Stress VaR
(financial risk, standard VaR, hidden risk)

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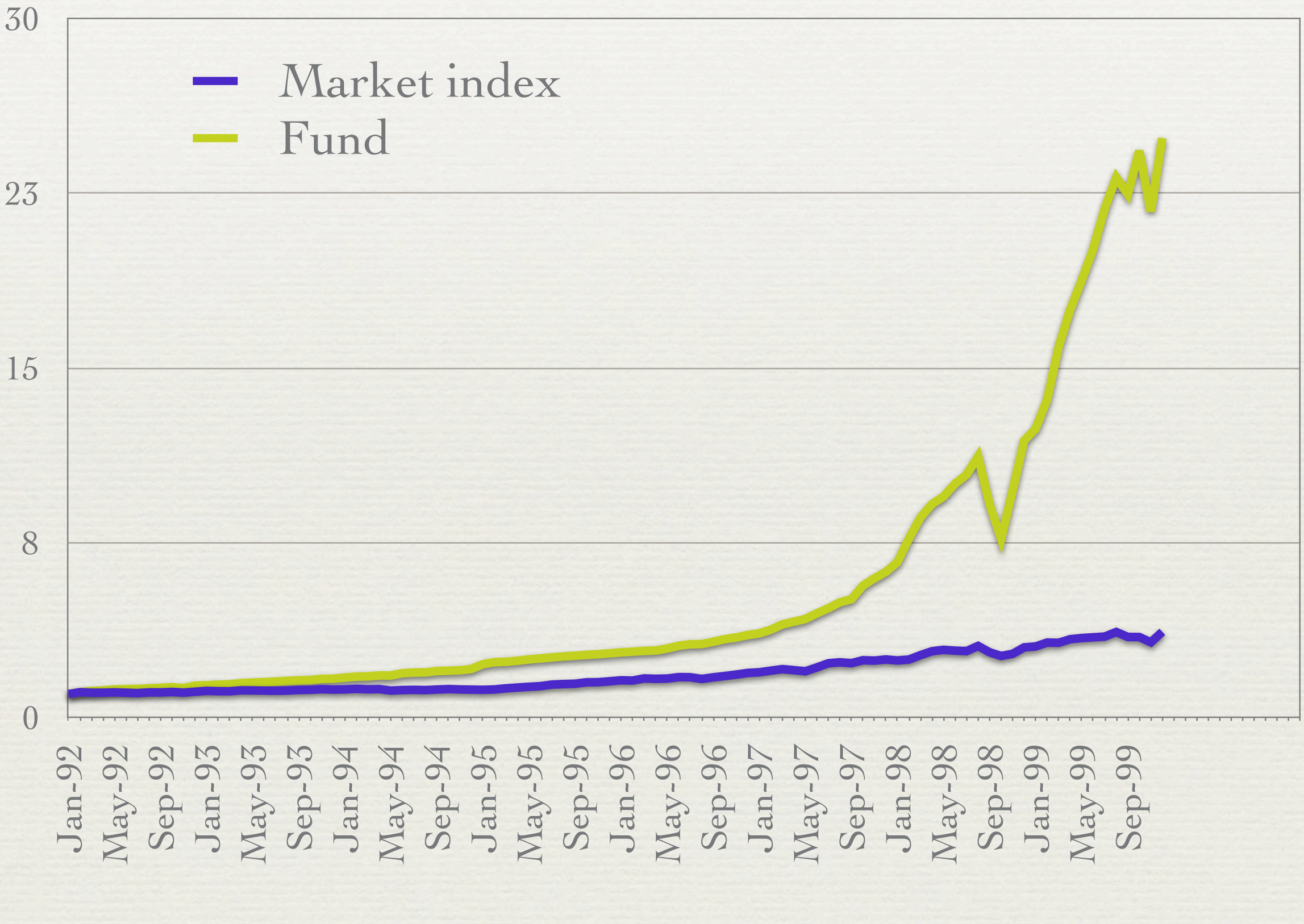
Example strategy



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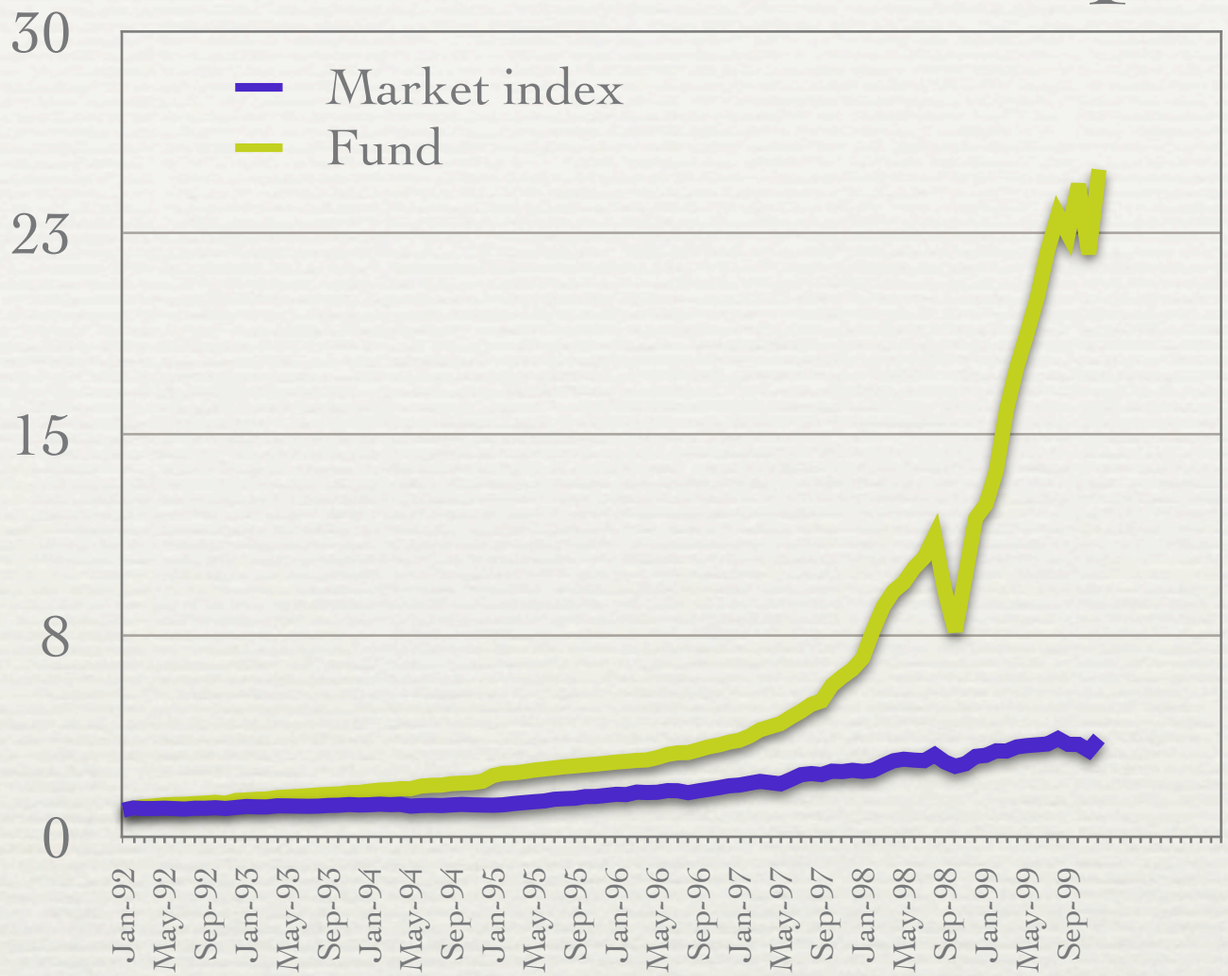

