Wick product instead of the usual pointwise multiplication we establish a new approach for solving optimal control problems based on the application of the Wiener-Itô chaos expansion method and the deterministic theory of optimal control. The proposed method can be applied to more general problems, eg. the state equations of the form

$$\dot{y} = \mathbf{A}y + \mathbf{T}\lozenge y + \mathbf{B}u, \qquad y(0) = y^0$$

in certain spaces of generalized stochastic processes.

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SOLVING ODES IN GSF SPACES: FIXED POINT METHODS AND DISTRIBUTIONS

LORENZO LUPERI BAGLINI

Generalized Smooth Functions are a minimal extension of Colombeau generalized functions to arbitrary domains of generalized points. They have been introduced by P. Giordano, M. Kunzinger and H. Vernaeve in [1]. A key property of GSF is their conceptual analogy with smooth functions: they are set-theoretical maps, they are closed by composition, they generalize most classical theorems of calculus and they have a good notion of being compactly supported. In this talk we show that this analogy holds also in the study of first order ODEs $y' = F(t, y)$ where F is a GSF. To convince the audience of this fact, we are going to show the GSf counterparts of certain classical theorems on ODEs, such as Picard-Lindelöf Theorem, in a framework based on the notion of asymptotic gauges (see [2]). Moreover, we will also present some ideas on how a characterization of distributions among GSF can be applied to obtain information about distributional solutions of ODEs, providing some examples.

This is a joint work ([3]) with P. Giordano, University of Vienna.

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Key words and phrases. Generalized smooth functions, Colombeau functions, ODE, fixed point methods, distributions.

CHARACTERIZATIONS OF POLYHARMONIC AND REAL ANALYTIC FUNCTIONS

GRZEGORZ ŁYSIK [1]

It is well-known that harmonic functions can be characterized by the mean value property. Namely, a function u continuous on an open set $\Omega \subset \mathbb{R}^n$ is harmonic on Ω if, and only if, for any closed ball $B(x, R) \subset \Omega$ the value of u at the center of the ball is equal to the integral mean of u over the ball.

We shall prove that polyharmonic functions on Ω can be characterized as those continuous functions on Ω for which integral mean over balls of radius R is expressed as an even polynomial of R with coefficients continuous on Ω. We also extend the above characterization to the case of real analytic functions. The novelty of our characterizations is that the conditions for polyharmonicity and real analyticity of u are expressed only in terms of metric and measure. This justifies introduction of definitions of polyharmonic and analytic functions on metric measure spaces.

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Key words and phrases. Polyharmonic functions, real analytic functions, integral means, metric measure spaces.

SEQUENTIAL APPROACH TO ULTRADISTRIBUTIONS

SNJEŽANA MAKSIMOVIĆ [1] AND STEVAN PILIPOVIĆ [2]

We introduce and analyze fundamental sequences of smooth functions partitioned into equivalence classes which we call s-ultradistributions. The spaces formed by these classes will be denoted as $\mathcal{U}'^{*}$. We prove the existence of an isomorphism between $\mathcal{U}^{*}$ and the space $\mathcal{D}'^{*}$ of ultradistributions of Beurling type in case $* = (p!^t)$ and of Roumieu type in case $* = \{p!^t\}$.

We also introduce and analyze the existance of product of two ultradistributions using a sequential approach to ultradistribution spaces.

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Key words and phrases. Fundamental sequences, Hermite expansions, ultradistributions of Beurling and Roumieu types.

FIXED POINTS IN ALGEBRAS OF GENERALIZED FUNCTIONS AND APPLICATIONS

JEAN-ANDRÉ MARTI

Fixed points of some operator F with a contraction property in some spaces (or algebras) E are classicaly involved to solve many problems in functional analysis. But the irregular cases suggest a generalized formulation which is the subject of the lecture and invites to define some operator Φ in a factor algebra $\mathcal{A}$ of generalized functions. $\mathcal{A}$ is constructed from a basic locally convex algebra $(\mathcal{E},\tau)$. The elements $x \in \mathcal{A}$ are classes $[x_\lambda]$ of some families $(x_\lambda)_{\lambda\in\Lambda}$ with "moderateness" linked to a factor ring $\mathcal{C}$ of so-called generalized numbers. Under some hypotheses, and for some operator Φ_λ in $\mathcal{E}$, Φ is well defined by

$$\mathcal{A} \ni [x_\lambda] = x \to \Phi(x) = [\Phi_\lambda(x_\lambda)] \in \mathcal{A}.$$

We suppose that each Φ_λ is a contraction in some $(\mathcal{E},\tau_\lambda)$ endowed with a locally convex toplology τ_λ depending on λ and then has a fixed point z_λ. This leads to define Φ as a contraction in $\mathcal{A}$. With some additional hypotheses, we can prove the moderateness of $(z_\lambda\,)_\lambda$ and find a fixed point of Φ: $z = \Phi(z) = [\Phi_\lambda(z_\lambda)] \in \mathcal{A}$.

