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Timely Issues in Risk Management. Uncertainty and Subjectivity in Quantitative Science.

Klaus Böcker

CQD Workshop, Zagreb, April 9, 2010

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A special report on financial risk

Number-crunchers crunched

The uses and abuses of mathematical models

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Correction to this article

IT PUT noses out of joint, but it changed markets for good. In the mid-1970s a few progressive occupants of Chicago's options pits started trading with the aid of sheets of theoretical prices derived from a model and sold by an economist called Fisher Black. Rivals, used to relying on their wits, were unimpressed. One model-based trader complained of having his papers snatched away and being told to "trade like a man". But the strings of numbers caught on, and soon derivatives exchanges hailed the Black-Scholes model, which used share and bond prices to calculate the value of derivatives, for helping to legitimise a market that had been derided as a gambling den.

Thanks to Black-Scholes, options pricing no longer had to rely on educated guesses.

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Disclaimer

The opinions expressed in this talk are those of the speaker and do not necessarily reflect the views of UniCredit Group.

Presented risk control and measurement concepts are not necessarily used by UniCredit Group or any affiliates.

Outline

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"How fast can we go?"

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- "Not faster than 68.483 km/h ?"


"How fast can we go?"
"Not faster than 68.483 km/h ?"
"Not faster than roughly about 50-60 km/h ?"

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How accurate is risk measurement?

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Example:

Pillar 3 report for EC (illustrative)

Economic capital [€ mn]	Dec 31, 2008
Market risk	3,564
Credit risk	9,981
Operational risk	2,612
Business risk	1,666
Total EC	**17,823**

How accurate is risk measurement?

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Total EC	**17,823**

The suggested absolute uncertainty of total EC is ± 1EUR.

This corresponds to a relative uncertainty of 0.0056 %.

How accurate is risk measurement? (cont'd)

Questions:

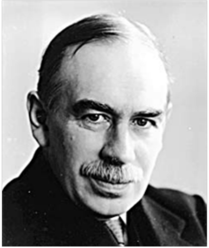
- What is the total total aggregated EC?
- What is the price of an ABS CDO?
- What is operational risk at a confidence level of 99.9%?
- What is the bank's loss given ΔGDP $= -1\%$?

Models

Claudia Schiffer

"A model is a model;
the reality is sometimes less perfect."

Modelers



John M. Keynes

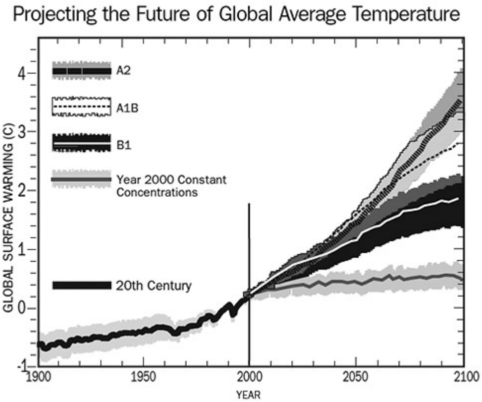
"It is better to be roughly right than precisely wrong."

Model uncertainty

Model uncertainty

Positive examples . . .

Model uncertainty: climatology



Model uncertainty: particle physics

Properties of the electron as reported by the *Particle Data Group*:

e $J = \frac{1}{2}$

Mass $m = (548.57990945 \pm 0.00000024) \times 10^{-6}$ u

Mass $m = 0.51099892 \pm 0.00000004$ MeV

$\left|m_{e+} - m_{e-}\right|/m < 8 \times 10^{-9}$, CL = 90%

$\left|q_{e+} + q_{e-}\right|/e < 4 \times 10^{-8}$

Magnetic moment $\mu = 1.0011596521859 \pm 0.0000000000038\ \mu_B$

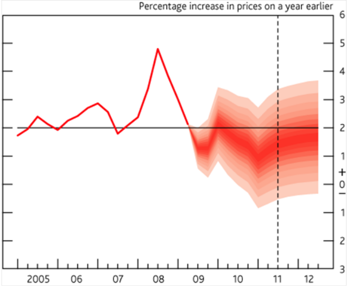
$(g_{e+} - g_{e-})\ /\ g_{\rm average} = (-0.5 \pm 2.1) \times 10^{-12}$

Electric dipole moment $d = (0.07 \pm 0.07) \times 10^{-26}\ e\,$cm

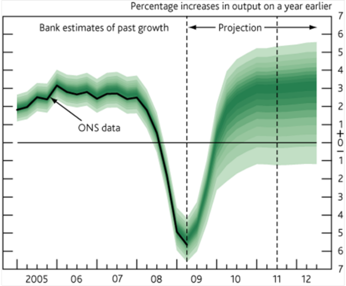
Mean life $\tau > 4.6 \times 10^{26}$ yr, CL = 90% [a]

Model uncertainty: Economy

Inflation Report, Bank of England (August 2009)



CPI Forecast



GDP Forecast

Model uncertainty

Negative examples . . .

