Factor Model Forecasts of Inflation in Croatia

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Motivation

1. Many evidences that there exist few unobservable variables - *common factors*, governing the whole economy. How to construct or identify these driving forces? (Stock and Watson, JBES, 2002)

2. Most empirical analysis of monetary policy wrongly presume that central bankers’ decisions are grounded on only few key macro variables while in practice the thousands of variables are being monitored. How to formalise this? (Bernanke and Boivin, JME, 2003)

3. Transition economies are dealing with data having quite short time spans (less than 10 years of reliable inflation data for Croatia). So, any sort of *data compression* methods - welcomed!
Objective

- This analysis tests whether information extracted from 144 economic variables can help in forecasting CPI inflation in Croatia
Forecasting Model

Model (James Stock & Mark Watson, JBES, 2002)

- Assume that for given observations of $N = 144$ variables $X_1, \ldots, X_N$ there exist $r$ ($\ll N$, hopefully just 2 or 3) factors $F_1, \ldots, F_r$ such that:

$$X_{it} = \lambda_{i1}F_{1t} + \ldots + \lambda_{ir}F_{rt} + \varepsilon_{it}, \quad i = 1, \ldots, N, \; t = 1, \ldots, T$$

- Forecasting model ($h$ - steps - ahead forecast):

$$X^T = \Lambda F^T + \varepsilon^T$$

$$y_{t+h} = \alpha_h + \beta_0 F_t + \ldots + \beta_p F_{t-p} + \gamma_0 y_t + \ldots + \gamma_q y_{t-q} + \epsilon_{t+h}$$

- (Too) many unknowns: $\Lambda$, $F$ and noise variances, so in order to estimate the system some restrictions need to be imposed!
Forecasting Model

Estimation

Assume $\Lambda^\tau \Lambda = I_r$ and minimise the sum of squares:

\[
V(F, \Lambda) = \sum_{i=1}^{N} \sum_{t=1}^{T} \varepsilon_{it}^2 = \|X^\tau - \Lambda \hat{F}^\tau\|^2_F,
\]  
(1)

Applying (multiple multivariate version) least squares we estimate factors:

\[
\hat{F}^\tau = (\Lambda^\tau \Lambda)^{-1} \Lambda^\tau X^\tau = (\Lambda^\tau \Lambda = I_r) = \Lambda^\tau X^\tau,
\]  
(2)

By plugging (2) into (1) we have:

\[
V(\hat{F}, \Lambda) = \|X^\tau - \Lambda \hat{F}^\tau\|^2_F = \|X\|^2_F - tr(\Lambda^\tau X^\tau X \Lambda).
\]  
(3)
Forecasting Model

Estimation

- Solve the problem:

\[
\max_\Lambda \text{tr}(\Lambda^\tau X^\tau X \Lambda), \quad \Lambda^\tau \Lambda = I_r
\]

- Setting \( \hat{\Lambda} \) equal to \( k \) eigenvectors of \( X^\tau X \) corresponding to \( k \) largest eigenvalues yields the principal components estimator \( \hat{F} = X \hat{\Lambda} \)
Results

- **Factor model**

\[ X^T = \Lambda F^T + \varepsilon^T \]

\[ y_{t+h} = \alpha_h + \beta_0 F_t + \ldots + \beta_p F_{t-p} + \gamma_0 y_t + \ldots + \gamma_q y_{t-q} + \epsilon_{t+h} \]

challenged against non-trivial AR benchmark in inflation forecasting exercise

- forecasts evaluated using MSE relative to benchmark
  (Relative MSE\(<1 \Rightarrow \text{factor model beats benchmark}): \)

<table>
<thead>
<tr>
<th>Horizon</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative MSE</td>
<td>0.91</td>
<td>0.81</td>
<td>0.72</td>
<td>0.92</td>
</tr>
</tbody>
</table>
Results

$R^2$ statistics between first two factors and 144 variables of Croatian economy
Results

$R^2$ statistics between tenth and eleventh factor and 144 variables of Croatian economy
Principal Component as a summary statistics

- Recent analysis by CNB deals with the impact of USD/EUR rate on CEECs inflation rates
- Effect seems to be quite strong, especially in countries fixing their ER to EUR (Croatia, Bulgaria, Estonia)
- Fixers’ inflation rates principal component highly correlated to USD/EUR ER
- Common force driving CEECs inflation rates?
Principal Component as a summary statistics

Fixers’ inflations principal component and EUR/USD exchange rate
Bai, J., and S. Ng (2002), *Determining the number of factors in approximate factor models*, Econometrica 70, pp. 191-221.