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Monthly mean	1.43	3.59
Monthly SD	3.55	5.75
Minimum month	-8.90	-18.30
Maximum month	14.00	27.00
Annual Sharpe ratio	1.39	2.16
No. of negative months	35	6
Correlation to Market index	1.00	0.61
Growth of 1 since inception	€ 3.67	€ 24.87

Numbers taken from A. Lo (2009)

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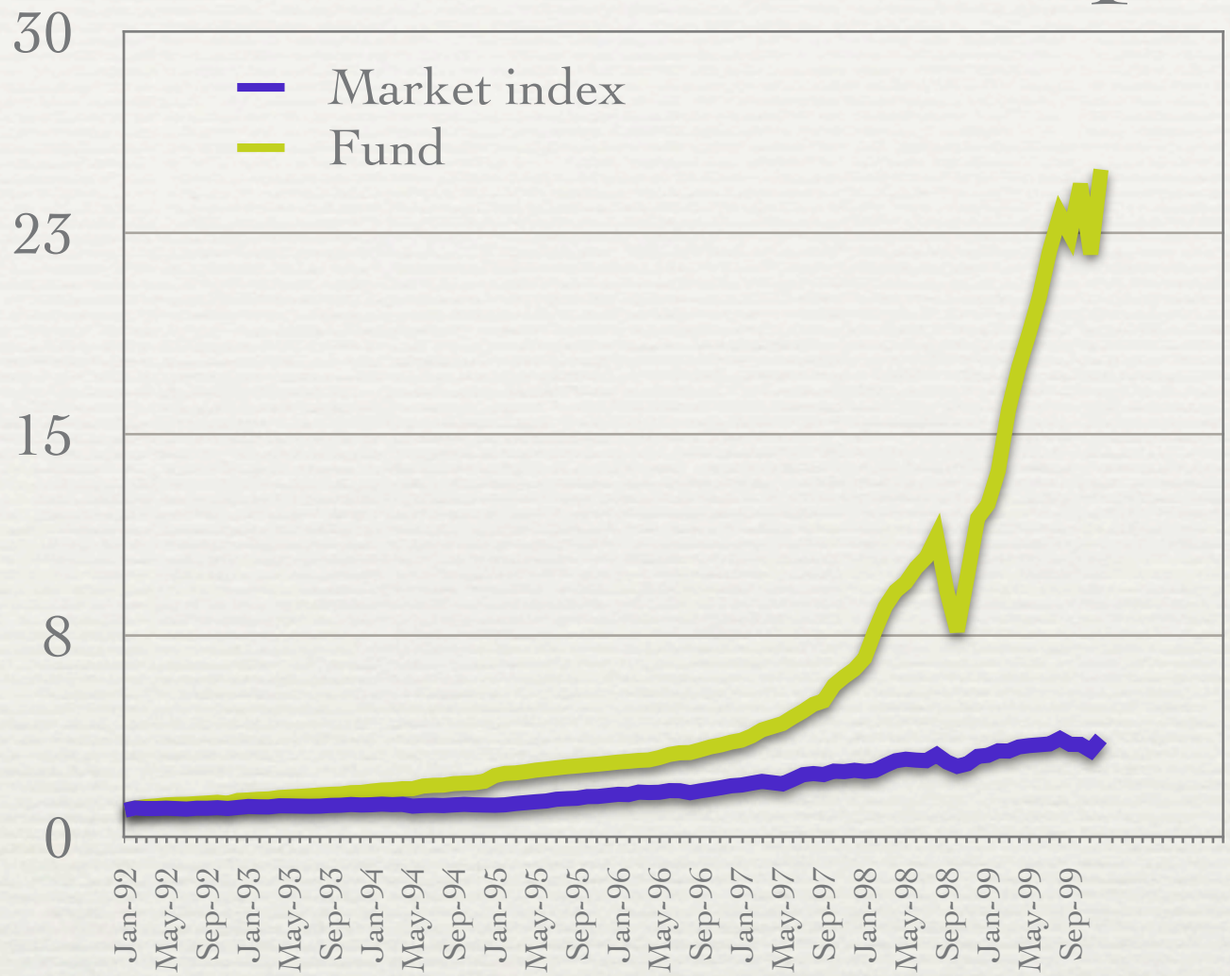

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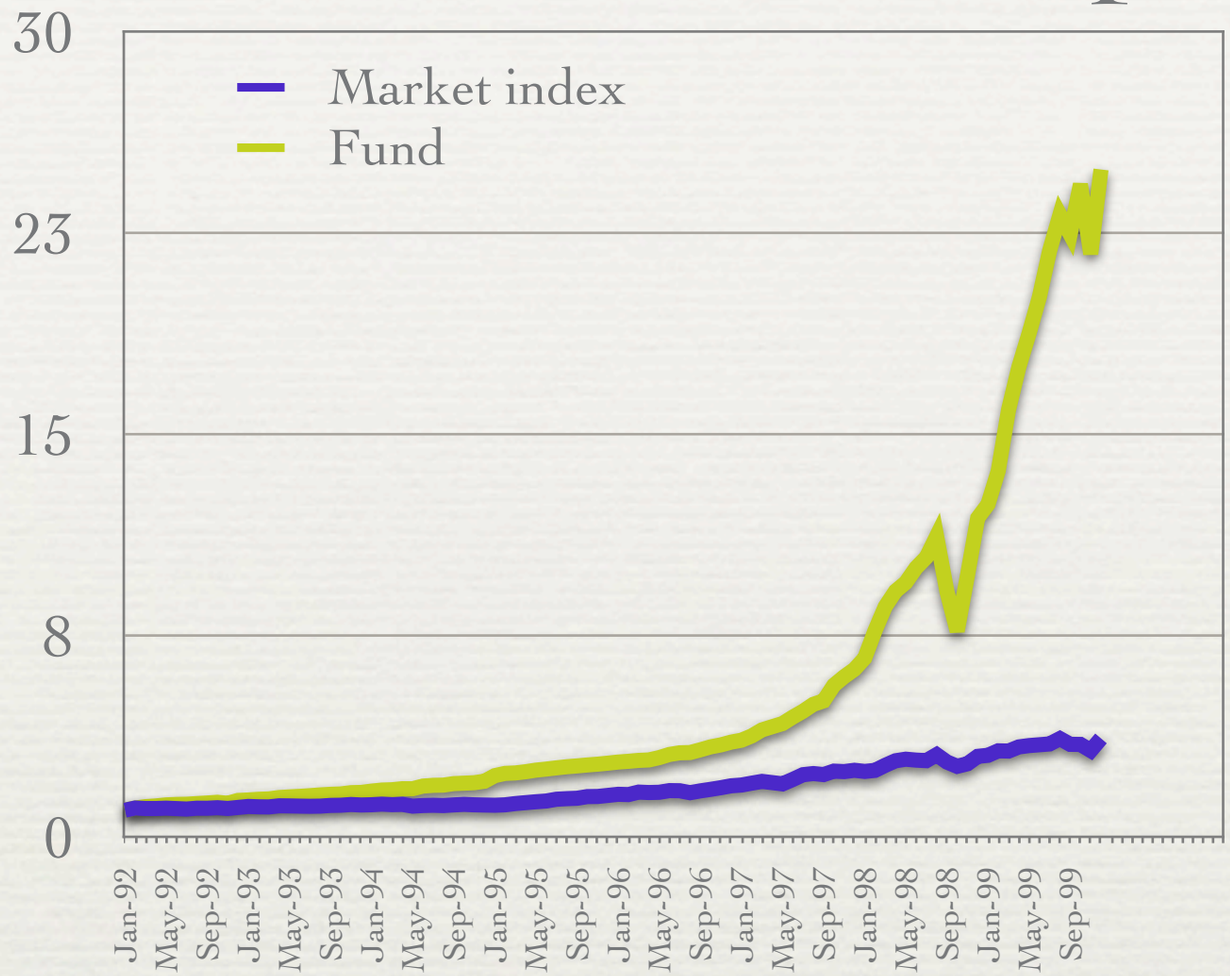