Model uncertainty: Risk measurement

Economic capital [€ mn]	**Dec 31, 2008**
Market risk	3,564
Credit risk	9,981
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Model uncertainty: Consequences

Some situations require a careful and cautious assessment of a situation. This means:

- Do not blindly rely on formalized mathematical models often based on strong and unrealistic assumptions.
- Account for model uncertainty by applying more than a single model.
- Account for parameter uncertainty within a given model.

Outline

The Dahlem Report

The Financial Crisis and the Systemic Failure of Academic Economics*

David Colander,
Department of Economics
Middlebury College
Middlebury, VE, USA

Hans Föllmer
Department of Mathematics
Humboldt University Berlin
Berlin, Germany

Armin Haas
Potsdam Institute for Climate Impact Research
Potsdam, Germany

Michael Goldberg
Whittemore School of Business & Economics
University of New Hampshire
Durham, NH, USA

Katarina Juselius
Department of Economics
University of Copenhagen
Copenhagen, Denmark

Alan Kirman
GREQAM, Université d'Aix-Marseille lll,
EHESS et IUF
Marseille, France

Thomas Lux[1]
Department of Economics
University of Kiel
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Kiel, Germany

Brigitte Sloth
Department of Business and Economics
University of Southern Denmark
Odense, Denmark

The Dahlem Report (cont'd)

Abstract: The economics profession appears to have been unaware of the long build-up to the current worldwide financial crisis and to have significantly underestimated its dimensions once it started to unfold. In our view, this lack of understanding is due to a misallocation of research efforts in economics. We trace the deeper roots of this failure to the profession's insistence on constructing models that, by design, disregard the key elements driving outcomes in real-world markets. The economics profession has failed in communicating the limitations, weaknesses, and even dangers of its preferred models to the public. This state of affairs makes clear the need for a major reorientation of focus in the research economists undertake, as well as for the establishment of an ethical code that would ask economists to understand and communicate the limitations and potential misuses of their models.

Researchers, quants, economists *"have an ethical responsibility to communicate the limitations of their models and the potential misuses of their research."* [Colander et al. (2008)]

Outline

Why we have used models

Why we have used models

Why we have used models

Models have been used for creating numbers and numbers and numbers . . .

- Risk controlling
- Risk measurement
- Capital ratios
- Risk appetite

Why we have used models (cont'd)

Models have been used to construct financial products (Financial Engineering) for

- hedging risk,
- pricing risk,

Why we have used models (cont'd)

Models have been used to construct financial products (Financial Engineering) for

- hedging risk,
- pricing risk,

and for

- increasing leverage

Ever more complex products have been developed with the main intension to place refined bets on the market, thereby significantly increasing systematic risk.

Why we also should use models

Models are essential for gaining a deeper understanding of a subject matter.

Why we also should use models

Models are essential for gaining a deeper understanding of a subject matter.

- Models provide insight into underlying relationships of a system.
- Models are "intelligence amplifiers".
- Models can lead to conclusions that sheer intuition cannot perceive.

Provided they are used the right way, models are still a sine qua non for successful risk management.

Outline

The loss distribution approach

Definition (Operational risk)

The risk of losses resulting from inadequate or failed processes, people and systems, or external events.

Definition (Standard LDA model)

- The loss severities $(X_k)_{k \in \mathbb{N}}$ are positive iid random variables with distribution function F describing the magnitude of each loss event.
- The number $N(t)$ of loss events in the time interval $[0, t]$ for $t \geq 0$ is random and is described by the frequency process $(N(t))_{t \geq 0}$.
- The severity process and the frequency process are assumed to be independent.
- The aggregate loss process is given by $S(t) = \sum_{k=1}^{N(t)} X_k$.

Operational Value-at-Risk

Let $G_t(x) = P(S(t) \leq x)$ be the aggregate loss distribution function at time $t \geq 0$.

Definition (Operational Value-at-Risk)

Operational VAR (OpVAR) up to time t at confidence level κ is defined as

$$\mathrm{VAR}_t(\kappa) = G_t^{\leftarrow}(\kappa)\,, \quad \kappa \in (0,1)\,,$$

with $G_t^{\leftarrow}(\kappa) = \inf\{x \in \mathbb{R} : \; G_t(x) \geq \kappa\}, \; 0 < \kappa < 1.$

Typically: $\kappa = 0.999, 0.9995, 0.9998$.