We extend the results to the case where Φ is an operator in the product $\mathcal{A}^m$ of algebras constructed on $\mathcal{E}^m$. The main result of that section is that *Any contraction* $\Phi : \mathcal{A}^m \to \mathcal{A}^m$ *has a fixed point in* $\mathcal{A}^m$. It leads to the Generalized Cauchy-Lipschitz problem : Solve

$$(GCL)\begin{cases} \partial x = f(.,x) \\ x(t_0) = \xi \end{cases}$$

with $x \in \operatorname{Im}\left(\mathfrak{C}^1_{\mathcal{C}}(J,\mathbb{R})\right)^m \subset \left(\mathfrak{C}^0_{\mathcal{C}}(J,\mathbb{R})\right)^m$ and $f \in \left(\mathfrak{C}^0_{\tau,\mathcal{C}}(J\times\mathbb{R}^m,\mathbb{R})\right)^m$ globally Lipschitz, for some ring of "generalized numbers" $\mathcal{C}$, with $t_0 \in J$ and ξ is a given element $\in \widetilde{\mathbb{R}^m}$. The "derivation" ∂ is a map from $\operatorname{Im}\left(\mathfrak{C}^1_{\mathcal{C}}(J,\mathbb{R})\right)^m$ to $\left(\mathfrak{C}^0_{\mathcal{C}}(J,\mathbb{R})\right)^m$. The algebra $\left(\mathfrak{C}^0_{\mathcal{C}}(J,\mathbb{R})\right)^m$ (resp.$\left(\mathfrak{C}^1_{\mathcal{C}}(J,\mathbb{R})\right)^m$) generalize $\left(\mathrm{C}^0(J,\mathbb{R})\right)^m$ (resp.$\left(\mathrm{C}^1(J,\mathbb{R})\right)^m$) and $\left(\mathfrak{C}^0_{\tau,\mathcal{C}}(J\times\mathbb{R}^m,\mathbb{R})\right)^m$ is a generalization of $\left(\mathrm{C}^0(J\times\mathbb{R}^m,\mathbb{R})\right)^m$ without use of derivatives, as in the classical formulation.

The main result of that section is *that it exists a ring of "generalized numbers"* C *such that* $f \in \left(\mathfrak{C}^0_{\tau,\mathcal{C}}(J\times\mathbb{R}^m,\mathbb{R})\right)^m$ *and a map* $\Phi : \left(\mathfrak{C}^0_{\mathcal{C}}(J,\mathbb{R})\right)^m \to \left(\mathfrak{C}^0_{\mathcal{C}}(J,\mathbb{R})\right)^m$ *with an unique fixed point solving* (GCL) *with* $t_0 \in R_+$ *and* $\xi \in \widetilde{\mathbb{R}^m}$. Uniqueness and relationship with the classical problem are discussed.

The last subsection shows a link between the Cauchy-Lipschitz theorem and the transport equation. We cite some results when the coefficients have a weak regularity of Sobolev type or with controlled irregularities. But it is not the case of distributions we wish to treat later with our generalized methods.

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Key words and phrases. Fixed points theory, algebras of generalized functions, Cauchy-Lipschitz theorem.

DECAY ESTIMATES FOR WAVE EQUATION WITH A POTENTIAL ON EXTERIOR DOMAINS

TOKIO MATSUYAMA [1]

Let Ω be an exterior domain in $\mathbb{R}^3$ such that the obstacle $\mathbb{R}^3 \setminus \Omega$ is compact and its boundary $\partial\Omega$ is of $C^{2,1}$. For the sake of simplicity, we assume that the origin does not belong to $\overline{\Omega}$.

We consider the initial-boundary value problem for the wave equations with a potential in the exterior domain Ω. More precisely, we are concerned with the following initial-boundary value problem, for a function $u = u(t, x)$:

$$\partial_t^2 u - \Delta u + V(x)u = F(t, x), \quad t \neq 0, \quad x \in \Omega, \tag{1}$$

with the initial condition

$$u(0, x) = f(x), \quad \partial_t u(0, x) = g(x), \tag{2}$$

and the boundary condition

$$u(t, x) = 0, \quad t \in \mathbb{R}, \quad x \in \partial\Omega, \tag{3}$$

where V is a real-valued measurable function on Ω satisfying

$$-c_0|x|^{-\delta_0} \le V(x) \le c_1|x|^{-\delta_0} \quad \text{for some } 0 < c_0 < \frac{1}{4},\ c_1 > 0 \text{ and } \delta_0 > 2.$$

In this talk I will inform the results on the local energy decay estimates and dispersive estimates for IBVP (1)–(3). Any geometrical assumption on domains such as non-trapping condition is not imposed in the theorems. As a by-product, Strichartz estimates will be obtained. The precise statements will be given in the talk.