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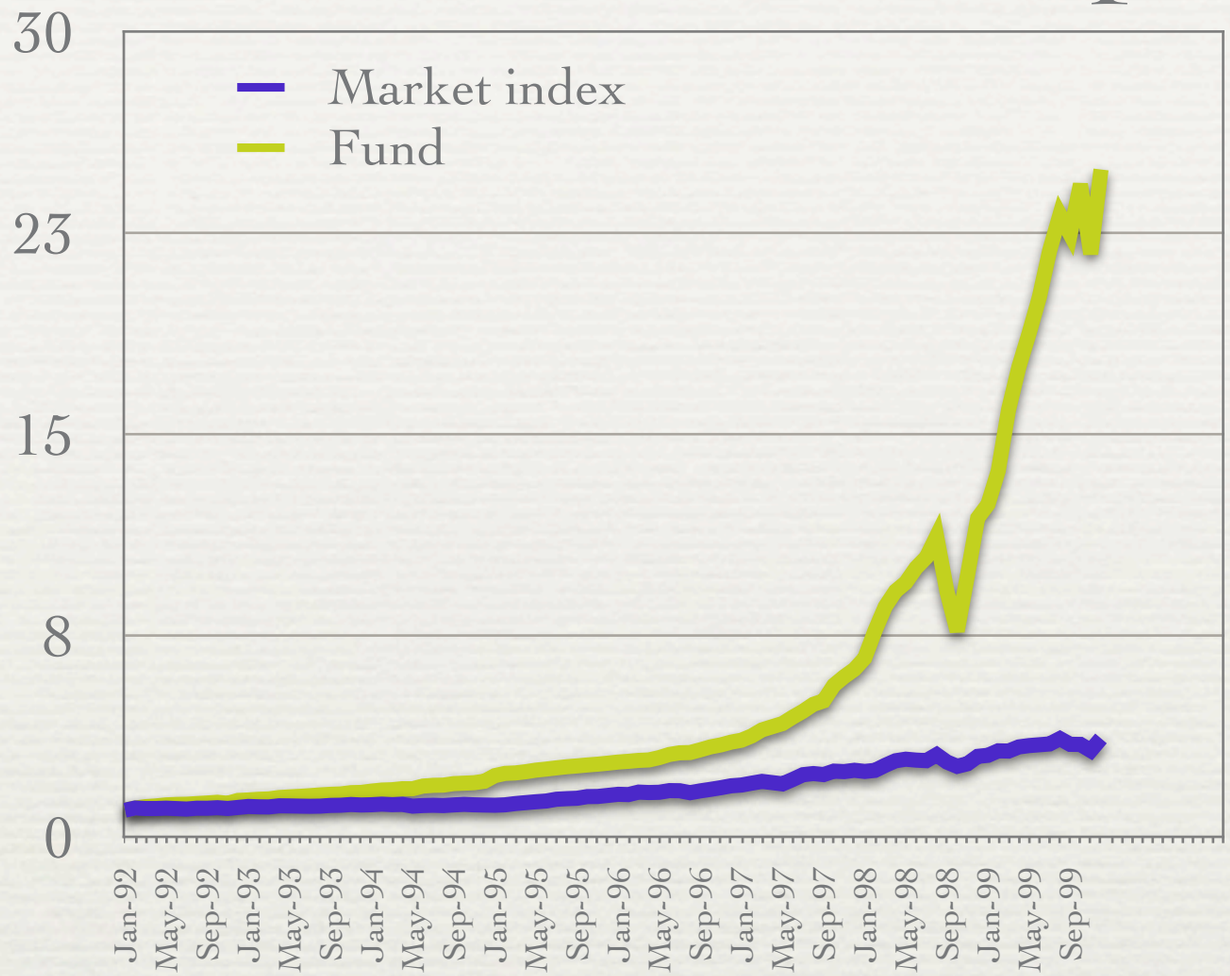

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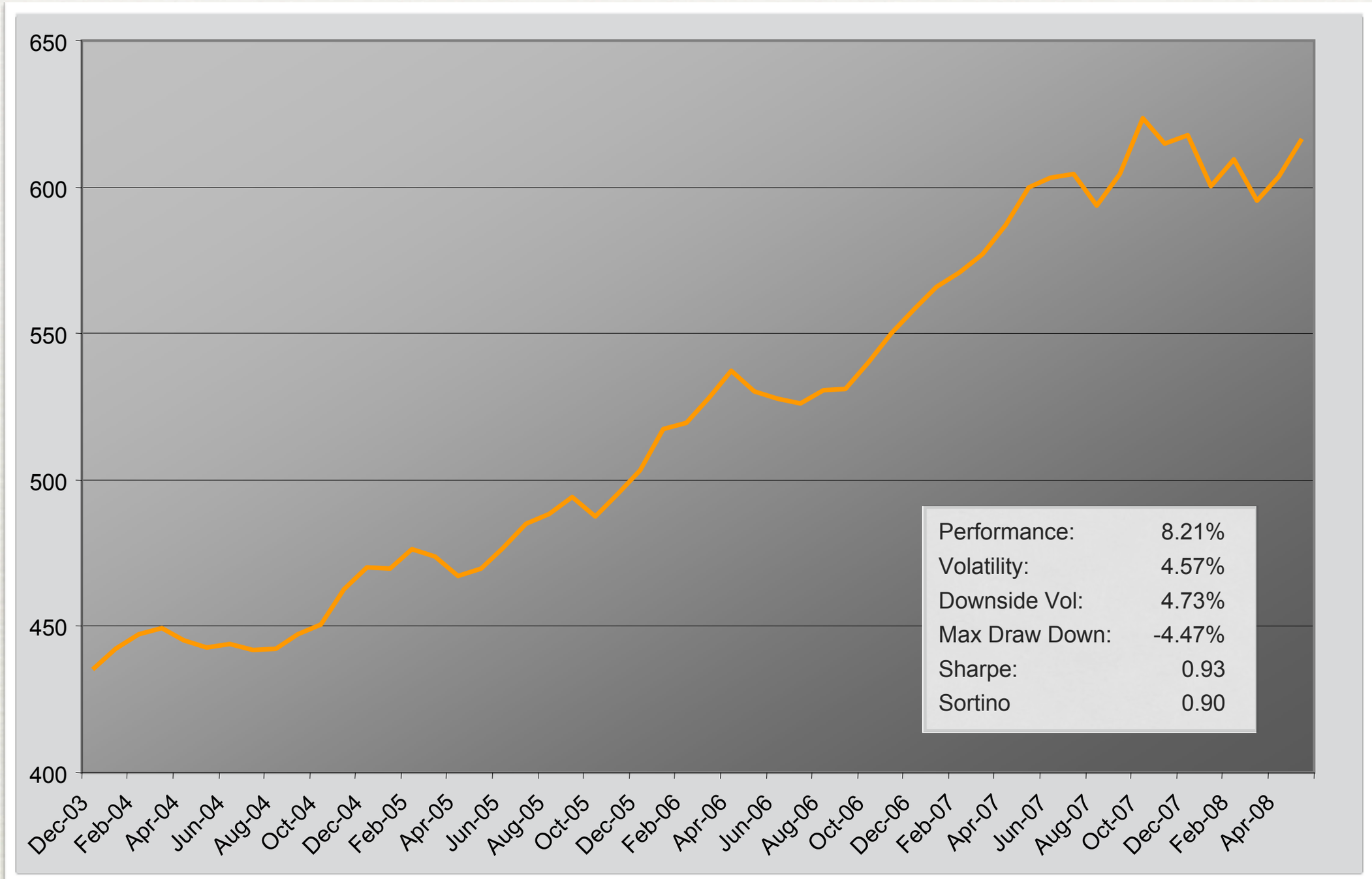
- Returns: lots of small positive returns, and rare HUGE losses
- Hidden Risk: Strategy provides market insurance against a large loss. Would you pay a managers fee to write insurances?