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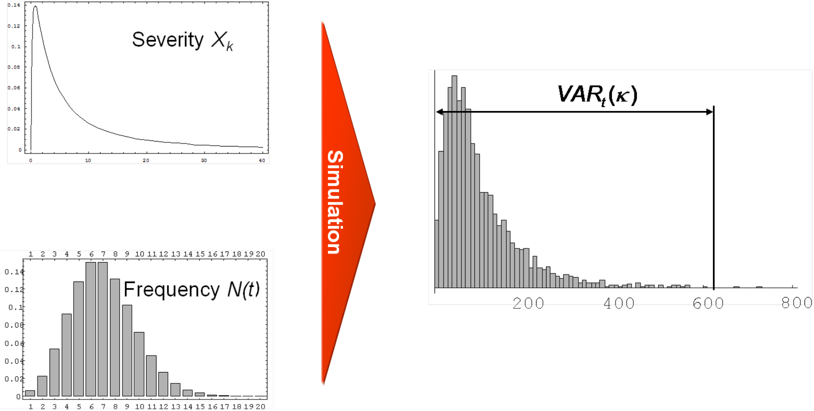
Questions:

- Time scaling of OpVAR?
- Interpretation of OpVAR?
- Accuracy???

Simulating operational risk



Heavy tails and subexponential distributions

Operational lossesnare heavyly tailed.
What does this mean mathematically?

Definition (Subexponential distributions)

Let $(X_k)_{k\in\mathbb{N}}$ be iid random variables with distribution function F.
Then F is said to be a subexponential distribution $(F \in \mathcal{S})$ if

$$\lim_{x\to\infty} \frac{P(X_1 + \cdots + X_n > x)}{P(\max(X_1, \ldots, X_n) > x)} = 1 \quad \text{for some (all) } n \geq 2.$$

This means:

As far as you are concerned with OpVAR at a high confidence level, forget about the small losses because only the severe losses matter (single loss approximation).

Approximating univariate OpVAR

In a Standard LDA model with fixed $t > 0$ (e.g. $t = 1$ year) and subexponential distribution severity F, we have that (Böcker and Klüppelberg (2005))

$$\mathrm{VAR}_t(\kappa) = G_t^{\leftarrow}(\kappa) = F^{\leftarrow}\left(1 - \frac{1-\kappa}{EN(t)}(1 + o(1))\right)\,, \quad \kappa \uparrow 1\,,$$

where $EN(t)$ is the expected number of losses in $[0, t]$.

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where $EN(t)$ is the expected number of losses in $[0, t]$.

Example:

Now assume that $EN(t) = 100$ and $\kappa = 0.9997$. Then,

$$\mathrm{VAR}_t(0.9997) \approx F^{\leftarrow}(0.999997)$$

corresponding to a 1 in 333,333 years event!

Outline

Risk compensation

Definition (Risk compensation)

- Individuals have an innate propensity to take risks.
- people endeavor to keep this level constant and, therefore, permanently balance their behavior in response to the perceived risk.

Example: anti-lock brakes on automobiles, car safety belts, cycle helmets for children and sky diving.

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Example: anti-lock brakes on automobiles, car safety belts, cycle helmets for children and sky diving.

- The feeling of greater security tempts people to be more reckless.
- Pseudo accuracy (caused by neglecting uncertainty) leads to the wrong subjective risk perception and may create a dangerous overconfidence in the decision maker.

Overconfidence

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- Overestimation of ones actual performance.
- Overplacement of ones performance relative to others.
- Excessive precision in ones beliefs.

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Example: (Ben-David, I. et al. 2007)

- Realized market returns are within the executives' 80 % confidence intervals only 38 % of the time.
- Companies with overconfident CFOs
  - use lower discount rates to value cash flows,
  - use more debt,
  - are less likely to pay dividends and more likely to repurchase shares,
  - use proportionally more long-term (as opposed to short-term) debt.

Outline

Statistical inference

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Aim:

- *"The business of statistics is to provide information or conclusion about uncertain quantities and to convey the extent of uncertainty in the answer"*[Berger (1980)].

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- *"The business of statistics is to provide information or conclusion about uncertain quantities and to convey the extent of uncertainty in the answer"*[Berger (1980)].
- Obtain statistical results in a way so that they can be easily utilized also by non-statisticians for decision making.

Parametric statistical modeling

- A statistical investigation is performed to obtain information about a certain state of nature θ (e.g. state of the economy, credit risk portfolio).
- The nature is approximated by a model and θ is a parameter (e.g. GDP, CPI, PD, LGD, volatility).

