Dividend problem for a general Lévy insurance risk process

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Economic point of view

- The word *dividend* comes from the Latin word *dividendum* meaning *the thing which is to be divided* and has got sense of *portion of interest on a loan, stock, etc.*
- Dividends are usually defined as the distribution of earnings in real assets among the shareholders of the firm (in proportion to their ownership).
- Dividends are paid from the firm's after-tax income. For the recipient, dividends are considered regular income and are therefore fully taxable.
- There are two sides of dividends policies in the modern corporate firms. The first are managers of the firm (insiders), the second are shareholders (outsiders). The interest of management and shareholders may not coincide. This has important consequences for dividend policy. There is a suggestion that former typically prefer a low payout in order to pursue growth maximizing strategies or consume additional benefits, while letters generally wish for a high payout since this will force the management to incur the inspection of the capital markets for each new project undertaken.
- We focus in this talk on the maximizing the cumulant dividend payments (we look at it only from the point of view of beneficiaries).

Cramér-Lundberg model

The reserve of an insurance company can be described by a Cramér-Lundberg process (Filip Lundberg (1903) and Harald Cramér (1933)):

$$X_t = x + ct - \sum_{k=1}^{N_t} C_k$$

where

C_k - sequence of independent, identically distributed random variables with distribution function F

N_t - independent Poisson process with intensity λ

c - the premium income per unit time

Spectrally negative Lévy process

X_t - spectrally negative Lévy process, which is not subordinator, that is
X_t - process with stationary and independent increments having only negative jumps

Process X_t models the risk-process of an insurance company before dividends are deducted.

Lévy-Khitchine formula:

$$Ee^{i\theta X_t} = e^{-\Psi(\theta)t},$$

where

$$\Psi(\theta) = -ic\theta + \frac{\sigma^2}{2}\theta^2 + \int_{(-\infty,-1)} \left(1 - e^{i\theta x}\right) \Pi(dx) + \int_{(-1,0)} \left(1 - e^{i\theta x} + i\theta x\right) \Pi(dx) \tag{1}$$

where we assume that $\int_{(-\infty,0)} (1 \wedge x^2)\, \Pi(dx) < \infty$

De Finetti problem

We assume that $X_t \to \infty$ a.s. that is $c - \int_{(-\infty,-1)} |x| \, \Pi(dx) > 0$.

"That is why de Finetti (1957) proposed another, economically motivated, criterion to the actuarial world. Instead of focussing on the safety aspect (measured by the probability of ruin) he proposed to measure the performance of an insurance portfolio by the maximal dividend payout that can be achieved over the lifetime of the portfolio. In particular, he proposed to look for the expected discounted sum of dividend payments until the time of ruin, where the discounting is with respect to some constant discount rate $q > 0$. Whereas de Finetti himself solved the problem to identify the optimal such dividend strategy in a very simple discrete random walk model, since then many research groups have tried to address this optimality question under more general and more realistic model assumptions and until nowadays this turns out to be a rich and challenging field of research that needs the combination of tools from analysis, probability and stochastic control."
(Albrecher and Thonhauser 2009)

De Finetti problem

X_t - spectrally negative Lévy process

π - a dividend strategy consisting of a **non-decreasing**, left-continuous $\mathbf{F}$-adapted process $\pi = \{L_t^\pi, t \geq 0\}$ with $L_0^\pi = 0$, where L_t^π represents the cumulative dividends paid out by the company up till time t

The risk process controlled by a dividend policy π is then given by

$$U_t^\pi = X_t - L_t^\pi$$

Ruin time:

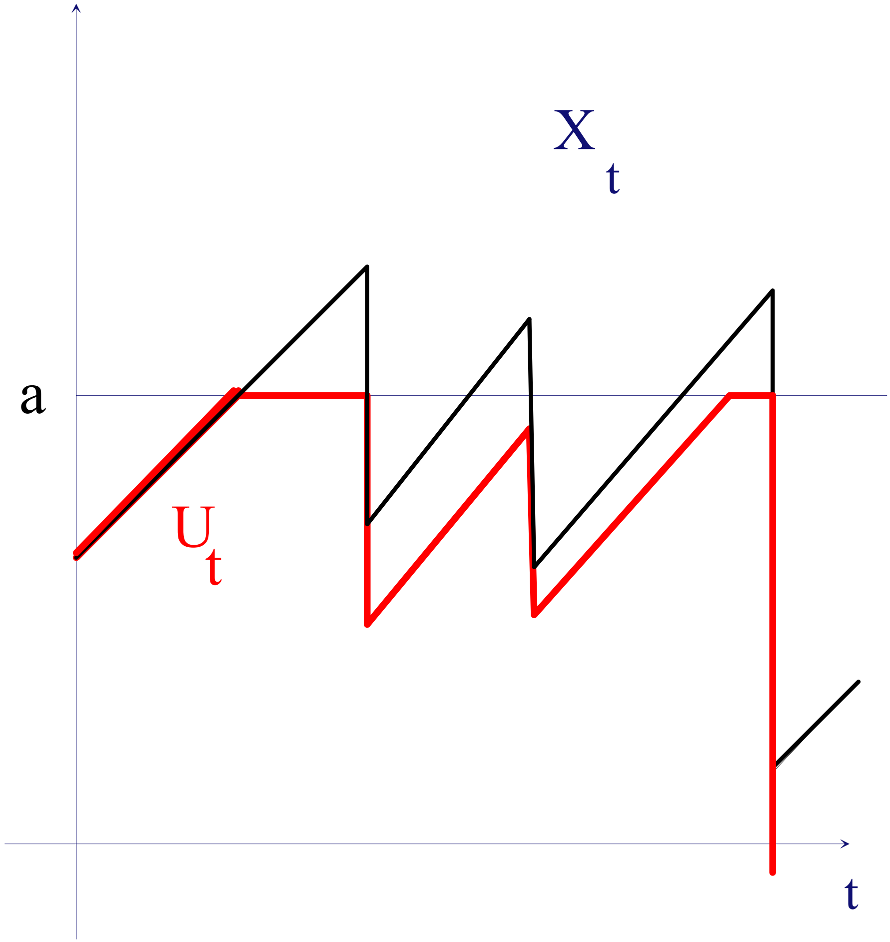
$$\sigma^\pi = \inf\{t \geq 0 : U_t^\pi < 0\}$$