This talk is based on the joint work with Vladimir Georgiev (Dipartimento di Matematica, Università di Pisa).

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Key words and phrases. Wave equation, local energy decay, dispersive estimates.

GENERALIZED SOLUTIONS TO STOCHASTIC PROBLEMS

IRINA V. MELNIKOVA

We consider the stochastic Cauchy problem

$$X'(t) = AX(t) + B\mathbb{W}(t), t \geq 0, X(0) = \zeta,$$

with A being the generator of a regularized semigroup in a Hilbert space H, white noise process $\mathbb{W}$ in another Hilbert space $\mathbb{H}$, and $B : \mathbb{H} \to H$, which may depend on X. The problem is ill-posed due to the condition on A and irregular properties of $\mathbb{W}$.

We pay special attention to the problem with differential operators $A = A(i\partial/\partial x)$ and compare generalized (in x) solutions in spaces of generalized functions constructed on the basis of the Gelfand-Shilov classification and generalized (in t) solutions in abstract distribution spaces constructed on the basis of the semigroup classification.

We consider constructions of Wiener and white noise processes according to some specific models.

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Key words and phrases. abstract distribution, Gelfand-Shilov spaces, white noise, Wiener process, semigroup, generator.

PIZZETTI-TYPE FORMULAS AND THEIR APPLICATIONS

SŁAWOMIR MICHALIK [1]

Let $\varphi(z)$ be a holomorphic function in a neighbourhood $D \subset \mathbb{C}^n$ of the origin, $P(\partial_z) \in \mathbb{C}[\partial_z]$ be a partial differential operator of order p with constant coefficients and μ be a finite complex Borel measure supported in the closed ball $B(0,R)$ in $\mathbb{R}^n$ of total mass 1.

We say that a generalised integral mean

$$M_\mu(\varphi; z, r) := \int_{\mathbb{R}^n} \varphi(z + ry)\, d\mu(y)$$

satisfies a *Pizzetti-type formula* for the operator $P(\partial_z)$ if

$$M_\mu(\varphi; z, r) = \sum_{j=0}^{\infty} \frac{P^j(\partial_z)\varphi(z)}{m(j)} r^{pj} \quad \text{for some function} \quad m \quad \text{satisfying} \quad m(j) \sim (j!)^p.$$

In the talk we will describe generalised integral means $M_\mu(\varphi; z, r)$ and operators $P(\partial_z)$ for which the Pizzetti-type formulas hold.

We will also discuss the applications of the Pizzetti-type formulas. In particular, we will show that if $M_\mu(\varphi; z, r)$ satisfies the Pizzetti-type formula for $P(\partial_z)$ then we are able to characterise summable formal power series solutions of the Cauchy problem

$$(\partial_t - P(\partial_z))u = 0, \qquad u(0, z) = \varphi(z)$$

in terms of holomorphic properties of the generalised integral mean $M_\mu(\varphi; z, t)$.

The presented results are based on [1].

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Key words and phrases. Linear partial differential equations, generalised integral means, Pizzetti's formula, k-summability.

ON THE PRODUCT IN THE GELFAND-SHILOV SPACES

SVETLANA MINCHEVA-KAMINSKA

We consider several sequential definitions of the product of distributions in the Gelfand-Shilov spaces $\mathcal{K}'(M_p)$ (see [3]) which are natural modifications of the Mikusiński-Shiraishi-Itano definitions of the product of distributions in $\mathcal{D}'$ (see [5, 6]). The definitions are based on various classes of delta-sequences. Using the Mikusiński-Antosik diagonal theorem (see [1, 2]) we prove the equivalence of the considered definitions of the product in $\mathcal{K}'(M_p)$. The result is a generalization of the theorem on the equivalence of certain sequential products of tempered distributions proved in [4].

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Key words and phrases. Product of (tempered) distributions (in $\mathcal{S}'$) in $\mathcal{D}'$, Gelfand-Shilov spaces $\mathcal{K}'(M_p)$, product of distributions in $\mathcal{K}'(M_p)$, Mikusiński-Antosik diagonal theorem.

H-DISTRIBUTIONS, DISTRIBUTIONS OF ANISOTROPIC ORDER AND SCHWARTZ KERNEL THEOREM

NENAD ANTONIĆ [1], MARKO ERCEG [2], AND MARIN MIŠUR [3]

H-distributions were introduced by Antonić and Mitrović as an extension of H-measures to the $\mathrm{L}^p - \mathrm{L}^q$ setting. Their variants have been successfully applied to problems in velocity averaging (Lazar-Mitrović 2012) and compensated compactness with variable coefficients (Mišur-Mitrović 2015). They have also been extended to the Sobolev space setting (Aleksić-Pilipović-Vojnović 2016).