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- A statistical investigation is performed to obtain information about a certain state of nature θ (e.g. state of the economy, credit risk portfolio).
- The nature is approximated by a model and θ is a parameter (e.g. GDP, CPI, PD, LGD, volatility).
- A vector $x = (x_1, \ldots, x_n)$ of observations is collected. The probability distribution of X depends on the sate of nature and thus on the model and θ.
- $f(x|\theta)$ is the joint density of the data X given a value of θ.
- Often, functions $\Delta(\theta)$ of the parameters θ are of interest (e.g. VaR, ES)

The key questions

What is f?

What is θ?

"The story that I have to tell is marked all the way through by a persistent tension between those who assert that the best decisions are based on quantification and numbers, determined by the patterns of the past, and those who base their decisions on more subjective degrees of belief about the uncertain future. This is a controversy that has never been resolved."

From the introduction to Against the Gods:
The remarkable story of risk, by Peter l. Bernstein

Patterns of the past: an example

The New York Times

Panicked Traders Take VW Shares on a Wild Ride

By LOUISE STORY, MICHAEL J. DE LA MERCED and CARTER DOUGHERTY
Published: October 28, 2008

October 26, 2008

*This article is by **Louise Story**, **Michael J. de la Merced** and **Carter Dougherty**.*

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Short Squeeze

Shares in Volkswagen more than quadrupled in this week, possibly because traders who had sold shares short had to cover their positions.

VOLKSWAGEN SHARE PRICE

Daily closes on the Frankfurt exchange

The auto industry is struggling, but for a few minutes on Tuesday, Volkswagen became the most valuable company in the world, one with a market value greater than Apple, Philip Morris and Intel combined.

That soaring value reflected engineering of a financial, rather than automotive, sort. It came as stock traders scrambled when Porsche, a rival seeking to build one of Europe's great car dynasties, revealed it had increased its holdings in VW, giving it an economic stake equal to about 75 percent of the company's voting shares.

Volkswagen's stock soared to as high as 1,005 euros a share, about $1,258, on Tuesday before closing at 918 euros. The shares ended last week at 210 euros.

Patterns of the past: an example (cont'd)

The New York Times

VW Shares Plunge, a Day After Surge

By CARTER DOUGHERTY
Published: October 29, 2008

October 29, 2008

FRANKFURT — Shares in Volkswagen fell by nearly half Wednesday after its main shareholder, Porsche, took steps to ease a quadrupling in the stock price that had pushed some of the world's biggest hedge funds to the wall.

The move came as the German financial supervisor, Bafin, announced a formal investigation into gyrations of VW stock that briefly made it the most valuable company in the world a day earlier. "We need to take a closer look if there was market manipulation," a Bafin spokeswoman, Anja Engelland, said.

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+ Porsche AG

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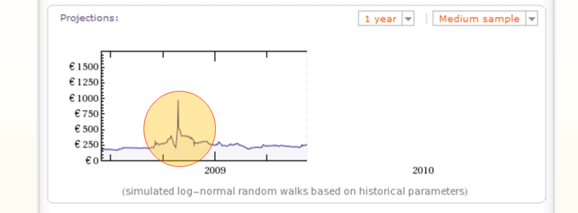
(500) DAYS OF SUMMER

Patterns of the past: an example (cont'd)

Patterns of the past: an example (cont'd)

Patterns of the past: an example (cont'd)

(lots of output regarding the share price of Volkswagen AG...)



Patterns of the past: an example (cont'd)

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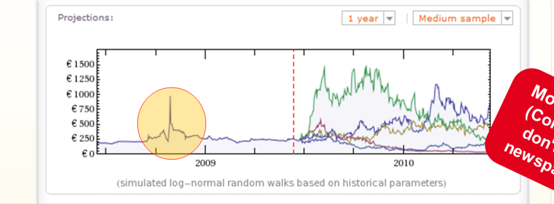

Model Risk!
(Computers
don't read
newspapers)

Different uncertainties

Aleatory Uncertainty

- Caused by randomness
- Based on historical data
- Quantified by statistical data analysis

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Aleatory Uncertainty

- Caused by randomness
- Based on historical data
- Quantified by statistical data analysis

Epistemic uncertainty

- Due to imperfect knowledge
- Typically associated with one-off, unrepeatable things
- Assessed by expert judgement

Expert judgement and subjective opinions

Principles for sound stress testing practices and supervision
(BIS Consultative Document, March 2009)

- "The management of most banks did not ... sufficiently take account of qualitative expert judgment to develop innovative ad-hoc stress scenarios."
- "The compilation of forward-looking scenarios requires combining the knowledge and judgment of experts across the organization."
- "The financial crisis has shown that estimating ex ante the probabilities of stress events is problematic. ... In this respect, the crisis has underscored the importance of giving appropriate weight to expert judgment in defining relevant scenarios with a forward looking perspective."