Discounted value of paid dividend:

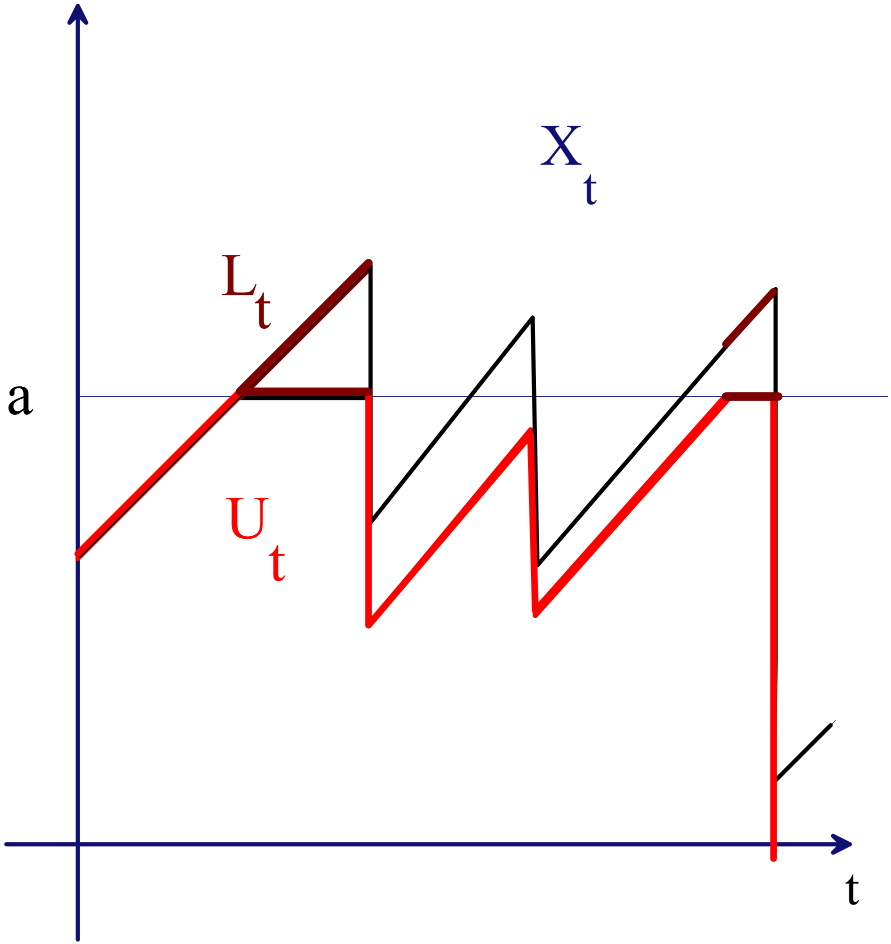
$$I_q^\pi = \int_0^{\sigma^\pi} \mathrm{e}^{-qt} dL_t^\pi$$

$v_*(x) = \sup_\pi \mathbb{E}_x \left[I_q^\pi\right]$ - the value function

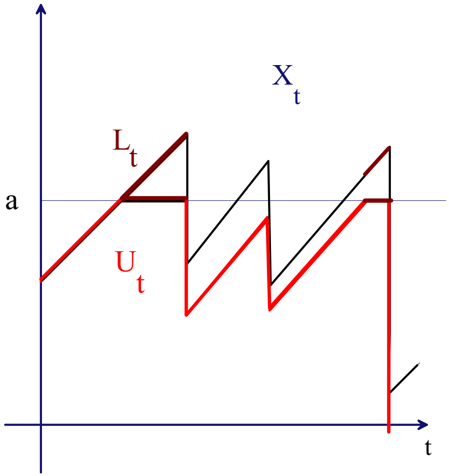
Barrier strategy π^a



Barrier strategy



Barrier strategy π^a



'If the barrier is too high, then we will wait too long for the risk process to hit the barrier and if we put the barrier too low then we derive the ruin too quickly.' We can then expect the existence of the 'optimal barrier'.