This talk is about recent efforts to give a precise description of H-distributions. We introduce the notion of anisotropic distributions – distributions of different order with respect to different coordinate directions. In order to show that H-distributions are anisotropic distributions of finite order with respect to every coordinate direction, we prove a Schwartz kernel theorem for anisotropic distributions.

The results that will be presented in this talk are a part of paper in progress [1].

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Key words and phrases. H-distributions, anisotropic distributions, kernel theorem.

ON A FRONT EVOLUTION IN POROUS MEDIA WITH A SOURCE

MAROJE MAROHIĆ [1], DARKO MITROVIĆ [2], AND ANDREJ NOVAK [3]

We analyze evolution of the interface between immiscible liquids of different densities in porous media. The liquids can be compressible (CO2 or natural gases) or incompressible (oil, water). We rigorously prove that, if the heavier liquid is on the top and there are no sink or source, a tip of the interface will move in the direction of the gravity (if the tip is directed toward the bottom) or the buoyancy (if the tip is directed toward the top). We also show how the sink/source influence propagation of the interface and provide numerical examples.

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Key words and phrases. Darcy law; immiscible fluids; density driven flow.

FULL AND SPECIAL COLOMBEAU ALGEBRAS

EDUARD A. NIGSCH

A basic space of generalized functions on $\Omega \subseteq \mathbb{R}^n$ which incorporates both the special and the full approach to Colombeau algebras is given by

$$C^\infty\big(\mathcal{L}(\mathcal{D}'(\Omega), C^\infty(\Omega))^I, C^\infty(\Omega)^I\big)$$

with $I = (0, 1]$. We discuss recent investigations into this basic space and corresponding structural properties of Colombeau algebras, in particular:

(1) *locality properties*, which serve to obtain the sheaf property and show how this basic space contains those of known special and full Colombeau algebras;
(2) *point values* and how generalized points from the special variant suffice to characterize functions in the full variant;
(3) *restriction* to open subsets and arbitrary submanifolds.

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Key words and phrases. Colombeau Algebra, Full, Special, Locality Properties, Point values, Restriction.

THE BURGERS EQUATION WITH POISSON WHITE NOISE AS INITIAL DATA

MICHAEL OBERGUGGENBERGER [1]

Entropy solutions to the inviscid Burgers equation can be obtained as zero viscosity limits by Hopf's method, which requires evaluating the minimum of a certain function. In particular, this method can be applied to initial data which are regularizations of Dirac measures or derivatives thereof. Letting the regularization parameter tend to zero, one obtains entropy solutions to the inviscid Burgers equation with singular initial data. This approach has been introduced by Todor Gramchev in 1990 (published later, e.g. in [1]). We take up this method to construct solutions to the stochastic Burgers equation with Poisson white noise as initial data.

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Key words and phrases. Conservation laws, delta waves, stochastic partial differential equations.

MICROLOCAL ANALYSIS OF FRACTIONAL TYPE WAVE EQUATIONS

LJUBICA OPARNICA

Fractional type wave equations describe wave phenomenas when viscoelasticity of a material or non-local effects of a material comes into an account. We determine the wave front sets of solutions to such equations.

For the space fractional wave equation we show that no spatial propagation of singularities occurs and for the (time) fractional Zener wave equation, we show an analogue of non-characteristic regularity, see [1]. For Eringen fractional wave equation, which models elastic wave dispersion in small scale structures as micro and nanostructures, there is no spatial propagation of singularities, [2].

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Key words and phrases. fractional differential equations, wave front set.

RUBIO DE FRANCIA'S INEQUALITY AND AN APPLICATION TO MULTIPLIERS

LJUDEVIT PALLE [1]

The first result extending the classical Littlewood-Paley inequality to other than the dyadic intervals was proved by L. Carleson [1]. We will present Rubio de Francia's inequality, which is a Littlewood-Paley inequality for arbitrary intervals. Instead of following the original proof in [4], a time-frequency perspective by M. T. Lacey [3] will be considered. At the end we will give an application to Fourier multiplier noted in [2].

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Key words and phrases. Littlewood-Paley theory, Calderón-Zygmund theory, time-frequency analysis, vector-valued inequalities, multipliers.

CUCKER-SMALE MODEL WITH FINITE RANGE OF COMMUNICATIONS

JEA-HYUN PARK

Many researches for collective motion of self-propelled units such as flashing of fireflies, chorusing of crickets, schools of fished, and flocks of starlings are proceeding actively in various field [1, 7, 8]. Especially, the terminology '*flocking*' represents collective behavior exhibited when a group of birds are foraging or in flight. It has received lots of attention to control formation of robots, e.g., unmanned aerial vehicles, sensor networks [5, 6].

Recently several mathematical models for flocking were introduced to research flocking phenomena. Among them, Cucker-Smale model has been taken actively interests by many researchers [2, 3, 4].