Frequency probability

The frequency probability of an event is the relative proportion of its occurrence in a long, ideally infinte, series of independent repetitions.

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Example (coin-tossing game)

Consider the relative number of “heads” in a (obviously repeatable) coin-tossing game. The strong law of large numbers ensures that the sample average of the independent tosses converges in probability towards the expected value, i.e. for the coin-tossing game we have that

$$P(\lim_{n\to\infty} \frac{H_n}{n} = \frac{1}{2}) = 1\,,$$

where H_n denotes the number of “heads” in a series of n trials.

Frequency probability (cont'd)

The main attraction of frequency probability is that it is impersonal and only relates to events of the objective world.

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There are, however, several difficulties with the frequency interpretation of probability.

Problems:

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Problems:

- it is circular because independence is defined in terms of probability,
- it gives probabilities only to long or infinite series of trials, not to nonrepeatable events,
- it is counterintuitive and contrasts the notion probability people are using in everyday reasoning.

Frequency probability (cont'd)

Examples:

- What is the probability of an earthquake in California with magnitude $m \geq 7$ in the next 20 years?
- What is the probability of a severe global H1N1 pandemic?
- What is the probability of a lasting rise in interest rates?

Subjective probability

The theory of subjective probability (personal probability) has been developed to overcome the shortcomings of the frequency interpretation of probability. Early advocate have been Bruno de Finetti, Frank Ramsey, Leonard Savage, and Dennis Lindley.

Definition (Subjective probability)

The probability of an event is a measure of the degree of belief that this event will occur.

Subjectivity in science

I.J. Good (1916 - 2009)

"The subjectivist states his judgements, whereas the objectivist sweeps them under the carpet by calling assumptions knowledge, and he basks in the glorious objectivity of science."

Subjectivity in science (cont'd)

Accept the subjective bias of knowledge and science.

- Subjectivity is an inherent and required part of the scientific method and statistical inference.

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Subjectivity in science (cont'd)

Accept the subjective bias of knowledge and science.

- Subjectivity is an inherent and required part of the scientific method and statistical inference.
- Informed scientific judgement should not be shunned as a nonobjective and therefore a poor methodological approach.
- Only very few statistical investigations are approximately "objective". Usually, choices of such features as the model will have a serious bearing on the results and the conclusion.

Outline

The Bayesian approach

Thomas Bayes

- Bayesian statistics appears as the calculus of uncertainty.
- In Bayesian statistics parameters θ are random variables.
- Bayesian techniques allow for prior beliefs about θ.

The Bayesian approach (cont'd)

Bayesian inference means updating the prior belief on θ after having observed the current data x,

$$\pi(\theta|x) = \frac{f(x|\theta)\pi(\theta)}{\int f(x|\theta)\pi(\theta)\, d\theta},$$

where $f(x|\theta)$ is the conditional probability distribution of data x.

Remarks:

- The prior belief is contained in the prior distribution $\pi(\theta)$.
- The prior distribution also models the uncertainty on the model parameters θ.
- The posterior distribution $\pi(\theta|x)$ reflects the updated beliefs about θ after observing x.

Bayesian inference–credible intervals

The posterior distribution contains all available information (prior and data!) about θ and is the basis of inference concerning θ and functions of it.

Since, in contrast to frequency statistics, the Bayesian approach provides a probability distribution $\pi(\theta|x)$ for the uncertain parameter, a natural confidence region appears.

Definition (Credible interval) For a given $\alpha \in (0, 1)$ we can find an interval (a, b) such that

$$1 - \alpha = P(a < \theta < b|x) = \int_a^b \pi(\theta|x)\, d\theta\,.$$

The interval (a, b) is called credible interval.

Bayesian inference–predictive distribution

Suppose you have a model $f(x|\theta)$ with posterior $\pi(\theta|x)$ and you want to predict a random variable $Y \sim g(y|\theta,x)$, i.e. based on the observation of $X \sim f(x|\theta)$.

Then, the predictive distribution of y is given by

$$g(y|x) = \int g(y|\theta,x)\pi(\theta|x)\, d\theta\,.$$

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Remarks:

- Note that $g(y|x)$ is independent of θ and just depends on the x (e.g. past realizations of the random variable X).
- The result provides an entire distribution for Y rather than a simple point estimate. Hence, parameter uncertainty can properly taken into account, for instance by reporting credible intervals.

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