Numbers taken from A. Lo (2009)

Outline

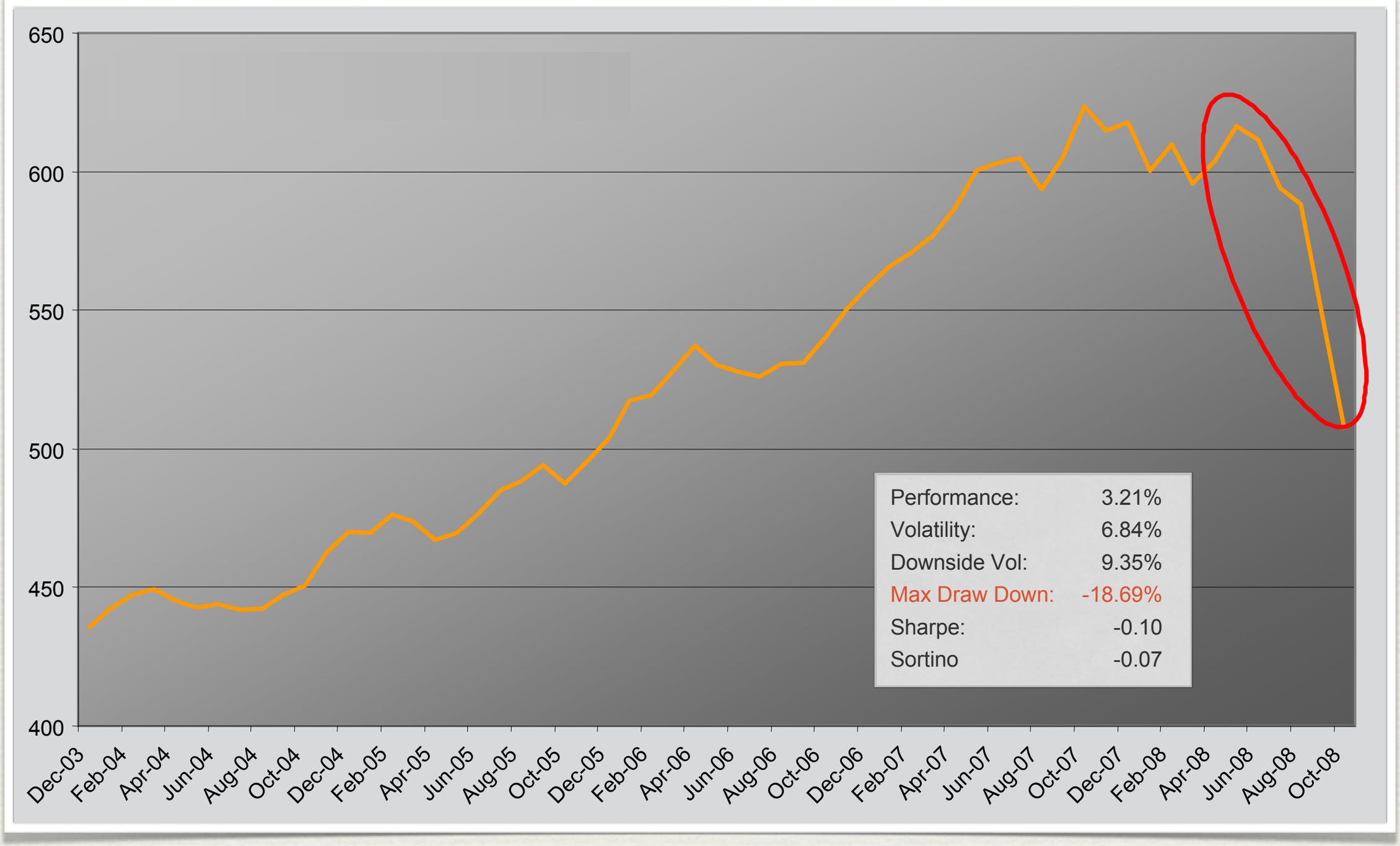
- How to quantify "hidden risks"
- Factor analysis and Stress VaR
- Portfolio performance using Stress VaR
- Conclusion
- Basel III capital requirements

Hidden risk



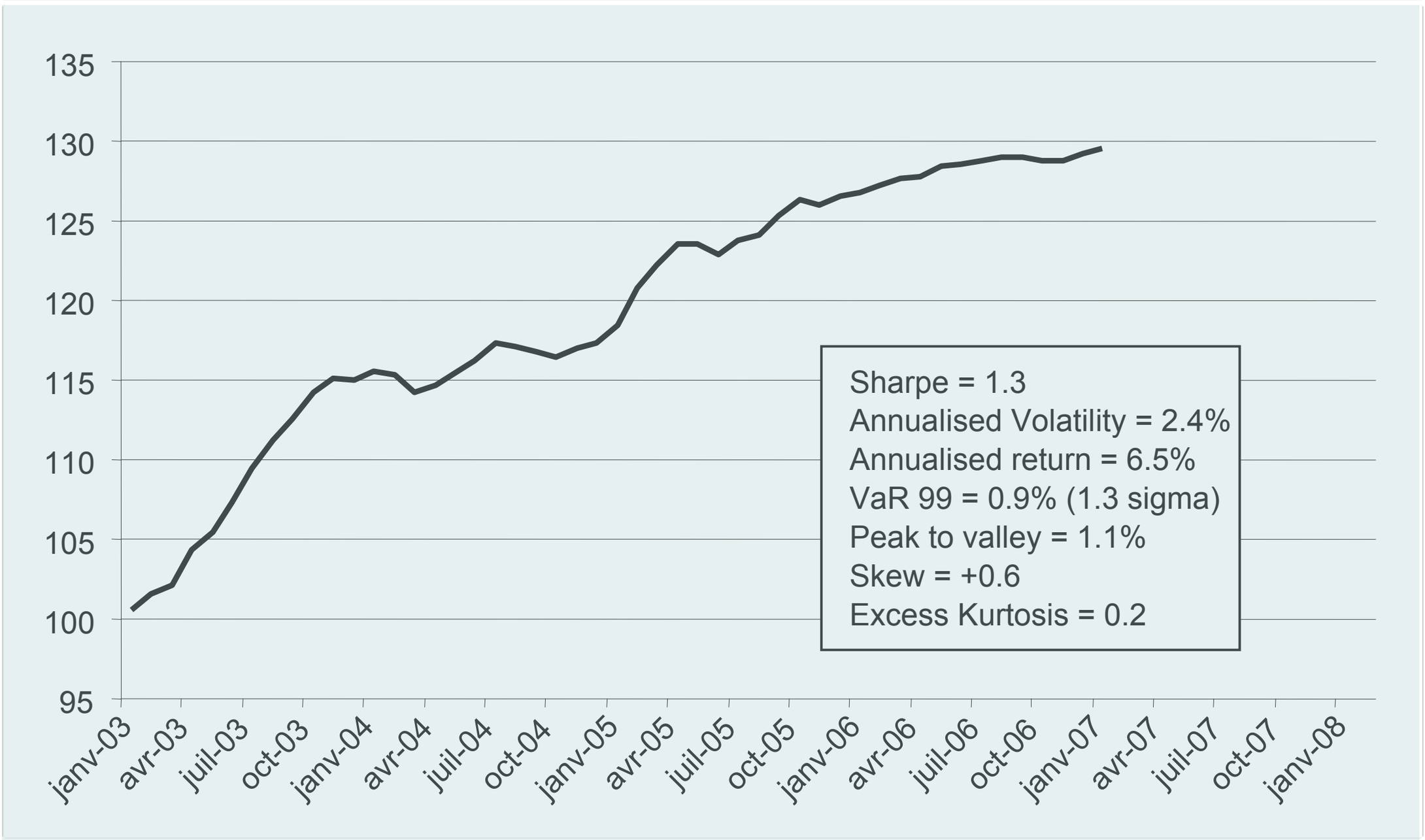
This fund seems to display all possible green lights for an investor... But will the performance last?