In this talk, we introduce Cucker-Smale model with finite range of communications and discuss flocking phenomena for this model which reads as

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \quad \frac{d\mathbf{v}_i}{dt} = c\sum_{j=1}^{N} \psi(\|\mathbf{x}_j - \mathbf{x}_i\|)(\mathbf{v}_j - \mathbf{v}_i), \quad 1 \le i \le N,$$

where $(\mathbf{x}_i(t), \mathbf{v}_i(t)) \in \mathbb{R}^{2d} \times [0, \infty)$ is the phase-space coordinate of i-th unit, c is a positive coupling constant, and the nonnegative functions ψ is a pairwise communication between i-th and j-th particles and, in this paper, it is definded by

$$\psi(s) = \begin{cases} 1, & s \le K \\ 0, & s > K, \end{cases}$$

for some positive value $K \in \mathbb{R}$.

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Key words and phrases. Cucker-Smale model, Synchronization, Collective motion.

BOSONIC MEAN FIELD LIMIT AND DISCRETE SCHRÖDINGER EQUATION

[1] BORIS PAWILOWSKI

We deal with approximations of the time-dependent linear many body Schrdinger equation with a particles interaction potential. We consider the bosonic Fock space in a finite dimensionial setting and introduce a discrete version of the Schrdinger equation. Mathematical tools include the reduced density matrices and Wigner measure techniques exploiting the formal analogy to semi-classical limits.

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Key words and phrases. Mean field limit, Schrödinger equation, semiclassical measure.

ANISOTROPIC SHUBIN OPERATORS AND EIGENFUNCTION EXPANSIONS IN NON-SYMETRIC GELFAND SHILOV SPACES

STEVAN PILIPOVIĆ

We derive new results on the characterization of weighted nonsymmetric Gelfand–Shilov spaces $S^{\mu}_{\nu}(\mathbf{R}^n)$, $\mu, \nu > 0$, $\mu + \nu \geq 1$ by Gevrey estimates of the L^p norms of iterates of anisotropic globally elliptic Shubin (or Γ) type operators, modelled by anisotropic harmonic oscillator $\mathcal{H}^{m,k}_n = (-\Delta)^{m/2} + (\|x\|^2)^{k/2}, \quad \|x\|^2 = x_1^2 + \ldots + x_n^2, k, m \in 2\mathbf{N}$, as well as by the decay of the Fourier coefficients in the eigenfunction expansions. In contrast to the symmetric case $\mu = \nu$ and $k = m$ (classical Shubin operators) we encounter resonance type phenomena involving the $\kappa := \mu/\nu$, namely we can characterize $S^{\mu}_{\nu}(\mathbf{R}^n)$, $\mu + \nu \geq 1$ by iterates and eigenfunction expansions defined by normal (m, k) anisotropic elliptic differential operators. In the nonresonant case $\kappa \notin \mathbf{Q}$ we characterize the non–quasianalytic Gelfand–Shilov spaces $S^{\mu}_{\nu}(\mathbf{R}^n)$, $\mu, \nu > 1$ by using operators with separation of variables symbols like $(-\Delta)^{m/2} + (1 + \|x\|^2)^{m/\kappa}$ and $(1 - \Delta)^{\kappa k/2} + \|x\|^k$. We stress that these results on such operators, which are neither differential nor p.d.o. with symbols from the usual classes like Γ or G, is a novelty without any counterpart in the case of compact manifolds. We outline also some applications of our results for deriving hypoellipticity–solvability for operators in scales of Banach spaces of Gelfand–Shilov spaces.

Joint work with Todor Gramchev, Marco Cappiello and Luigi Rodino.

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Key words and phrases. Gelfand Shilov type spaces.

THE STOKES PHENOMENON FOR CERTAIN PARTIAL DIFFERENTIAL EQUATIONS WITH MEROMORPHIC INITIAL DATA

SŁAWOMIR MICHALIK [1] AND BOŻENA PODHAJECKA[2]

We study the title Stokes phenomenon (named after its discoverer George Gabriel Stokes), which is the well-known fact that the formal solution of PDE can have different asymptotic expansions in different sectors of the complex plane.
We focus our attention to investigate this phenomenon for the solutions of the 1-dimensional complex heat equation and its generalizations with meromorphic initial conditions. We are interested in finding the Stokes lines, the anti-Stokes lines and jumps across the Stokes lines. The important point to note here is that we can describe these jumps in terms of hyperfunctions. We emphasize also that our principal tool used to characterize the Stokes phenomenon is the theory of Borel summability.

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Key words and phrases. Stokes phenomenon, Heat equation, Borel summability.

FRACTIONAL EVOLUTION EQUATIONS WITH GENERALIZED OPERATORS

MILOŠ JAPUNDŽIĆ [1] AND DANIJELA RAJTER-ĆIRIĆ [2]

We consider inhomogeneous fractional evolution equations with Caputo fractional derivatives and generalized Colombeau operators. In order to investigate those equations, we introduce Colombeau solution operators. By using the corresponding Colombeau theory we prove the existence and uniqueness result for the problem of consideration.