Distributional identity

For the barrier strategy with the barrier a:

$$L_t = a \vee \overline{X}_t - a, \qquad \text{where } \overline{X}_t = \sup_{s \le t} X_s$$

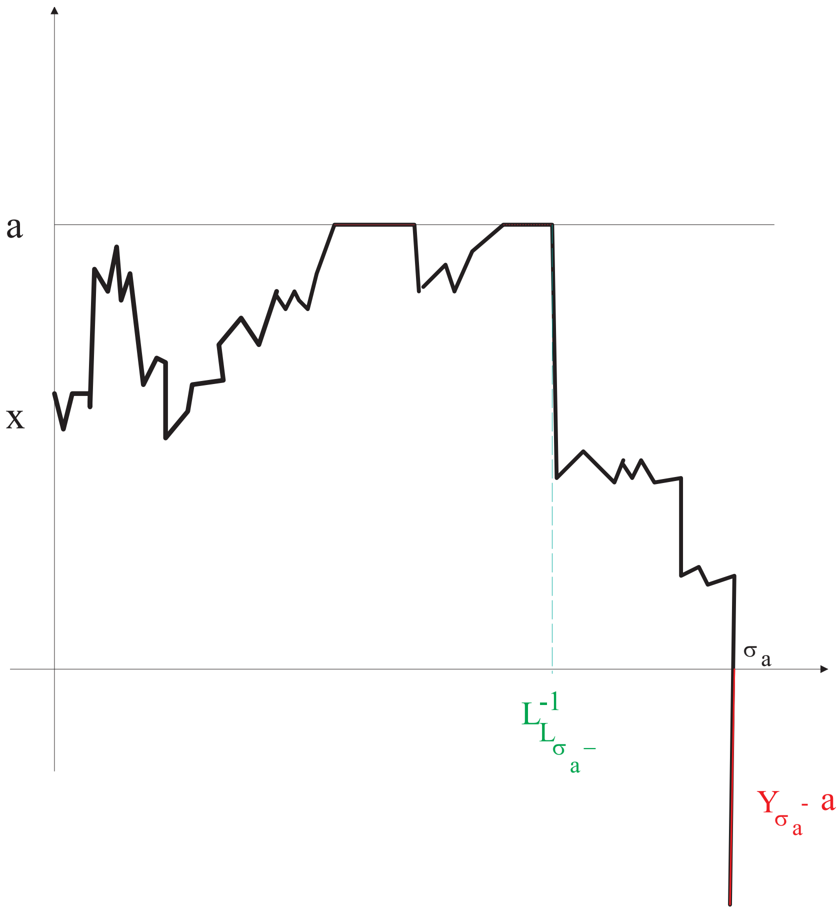
and

$$\sigma_a = \inf\{t > 0 : Y_t > a\},$$

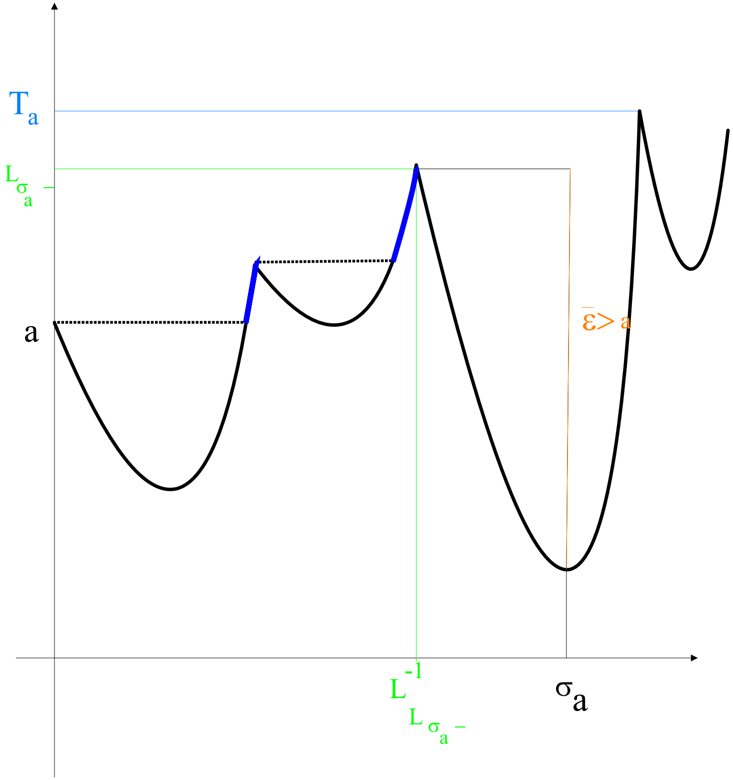
where

$$Y_t = (a \vee \overline{X}_t) - X_t$$

Controlled ruin proces once again



Discounted local time



Scale functions

Laplace exponent: $\psi(\theta)$:

$$\mathbb{E}[\mathrm{e}^{\theta X_t}] = \mathrm{e}^{t\psi(\theta)}$$

$\Phi(q)$ - greatest root of equation $\psi(\theta) = q$

First scaling function: $W^{(q)} : [0, \infty) \to [0, \infty)$:

$$\int_0^\infty \mathrm{e}^{-\theta x} W^{(q)}(y) dy = (\psi(\theta) - q)^{-1}, \qquad \theta > \Phi(q)$$