Hidden risk



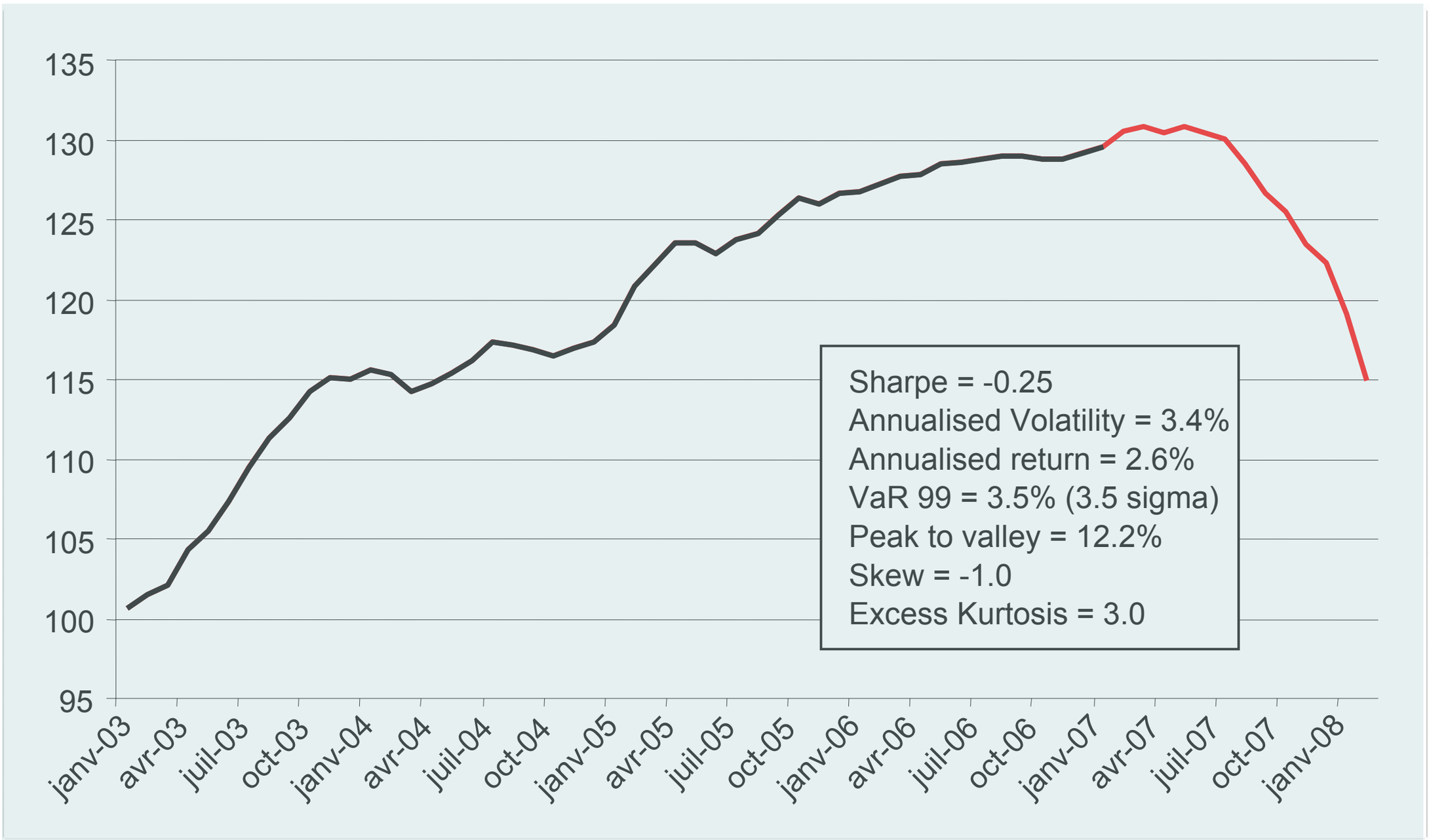
Losses during the crisis exceeded 4 times Max Drawdown. Could such a loss be anticipated by looking only at the past fund performances?

Hidden risk



This performance series would attract any investor who is solely focused on past performances. The sequel shows how the investor might be disappointed.

Hidden risk



What happened? "Black Swan"??

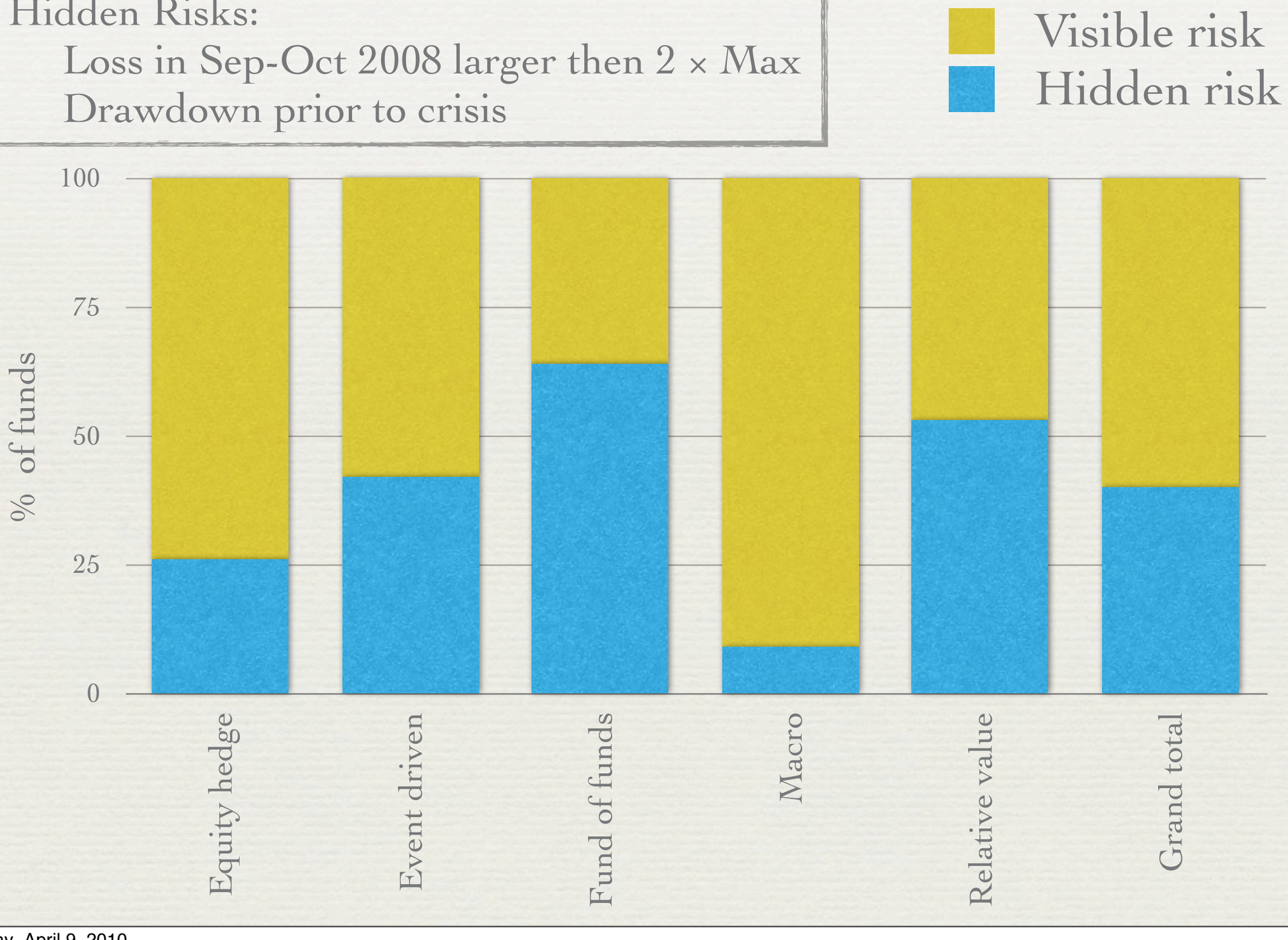
Hidden risk and "Black Swans"