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Key words and phrases. fractional evolution equation, generalized operator, solution operator.

ON IMPULSIVE GRAVITATIONAL WAVES WITH COSMOLOGICAL CONSTANT

CLEMENS SÄMANN

In this talk we will give an overview on recent work on impulsive gravitational waves on constant curvature backgrounds with cosmological constant. The investigation of geodesics in these spacetimes makes use of generalized functions in the sense of Colombeau and of (weak) solutions to ODEs in the sense of Filippov, combining non-smooth methods and geometry in a very fruitful way.

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Key words and phrases. geodesics, impulsive gravitational waves, non-smooth spacetimes.

QUASIAVERAGING OPERATORS

DIMITRIS SCARPALEZOS

Quasiaveraging operators , average strong association and comparison of regularities here is introduced in the frame of Colombeau generalized functions (or rather in the frame of an "integral on parameter" extension of those ideas)) a notion of average asociation analogous to cesaro convergence for sequences one important problem in this theory is the comparison of regularities between those new generalized functions and distributions to which they are associated in various senses of the word. To obtain results in this direction a notion of "quasiaveraging transform is introduced" the comparison of regularities is investigated in the cases of real anlytic regulariteis , Zygmund type regularities, and Besov type regularities.

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STOCHASTIC TRANSPORT WITH HIGHLY IRREGULAR TRANSPORT SPEED MODELED BY THE GOUPILLAUD MEDIUM

FLORIAN BAUMGARTNER [1], MICHAEL OBERGUGGENBERGER[2], AND MARTIN SCHWARZ [3]

In this talk we consider the one-dimensional transport equation with a spatially random transport speed $c(x)$.

$$
\begin{aligned}
u_t(x,t) + c(x)u_x(x,t) &= 0 \\
u(x,0) &= u_0(x)
\end{aligned}
$$

The randomness is modeled by a piecewise constant medium, called Goupillaud medium, such that the transport time Δt through each layer is constant. As a consequence solving the characteristic equation becomes a geometric problem.

We will elaborate the details for refinement of the medium. Furthermore, it will be shown that the characteristic curve converges to a strictly increasing Lévy process as $\Delta t \to 0$. Although this limit is not continues, one can expect a convergence of the solution in some sense.

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Key words and phrases. Goupillaud media, transport equation, Lévy process,

OPERATORS OF STOCHASTIC ANALYSIS IN INFINITE DIMENSIONS

DORA SELEŠI

The polynomial chaos expansion of stochastic processes allows to represent classical stochastic processes via orthogonal polynomial bases in a Hilbert space and to define various weak topologies to construct larger spaces of generalized stochastic processes. The method also allows to split a stochastic differential equation into an infinite system of deterministic partial differential equations that can be solved by various techniques. We will apply these techniques to solve some equations involving the three basic operators of stochastic variational calculus: the Malliavin derivative, the Skorokhod integral and the Ornstein-Uhlenbeck operator. The most interesting feature is a nice connection between the harmonic oscillator, the Ornstein-Uhlenbeck operator and the multiplication operator.

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Key words and phrases. Malliavin derivative, Skorokhod integral, Ornstein-Uhlenbeck operator, harmonic oscillator.

CONICAL SCHWARTZ FUNCTIONS WITH VANISHING MOMENTS

CHRISTIAN SPREITZER [1]

Smooth functions with vanishing moments play a significant role in the embedding of Schwartz distributions in spaces of Colombeau generalised functions as well as in wavelet theory. A non-zero test function (smooth and compactly supported) can only have finitely many vanishing moments. However, any test function φ_0 can be approximated in $W^{k,p}$-norms ($1 \leq k < \infty, 1 \leq p \leq \infty$) by a Schwartz function φ satisfying $\int_{\mathbb{R}^n} x^\alpha \varphi(x)\mathrm{d}x = 0$ for all $\alpha \in \mathbb{N}_0^n$ with $|\alpha| > 0$. Moreover, given an arbitrary open cone Γ in $\mathbb{R}^n$, it is possible to have $\mathrm{supp}(\varphi) \subseteq \mathrm{supp}(\varphi_0) \cup \Gamma$. If $\mathrm{supp}(\varphi_0) \subseteq \Gamma$, then $\mathrm{supp}(\varphi) \subseteq \Gamma$ and we call φ a conical Schwartz function. We explicitly construct a function φ with the desired properties. This result is related to the concept of generalised mollifiers developed in [1], [2] and [3].

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Key words and phrases. Mollifiers, moment conditions, embedding of distributions in Colombeau algebras.