$W^{(q)}$ is differentiable (not necessary continuously) and $W(x) = W^{(0)}(x)$
Second scaling function:

$$Z^{(q)}(y) = 1 + q \int_0^y W^{(q)}(z)\, dz$$

The choice of the optimal barrier

For the barrier strategy the upper index π will be skipped.
Under this strategy the value function

$$v_a(x) = \mathbb{E}_x I_q = \begin{cases} \frac{W^{(q)}(x)}{W^{(q)\prime}(a)} & 0 \le x \le a \\ x - a + \frac{W^{(q)}(a)}{W^{(q)\prime}(a)} & x > a \end{cases}$$

Hence optimal barrier is:

$$a^* = \inf\{c > 0 : W^{(q)\prime}(c) \le W^{(q)\prime}(x) \text{ for all } x\}$$

where $\inf \emptyset = \infty$

If $W^{(q)} \in C^2(0, \infty)$, then

$$W^{(q)\prime\prime}(a^*) = 0$$

Optimality of the barrier strategy

Hamilton-Jacobi-Bellman's (HJB) system of equations:

$$\max\{\Gamma f(x) - qf(x), 1 - f'(x)\} = 0, \qquad x > 0,$$

where Γ denotes the extended generator of X

Theorem 1. (Avram, Palmowski and Pistorius AAP 17 2007) Assume that $\sigma > 0$ or that X has bounded variation or, otherwise, suppose that $v_{a^*} \in C^2(0,\infty)$. In classical dividend setting $a^* < \infty$ and the following hold true:

(i) π_{a^*} is the optimal strategy in the set $\Pi_{\leq a^*}$ of all bounded by a strategies and $v_{a^*} = \sup_{\pi \in \Pi_{\leq a}} v_\pi$.

(ii) If $(\Gamma v_{a^*} - qv_{a^*})(x) \leq 0$ for $x > a^*$, the value function and optimal strategy are given by $v_* = v_{a^*}$ and $\pi_* = \pi_{a^*}$, respectively.

Brownian motion with drift

$$X_t = \sigma B_t + \mu t$$

$$W^{(q)}(x) = \frac{1}{\sigma^2 \delta}[\mathrm{e}^{(-\omega+\delta)x} - \mathrm{e}^{-(\omega+\delta)x}]$$

$$Z^{(q)}(y) = y + \frac{2q}{\sigma^2} + \frac{q}{\sigma^2\delta}\left[\frac{1}{\omega+\delta}\mathrm{e}^{-(\omega+\delta)y} - \frac{1}{\delta-\omega}\mathrm{e}^{(-\omega+\delta)y}\right]$$

where

$$\delta = \sigma^{-2}\sqrt{\mu^2 + 2q\sigma^2}$$

and

$$\omega = \mu/\sigma^2$$

Hence:

$$a^* = \log\left|\frac{\delta+\omega}{\delta-\omega}\right|^{1/\delta}$$

Jeanblanc and Shiryaev 1995, Gerber and Shiu 2004

Exception or rule ?

$$(\Gamma v_{a^*} - q v_{a^*})(x) \leq 0 \quad \text{for} \quad x > a^*$$

where

$$\begin{aligned}\Gamma f(x) \;&=\; \frac{\sigma^2}{2} f''(x) + c f'(x) \\ &+ \int_{-\infty}^{0} \left[f(x+y) - f(x) - f'(x) y 1_{\{|y|<1\}} \right] \Pi(dy)\end{aligned}$$

for $f \in C^2(0, \infty)$ and

Π is a Lévy measure of process X

σ^2 is a Gaussian coefficient

Azcue and Muler 2005

Cramér-Lundberg model with Gamma distributed claims:

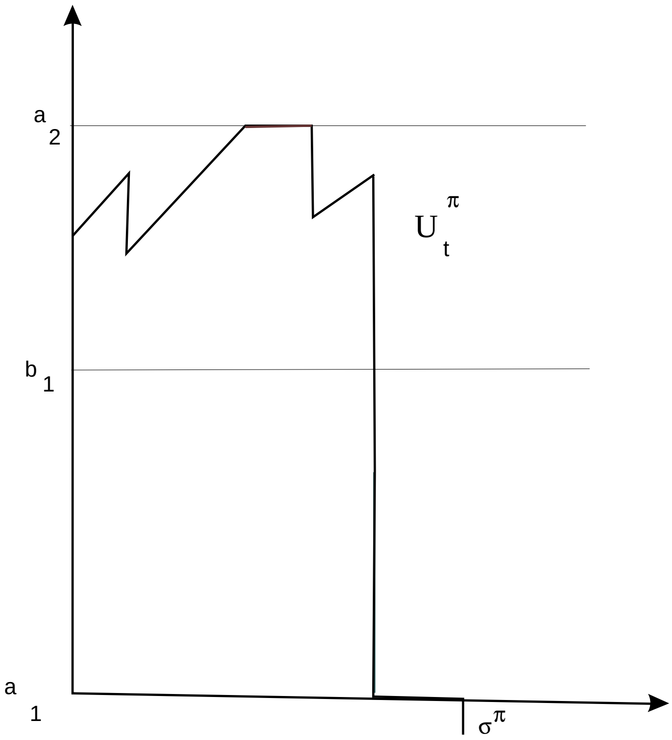
$$F(dx) = xe^{-x}dx,$$

the discount rate $q = 0.1$, the intensity $\lambda = 10$ of arrival Poisson process N_t, the premium rate $c = 2(1 + 0.07)\lambda$.