- *Ex-post:* "unexpected" large losses
  - Hidden risk appears when observed losses exceed anything that could have been extrapolated from past performances metrics, merely by using simple performance analysis tools
- *Ex-ante*: possible sources of hidden risk
  - Return smoothing, fraud, etc.
  - Correlation breaks, 'Time bombs'
  - Holding illiquid assets

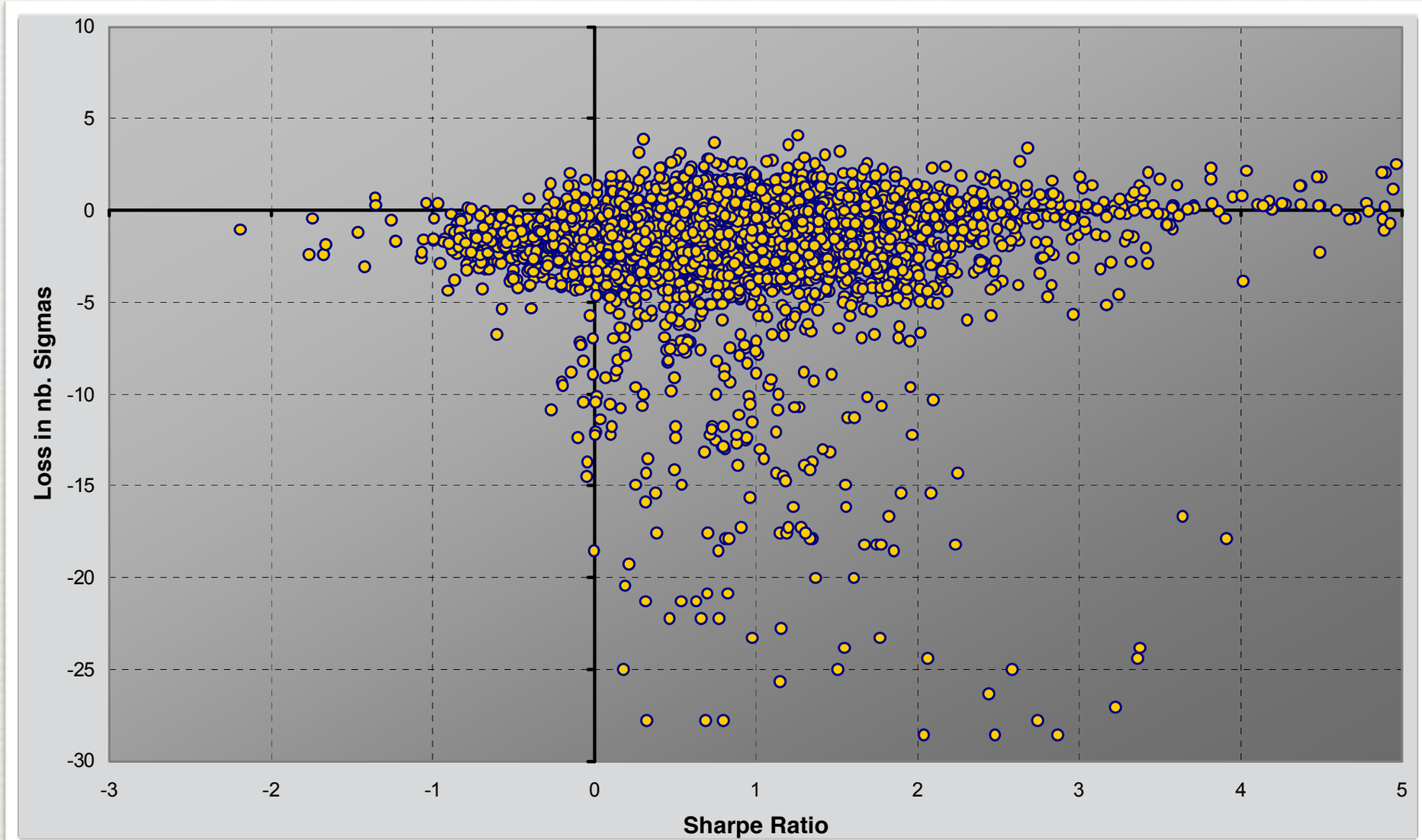
Ex-post statistics on hidden risk realization in Fall 2008

Hidden Risks:

Loss in Sep-Oct 2008 larger then 2 × Max Drawdown prior to crisis



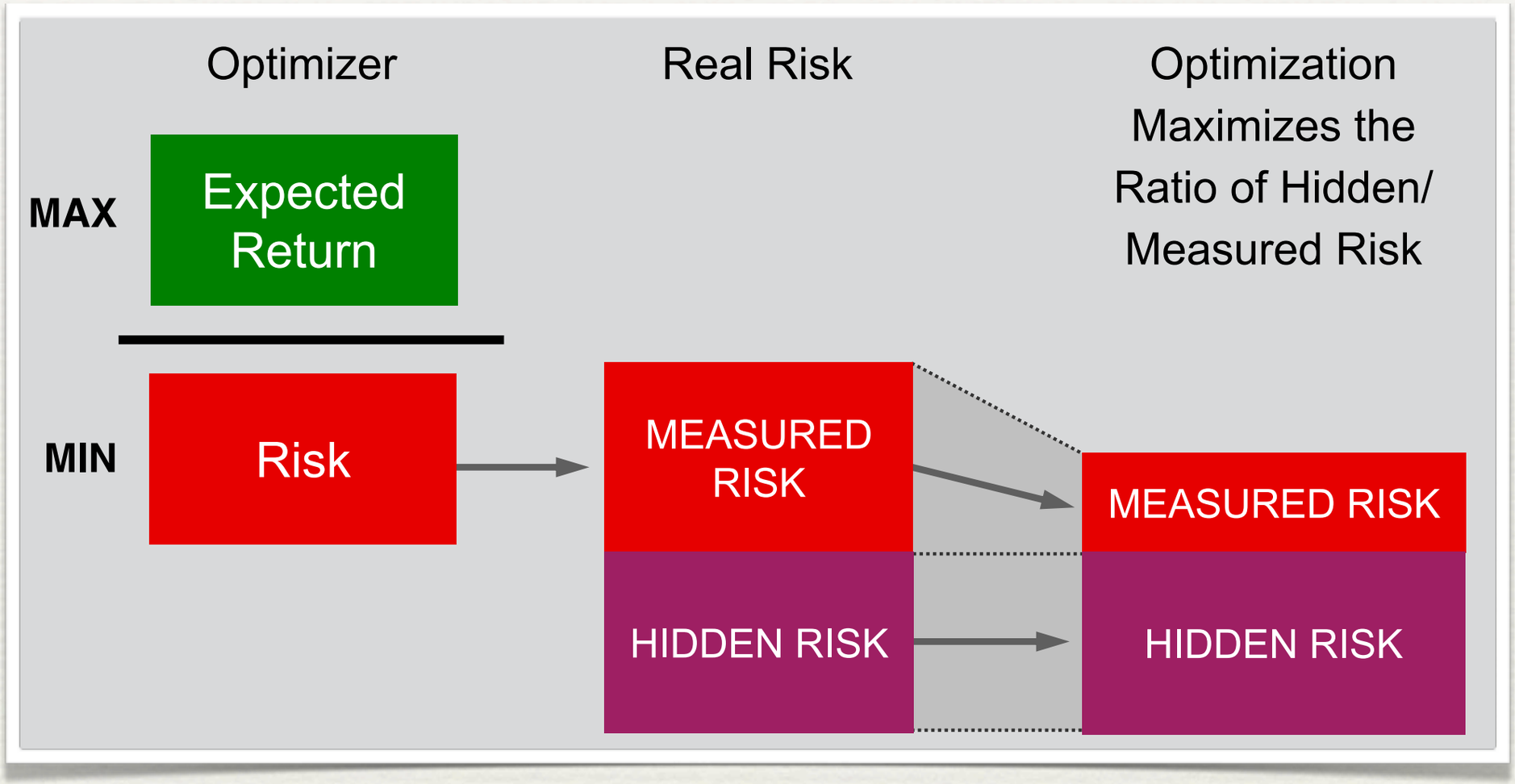
Sharpe ratio as a predictor



Still using the same sample of 3,098 funds, the X axis is the Sharpe Ratio over the period Jan 04 – Dec 07, the Y axis is the performance during Sep-Oct 08 divided by the volatility prior to the crisis.

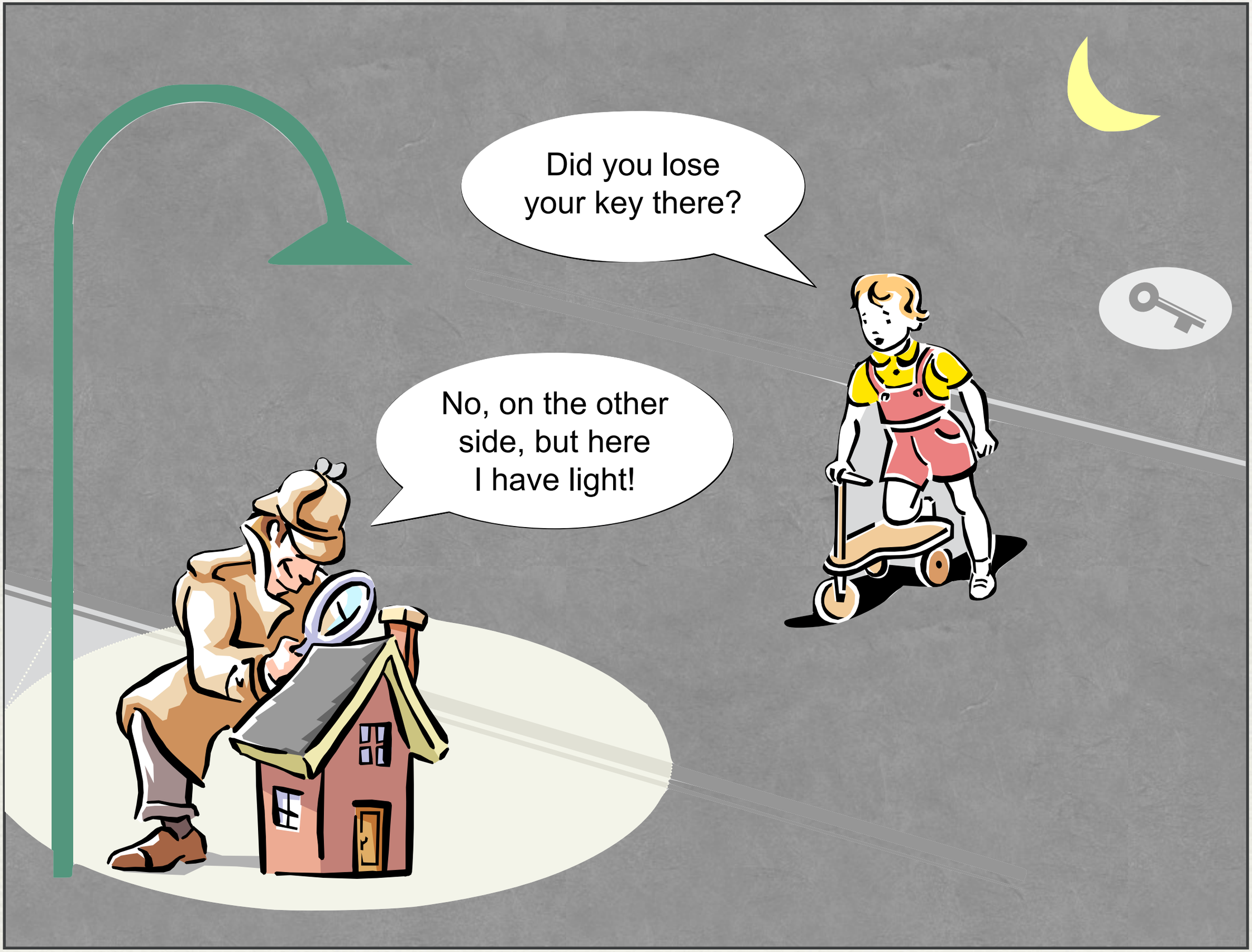
Clearly, the Sharpe ratio is a very poor predictor of losses during the crisis!

Optimizers failed, however advanced...