L^p-BOUNDEDNESS OF SPECTRAL MULTIPLIERS FOR SCHRÖDINGER OPERATORS ON OPEN SETS

TSUKASA IWABUCHI [1], TOKIO MATSUYAMA [2], AND KOICHI TANIGUCHI [3]

Let H_V be a self-adjoint extension of the Schrödinger operator $-\Delta + V(x)$ with the Dirichlet boundary condition on an arbitrary open set Ω of $\mathbb{R}^d$, where $d \geq 1$ and the negative part of potential V belongs to the Kato class on Ω. The purpose of this talk is to prove L^p-boundedness of spectral multipliers $\varphi(H_V)$ for any rapidly decreasing function φ on $\mathbb{R}$, where $\varphi(H_V)$ is defined via the spectral theorem. As a by-product, L^p-L^q-estimates and gradient estimates for $\varphi(H_V)$ are also obtained.

Furthermore, we consider the application of this boundedness to Besov spaces generated by H_V.

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Key words and phrases. Spectral multipliers, Schrödinger operators, Kato class.

WELL-POSEDENESS OF THE SUB-LAPLACIAN WAVE EQUATION ON STRATIFIED LIE GROUPS AND SUB-LAPLACIAN GEVREY SPACES

MICHAEL RUZHANSKY [1] AND CHIARA TARANTO [2]

In a recent work [3], C. Garetto and M. Ruzhansky investigate the Cauchy problem for the time-dependent wave equation for sums of squares of vector fields on compact Lie groups. In particular, they establish the well-posedness in spaces that compare to the Gevrey spaces. In this talk a generalisation of their result to all stratified Lie groups is presented. Furthermore, modelled on the spaces of *Gevrey-type* appearing in [3], we define the *sub-Laplacian Gevrey spaces* on manifolds and partially characterise these spaces. Finally we consider the case of the Heisenberg group, which allows us to give a full characterisation for the *sub-Laplacian Gevrey spaces*.

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Key words and phrases. sub-Laplacian, wave equation, Gevrey spaces, Heisenberg groups.

ASYMPTOTICALLY ALMOST AUTOMORPHIC GENERALIZED FUNCTIONS

FATIMA ZOHRA TCHOUAR [1]

Almost automorphic functions, as a generalization of almost periodic functions, were introduced by S. Bochner in [1]. Asymptotically almost periodic functions are due to M. Fréchet, see [7]. L. Schwartz introduced in [8] almost periodic distributions, and in [6], I. Cioranescu extended the concept of asymptotically almost periodic functions to Schwartz distributions. In [3], the authors introduced almost automorphic distributions, and in [5] asymptotically almost automorphic distributions as a continuity of the work on almost automorphic distributions. An algebra of almost automorphic generalized functions has been introduced and studied in [4], this algebra contains almost periodic generalized functions of [2] and also almost automorphic distributions of [3].

The aim of this work is to introduce and to study an algebra of asymptotically almost automorphic generalized functions containing asymptotically almost periodic functions as well as asymptotically almost automorphic distributions of [5].

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Key words and phrases. Asymptotically almost automorphic functions, asymptotically almost automorphic distributions, Colombeau algebra, asymptotically almost automorphic generalized functions.

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FOUNDATIONS OF QUANTUM MECHANICS IN A NON-SEPARABLE HILBERT SPACE

TODOR D. TODOROV

The axioms of the non-relativistic quantum mechanics are formulated within a non-separable Hilbert space $\mathcal{H}$. The space $\mathcal{H}$ is embedded between a (conventional) separable Hilbert space, H, and a (Colombeau type) generalized Hilbert space, $\widehat{H}$. In sharp contrast to the Gelfand rigged Hilbert space and the p-adic Hilbert space, $\mathcal{H}$ is a complete inner vector space over a field $\widehat{\mathbb{R}}(i)$, where $\widehat{\mathbb{R}}$ is a non-Archimedean real closed (and thus totally ordered) field. The latter allow the construction of probability measure associated with the Hermitian operators (observables) in $\mathcal{H}$.

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BEYOND GEVREY REGULARITY

STEVAN PILIPOVIĆ, NENAD TEOFANOV, AND FILIP TOMIĆ

We define and study classes of smooth functions which are less regular than Gevrey functions. To that end we introduce two-parameter dependent sequences which do not satisfy Komatsu's condition (M.2)', known as "stability under differential operators". Our classes therefore have particular behavior under the action of ultradifferentiable operators. On a more advanced level, we study microlocal properties and present our main result:

$$\mathrm{WF}_{0,\infty}(P(D)u) \subseteq \mathrm{WF}_{0,\infty}(u) \subseteq \mathrm{WF}_{0,\infty}(P(x,D)u) \cup \mathrm{Char}(P),$$

where u is a Schwartz distribution, $P(x,D)$ is a partial differential operator with coefficients in our classes and $\mathrm{WF}_{0,\infty}$ is the wave front set described in terms of new regularity conditions.

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Key words and phrases. Ultradifferentiable functions, Gevrey classes, ultradistributions, wave-front sets.