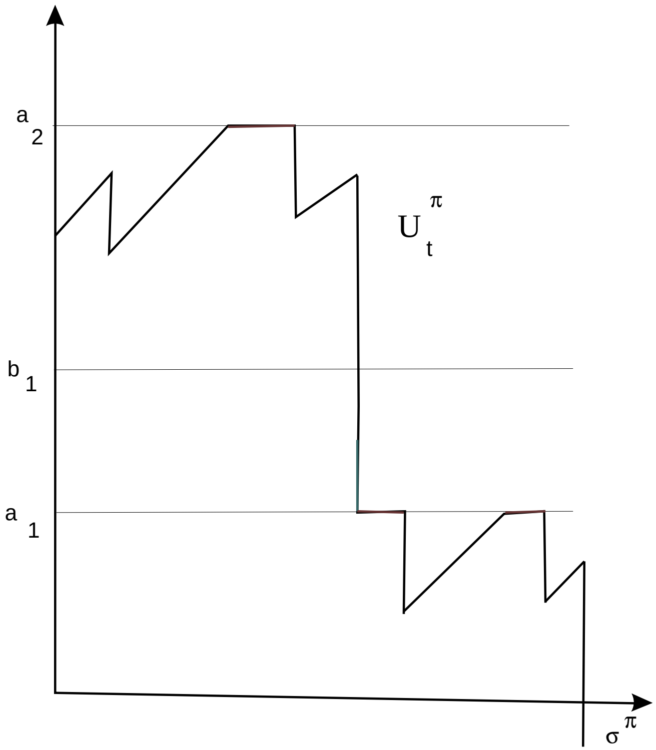
Then

$$v_*(x) = \begin{cases} x + 2.119 & x \in [0, 1.803) \\ 0.0944e^{-1.4882x} - 9.431e^{-0.07953x} + 11.257e^{-0.03957x} & x \in [1.803, 10.22) \\ x + 2.456 & x \geq 10.22 \end{cases}$$

Band strategy



Band strategy



Impulse control

$$\pi = \{(J_k, T_k), k \geq 0\}$$

where $0 \leq T_1 \leq T_2 \leq \ldots$ is an increasing sequence of $\mathbf{F}$-stopping times representing the times at which a dividend payment is made and J_i be a sequence of positive $\mathcal{F}_{T_i}$-measurable random variables representing the sizes of the dividend payments

K - a fixed cost

The controlled risk process

$$\overline{U}_t^{\pi} = X_t - L_t^{\pi} - KN_t^{\pi},$$

where

$$N_t^{\pi} = \#\{k : T_k \leq t\} \qquad L_t^{\pi} = \sum_{k=1}^{N_t^{\pi}} J_k$$

Band strategies

The value function:

$$\underline{v}_\pi(x) = \mathbb{E}_x\left[\int_0^{\sigma^\pi} \mathrm{e}^{-qt} dL_t^\pi - K\int_0^{\sigma^\pi} \mathrm{e}^{-qt} dN_t^\pi\right]$$

A band strategy with $a^- < a^+$:

1. Reducing the risk process U to level a^- if $x > a^+$

2. Each time when U hits the upper level a^+ make a payment of size $a_+ - a_-$

$$a^*(d) = \inf\{a \geq 0 : W^{(q)}(a+d) - W^{(q)}(a) \leq W^{(q)}(x+d) - W^{(q)}(x)\ \forall x \geq 0\}.$$

$$d^* = \inf\{d \geq 0 : W^{(q)}(a^*(d)+d) - W^{(q)}(a^*(d)) - (d-K)W^{(q)\prime}(a^*(d)+d) = 0\}$$

Optimal levels:

$$a_-^* = a^*(d^*) \qquad\qquad a_+^* = a^*(d^*) + d^*$$

Penalty function

$$v^\pi(x) = B^\pi(x) + H^\pi_w(x),$$

where

$$B^\pi(x) = \mathbb{E}_x \left[\int_0^{\sigma^\pi} \mathrm{e}^{-qt} dL^\pi_t \right]$$

and H^π_w denotes the Gerber-Shiu penalty function

$$H^\pi_w(x) = \mathbb{E}_x \left[\mathrm{e}^{-q\sigma^\pi} w(U_{\sigma^\pi}) \right]$$

associated to a penalty $w : \mathbb{R} \to \mathbb{R}_- \cup \{0\}$ ($w(x) = 0$ for $x \geq 0$). Furthermore, we assume w is an increasing function on $\mathbb{R}_-$, left-differentiable at 0. We want to find

$$v_*(x) = \sup_\pi v^\pi(x)$$

Band strategies

Definition 1. According to the band strategy $\pi_{\underline{b},\underline{a}}$ a lump-sum payment $U_t^{\underline{b},\underline{a}} - a_i$ is made if $U_t^{\underline{b},\underline{a}}$ is in (a_i, b_i), while no dividends are paid while $U_t^{\underline{b},\underline{a}}$ is in $[b_{i-1}, a_i)$ and the "overflow" of $U_t^{\underline{b},\underline{a}}$ over a_i are paid out as dividends.

Theorem 2. (Avram, Palmowski and Pistorius 2009) For $i \geq 1$, it holds that

$$v_{\underline{b},\underline{a}}(x) = \begin{cases} v_{\underline{b},\underline{a}}(b_{i-1}-) + \frac{W^{(q)}(x-b_{i-1})}{W^{(q)\prime}(a_i-b_{i-1})}\left[1 - D_{i-1}(a_i - b_{i-1})\right] \\ \qquad\qquad + \overline{D}_{i-1}(x - b_{i-1}) & \text{if } x \in [b_{i-1}, a_i) \\ \\ v_{\underline{b},\underline{a}}(a_i-) + x - a_i & \text{if } x \in [a_i, b_i), \end{cases}$$

where $v_{\underline{b},\underline{a}}(x) = w(x)$ for $x < 0$ and $D_i(y) = H_i(y) + F_i(y)$ ($i \geq 1$) with

$$\begin{aligned} H_i(y) &= Z^{(q)}(y) - [\psi'(0) - q v_{\underline{b},\underline{a}}(b_i-)]W^{(q)}(y), \\ F_i(y) &= -\int_0^y W^{(q)\prime}(y-z)K_i(z)dz - W^{(q)}(0)K_i(y), \\ K_i(y) &= \int_y^\infty \left(v_{\underline{b},\underline{a}}(b_i + y - z) - [v_{\underline{b},\underline{a}}(b_i-) + y - z]\right)\nu(dz), \end{aligned}$$

and $K_0(y) = \int_y^\infty (w(y-z) - w(0-) - w'(0)(y-z))\,\nu(dz)$, $H_0(y) = w'(0-)Z^{(q)}(y) - [w'(0-)\psi'(0) - w(0-)q]W^{(q)}(y)$, $\Pi(-\infty, -x) = \nu(x, \infty)$.

Band strategies

Level a_i is determined by the smooth fit condition of singular control:

$$0 = \lim_{x \downarrow a_i} v''_{\underline{a},\underline{b}}(x) = \lim_{x \uparrow a_i} v''_{\underline{a},\underline{b}}(x), \tag{2}$$

and similarly the level $b_i > 0$ is determined by the smooth fit condition

$$1 = \lim_{x \uparrow b_i} v'_{\underline{a},\underline{b}}(x) = \lim_{x \downarrow b_i} v'_{\underline{a},\underline{b}}(x), \tag{3}$$

if X has unbounded variation (or, equivalently, if X when starting at 0 immediately enters the positive half-axis almost surely), and determined by the continuous fit condition

$$v_{\underline{a},\underline{b}}(b_i-) = \lim_{x \uparrow b_i} v_{\underline{a},\underline{b}}(x) = \lim_{x \downarrow b_i} v_{\underline{a},\underline{b}}(x) \tag{4}$$

if X has bounded variation (or, equivalently, if it takes a strictly positive time for X to enter the positive half-axis almost surely).

Erlang $(2, \mu)$ claims

A numerical example for Cramér-Lundberg model

Taking $w(x) = 0.2x$ (penalty function), $\lambda = 10$ (intensity of claims arrivals), $\mu = 1$, $c = 21.4$ (premium rate), $q = 0.1$ (discounting rate) the optimal strategy is a 2-band strategy:

$$v_*(x) = \begin{cases} 0.2x & \text{for } x < 0 \\ x + 1.72277 & \text{for } 0 \le x \le 1.211 \\ 11.1287e^{0.039567x} - 9.6499e^{-0.079355x} & \\ +0.149139e^{-1.48825x} & \text{for } 1.211 < x \le 10.5051 \\ x + 2.16631 & \text{for } x > 10.5051 \end{cases}$$

Refraction

Process U solves equation:

$$U_t = X_t - \delta \int_0^t \mathbf{1}_{\{U_s > a\}} \, ds$$

Discounted cumulant dividends (Kyprianou and Loeffen 2010 and Gerber and Shiu 2006):

$$E_x \int_0^{\sigma_a} e^{-qt} \mathbf{1}_{\{U_s > a\}} \, ds = - \int_0^{0 \wedge (x-a)} \mathbf{W}^{(q)}(z) \, dz$$
$$+ \frac{W^{(q)}(x) + \delta \mathbf{1}_{\{x>a\}} \int_x^a \mathbf{W}^{(q)}(x-y) W^{(q),\prime}(y) \, dy}{\phi(q) \int_0^\infty e^{-\phi(q)y} W^{(q),\prime}(y+a) \, dy}$$

where

$$\phi(q) = \sup\{\psi(\theta) - \delta\theta = q\}$$

and $\mathbf{W}^{(q)}$ and $\mathbf{Z}^{(q)}$ are the scale function associated with process $X_t - \delta t$.

Taxes

$$U_t^\gamma = X_t - \int_0^t \gamma(\overline{X}_s)\, d\overline{X}_s$$

The process U^γ models the surplus process of an insurance company that pays out taxes according to a loss-carried-forward tax scheme, using a surplus-dependent rate $\gamma(\cdot)$. In other words, tax are collected when the company has recovered from its previous losses, i.e., is in a so-called profitable situation. Finally, note that when $\gamma(\cdot) = \gamma \in [0, 1]$, this model amounts to the situation studied in Albrecher et al. 2008 where the tax rate is constant, and when $\gamma = 1$, we retrieve the model where the company pays out as dividends any capital above its initial value $U^\gamma = x$ as in a risk model with a horizontal barrier strategy at level u (see e.g. Renaud and Zhou 2007).

$$E_x \int_0^{\sigma_a} e^{-qt}\gamma(\overline{X}_s)\, d\overline{X}_s = \int_x^\infty \exp\left\{ - \int_x^t \frac{W^{(q),\prime}(\overline{\gamma}(s))\, ds}{W^{(q)}(\overline{\gamma}(s))\, ds} \right\} \gamma(t)\, dt$$

where $\overline{\gamma}(y) = y - \int_x^y \gamma(s)\, ds$.