ON THEORY AND APPLICATIONS OF TIME-FREQUENCY ANALYSIS

VILLE TURUNEN

When and how often something happens in a signal? By properly quantizing these questions, we obtain the Born–Jordan time-frequency transform, defining a sharp phase-space energy density. We study properties of different time-frequency transforms, and also present computed examples from acoustic signal processing, quantum mechanics and medical sciences.

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Key words and phrases. Time-frequency analysis, Cohen's class, Born-Jordan transform.

(Q-)EXPONENTIAL C-DISTRIBUTION AND C-ULTRADISTRIBUTION SEMIGROUPS IN LOCALLY CONVEX SPACES; EXAMPLES

MARKO KOSTIĆ [1], STEVAN PILIPOVIĆ [2], AND DANIEL VELINOV [3]

The talk is devoted on the (q-) exponential C-distribution semigroups and (q-) exponential C-ultradistribution semigroups in the setting of sequentially complete locally convex spaces. Additionally, differential and analytic properties of C-distribution semigroups and C-ultradistribution semigroups are under consideration. We contribute our work and the work of many other authors, providing additionally plenty of various examples and applications of obtained results.

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Key words and phrases. C-distribution semigroups, C-ultradistribution semigroups, integrated C-semigroups, convoluted C-semigroups, well-posedness, locally convex spaces.

REGULARISATION v APPROXIMATION

JAMES VICKERS [1] AND YAFET SANCHEZ SANCHEZ [2]

In this talk we look at solutions to the wave equation in a low regularity situation where the metric (or equivalently the symbol of the differential operator) is singular. We compare and contrast solution methods based on the regularising the symbol and obtaining a Colombeau solution of the equation with methods based on approximation techniques.

In particular we will consider the use of Galerkin approximation methods in which one replaces the wave equation by a system of ODEs whose solution converges to a suitable notion of weak solution of the wave equation. We also consider the use of the vanishing viscosity method in which we approximate the hyperbolic initial value problem by a parabolic initial value problem and again show convergence to a weak solution. See for example [1] for examples of both these techniques. Note however, Evans assumes greater regularity of the symbol and obtains more regular solutions as a result. In our case we have adapted the argument to the low-regularity setting and only obtained weak solutions.

A feature of the approximation method is that we have changed the nature of the equation under consideration to one where we have better analytical control over the solutions (ODEs for the Galerkin approximation and a parabolic PDE for the viscosity method). In contrast by using regularisation one solves an equation of the same type and obtains a Colombeau solution. The issue then is to relate the weak solutions obtained through approximation methods to the weak equivalence class of the Colombeau solution.

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Key words and phrases. Low Regularity, Colombeau, weak equivalence, Galerkin, Vanishing viscosity.

H-DISTRIBUTIONS WITH NONZERO ORDER MULTIPLIERS

JELENA ALEKSIĆ [1], STEVAN PILIPOVIĆ[2], AND IVANA VOJNOVIĆ [3]

We construct H-distributions associated to weakly convergent sequences in Bessel space H^p_{-s} and show how these tools can serve to analyze possible strong convergence.

Usually, the strong convergence of weakly convergent sequence is tested on weakly convergent sequences in dual space H^q_s, $q = p/(p-1)$.

Using multipliers of nonzero order test space is not limited to the dual and can be smaller.

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EIGENEXPANSIONS OF ULTRADIFFERENTIABLE FUNCTIONS AND ULTRADISTRIBUTIONS IN $\mathbb{R}^n$

ĐORĐE VUČKOVIĆ [1]

In this talk we will show a characterization of $\mathcal{S}^{\{M_p\}}_{\{M_p\}}(\mathbb{R}^n)$ and $\mathcal{S}^{(M_p)}_{(M_p)}(\mathbb{R}^n)$, the general Gelfand-Shilov spaces of ultradifferentiable functions of Roumieu and Beurling type, in terms of decay estimates for the Fourier coefficients of their elements with respect to eigenfunction expansions associated to normal globally elliptic differential operators of Shubin type. Moreover, we will show that the eigenfunctions of such operators are absolute Schauder bases for these spaces of ultradifferentiable functions.

Our characterization extends earlier results by Gramchev et al [2] for Gevrey weight sequences. It also generalizes to $\mathbb{R}^n$ recent results by Dasgupta and Ruzhansky [3], which were obtained in the setting of compact manifolds.

This talk is based on collaborative work with J. Vindas [1].

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COMPLEX POWERS OF C-SECTORIAL OPERATORS

MILICA ŽIGIĆ [1]

We define complex powers of C-sectorial operators in the setting of sequentially complete locally convex spaces. The constructed powers are considered as the integral generators of equicontinuous analytic C-regularized resolvent families. The obtained results are incorporated in the study of incomplete higher order Cauchy problems. This is a joint work with C. Chen, M. Kostić, M. Li.

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