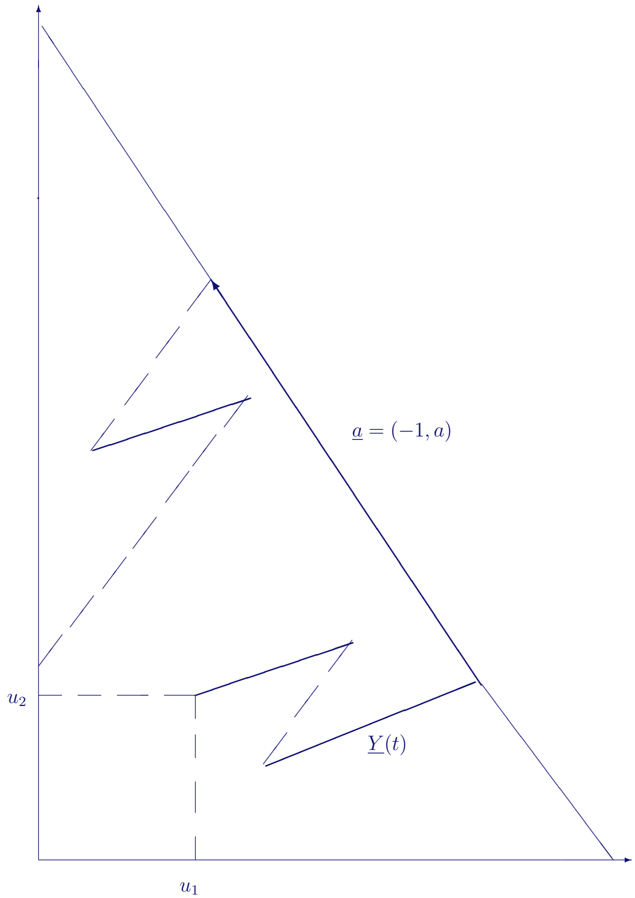
Two-dimensional risk process

Consider now a particular two-dimensional risk model in which two companies split the amount they pay out of each claim in fixed proportions (for simplicity we assume that they are equal), and receive premiums at rates c_1 and c_2, respectively (so-called proportional reinsurance). That is,

$$\underline{X}_t = (X_1(t), X_2(t)) = \left(u_1 + c_1 t - \beta_1 \sum_{i=1}^{N_t} C_i, u_2 + c_2 t - \beta_2 \sum_{i=1}^{N_t} C_i \right).$$

Without los of generality we will assume that $\beta_1 = \beta_2 = 1$ and $c_1 > c_2$.

Two-dimensional risk process



Two-dimensional risk process

Controlled risk process:

$$\underline{U}_t = (U_1(t), U_2(t)) = \underline{X}_t - \underline{L}_t$$

where

$$\underline{L}(t) = \left(\delta_1 \int_0^t \mathbf{1}_{\{\underline{Y}(t) \in \mathcal{B}\}},\ \delta_2 \int_0^t \mathbf{1}_{\{\underline{Y}(t) \in \mathcal{B}\}} \right)$$

describes the two-dimensional linear drift at rate $\underline{\delta} = (\delta_1, \delta_2) > (0,0)$ which is subtracted from the increments of the risk process whenever it enter the fixed set:

$$\mathcal{B} = \{(x,y) : x, y \geq 0 \quad \text{and} \quad y \geq b - ax\}, \qquad a, b > 0.$$

The case $\underline{\delta} = \underline{c} - \underline{a}$ for $\underline{c} = (c_1, c_2)$ and $\underline{a} = (-1, a)$ corresponds to the reflecting the risk process at the line $y = b - ax$. Let

$$v_n(u_1, u_2) = v_n(\underline{u}) = \mathbb{E}_{\underline{u}} \left[(1,1) \cdot \int_0^\sigma e^{-qt}\, d\underline{L}(t) \right]^n$$

where $\sigma = \inf\{t \geq 0 : U_1(t)U_2(t) < 0\}$.

Two-dimensional risk process

Theorem 3. (Czarna and Palmowski 2009)

$$\underline{c} \cdot \frac{\partial v_n}{\partial \underline{u}}(\underline{u}) - (\lambda + nq) v_n(\underline{u}) + \lambda \int_0^{\min(u_1,u_2)} v_n(\underline{u} - (1,1)v)\, dF(v) = 0$$

with the boundary conditions:

$$n\delta_0\, V_{n-1}(\underline{u}) = \underline{\delta} \cdot \left.\frac{\partial v_n}{\partial \underline{u}}\right|_{\underline{u} \in \mathcal{B}}, \qquad \underline{u} \in \mathcal{B}$$

$$\lim_{b \to \infty} v_n(\underline{u}) = 0, \qquad \underline{u} \in \mathcal{B}^{\mathrm{c}}$$

$$v_n(0, b) = 0.$$

Numerical analysis

Assume that we have $\mathrm{Exp}(\mu)$ claims with $\mu = 2$ and that $c_1 = 4$, $c_2 = 3$, $\lambda = 1$, $q = 0.1$.

Note that always there exists optimal choice of linear barrier (choice of its upper left end $(0, b)$ and it slope a). This choice depends on the initial reserves (u_1, u_2). For $(u_1, u_2) = (1, 2)$ the optimal barrier is determined by $b = 14$ and $a = 0.1$ and for $(u_1, u_2) = (2, 3)$ the optimal barrier is determined by $b = 15$ and $a = 0.1$. This is contrast to the one-dimensional case where the choice of the barrier is given only via the premium rate and the distribution of the arriving claims.

a	b					
	6	8	14	15	20	28
0.1	19.85	27.20	34.95	34.93	32.48	25.89
0.2	16.33	24.31	33.82	34.19	33.32	28.03
0.5	11.76	17.74	28.98	30.01	32.54	31.21
1	7.22	11.40	21.35	22.59	27.17	30.07

Expected value of dividend payments depending on a and b for fixed $(u_1, u_2) = (1, 2)$.

THANK YOU

- for the Invitation !
- for Your Attention !

Bibliography

- Albrecher H. and Thonhauser S. (2009) Optimality Results for Dividend Problems in Insurance. *RACSAM Rev. R. Acad. Cien. Serie A. Mat.* **103(2)**, 295-320.
- Albrecher H. and Kainhofer R. (2002) Risk theory with a non-linear dividend barrier. *Computing* **68(4)**, 289-311.
- Albrecher H., Hartinger J. and Tichy R. (2005) On the distribution of dividend payments and the discounted penalty function in a risk model with linear dividend barrier. *Scandinavian Actuarial Journal*, 103-126.
- Albrecher, H., Renaud, J.-F. and Zhou, X. (2008) A Lévy insurance risk process with tax. *Journal of Applied Probability* **45(2)**, 363-375.
- Asmussen, S., Avram, F. and Pistorius, M. (2004) Russian and American put options under exponential phase-type Lévy models. *Stochastic Process. Appl.* **109(1)**, 79–111.
- Avram, F., Kyprianou, A.E. and Pistorius, M.R. (2004) Exit problems for spectrally negative Lévy processes and applications to (Canadized) Russian options. *Ann. Appl. Probab.* **14**, 215–238.

Bibliography

- Avram, F., Palmowski, Z. and Pistorius, M. R. (2007) On the optimal dividend problem for a spectrally negative Lévy process. *Ann. Appl.*P. 17, 156-180.
- Avram, F., Palmowski, Z. and Pistorius, M. R. (2010) On optimal dividend distribution for a Lévy risk-process in the presence of a Gerber-Shiu penalty function. Submiited for publication.
- Czarna, I. and Palmowski Z. (2009) De Finetti's dividend problem and impulse control for a two-dimensional insurance risk process. Submitted for publication.
- De Finetti, B. (1957) Su un'impostazion alternativa dell teoria collecttiva del rischio. *Transactions of the XVth International Congress of Actuaries* 2, 433-443.
- Dickson, D. C. M. and Waters, H. R. (2004) Some optimal dividends problems. *Astin Bull.* 34, 49-74.

Bibliography

- Gerber, H. and Shiu, E. (2006) On optimal dividends: From reflection to refraction. *Journal of Computational and Applied Mathematics* **186**, 4-22.
- Gerber, H. (1981) On the probability of ruin in the presence of a linear dividend barrier. *Scandinavian Actuarial Journal*, 105-115.
- Gerber, H.U. and Shiu, E.S.W. (2004) Optimal dividends: analysis with Brownian motion, *North American Actuarial Journal* **8**, 1-20.
- Jeanblanc, M. and Shiryaev, A.N. (1995) Optimization of the flow of dividends, *Russian Math. Surveys* **50**, 257-277.
- Kyprianou, A. and Palmowski, Z. (2007) Distributional study of De Finetti's dividend problem for a general Lévy insurance risk process. *Journal of Applied Probability* **44(2)**, 428-443.
- Kyprianou, A., Rivero, V. and Song, R. (2008) Convexity and smoothness of scale functions and de Finetti's control problem. To appear in *Journal of Theoretical Probability*.
- Kyprianou A. and Loeffen, R. (2010) Refracted Lévy processes. To appear in *Annales de l'Institut Henri Poincaré.*

Bibliography

- Loeffen R. (2008) On optimality of the barrier strategy in de Finetti's dividend problem for spectrally negative Lévy processes. *Annals of Applied Probability* **18(5)**, 1669-1680.
- Loeffen, R. (2009) An optimal dividends problem with transaction costs for spectrally negative Lévy processes. *Insurance: Mathematics and Economics*, **45(1)**, 41-48.
- Loeffen, R. and Renaud, J.F. (2009) De Finetti's optimal dividends problem with an affine penalty function at ruin. To appear in *Insurance: Mathematics and Economics.*
- Løkka, A. and Zervos, M. (2005) Optimal dividend and issuance of equity policies in the presence of proportional costs. *Preprint.*
- Renaud, J.F. (2009) The distribution of tax payments in a Lévy insurance risk model with a surplus-dependent taxation structure. *Insurance: Mathematics and Economics* **45**, 242-246.
- Harrison, J. M. and Taylor, A.J. (1978) Optimal control of a Brownian storage system, *Stoch. Process. Appl.* **6**, 179–194.