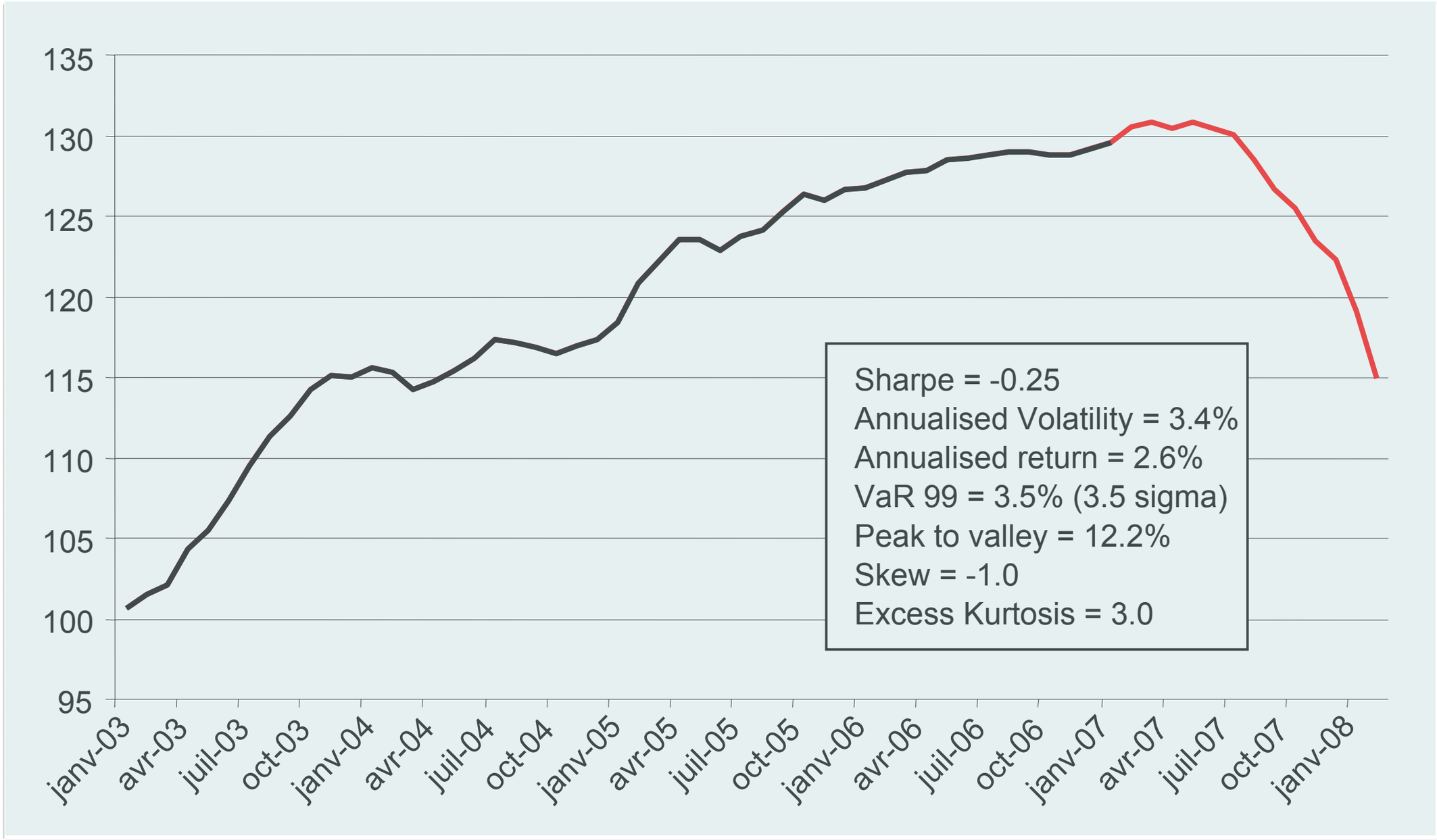


Optimizers, however sophisticated, simply maximize expected return while minimizing measured risk. Therefore, by design, optimizers maximize the proportion of unmeasurable risk – i.e. hidden risk – leading automatically to portfolios which eventually deliver very nasty surprises....

What are you looking for?

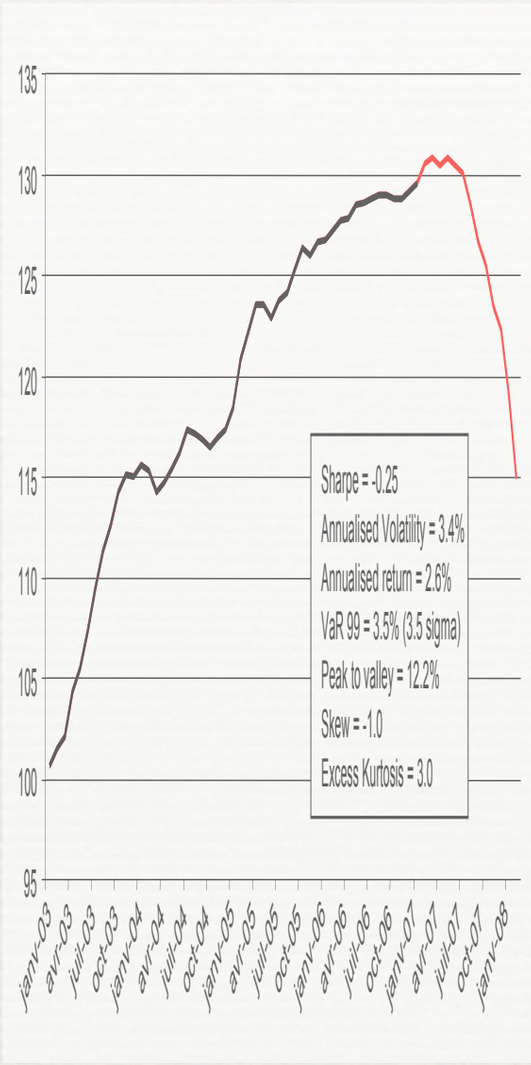


What is the risk of this fund??



Could such a loss be anticipated, only looking at past fund performance? Yes, with nonlinear factor analysis.

Factor analysis



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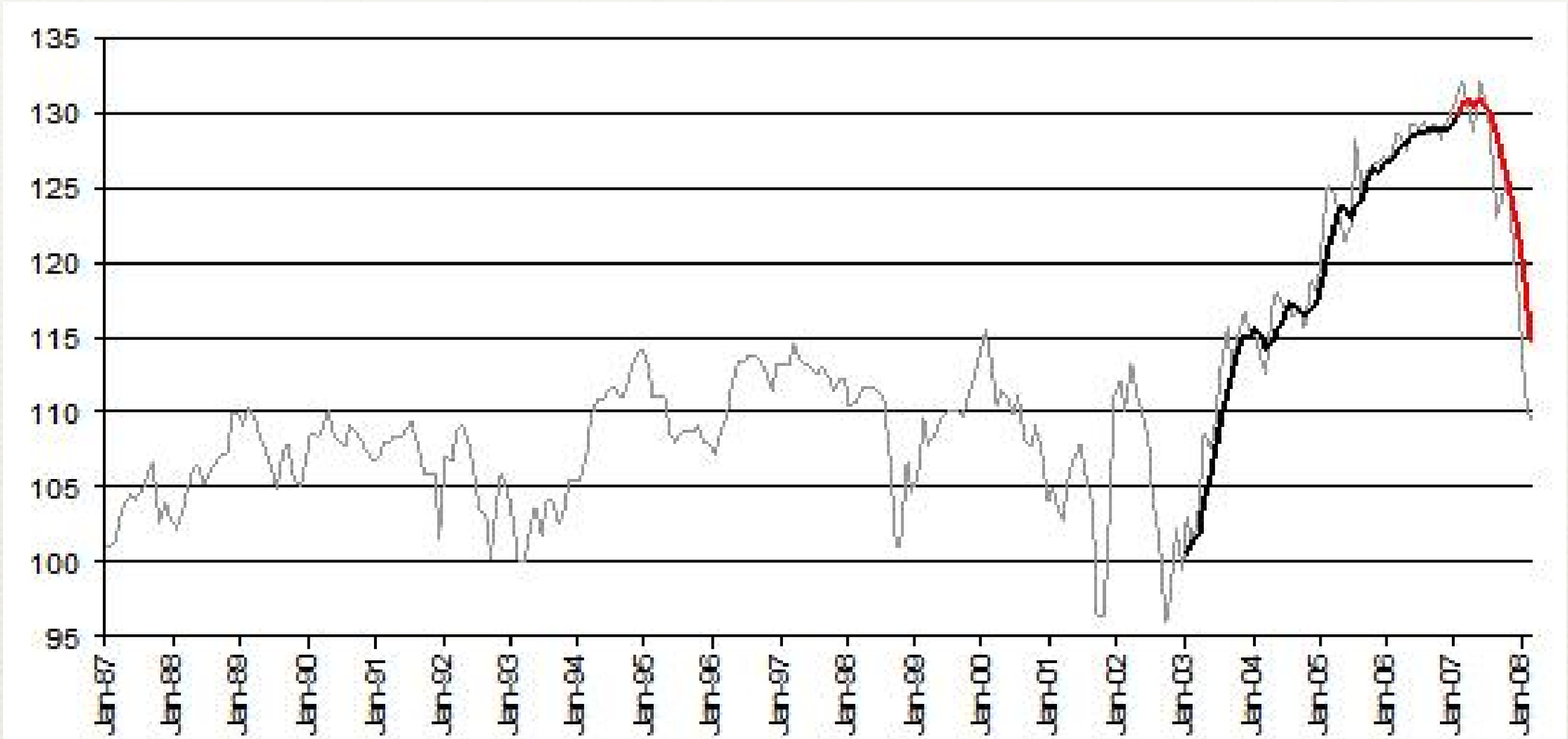
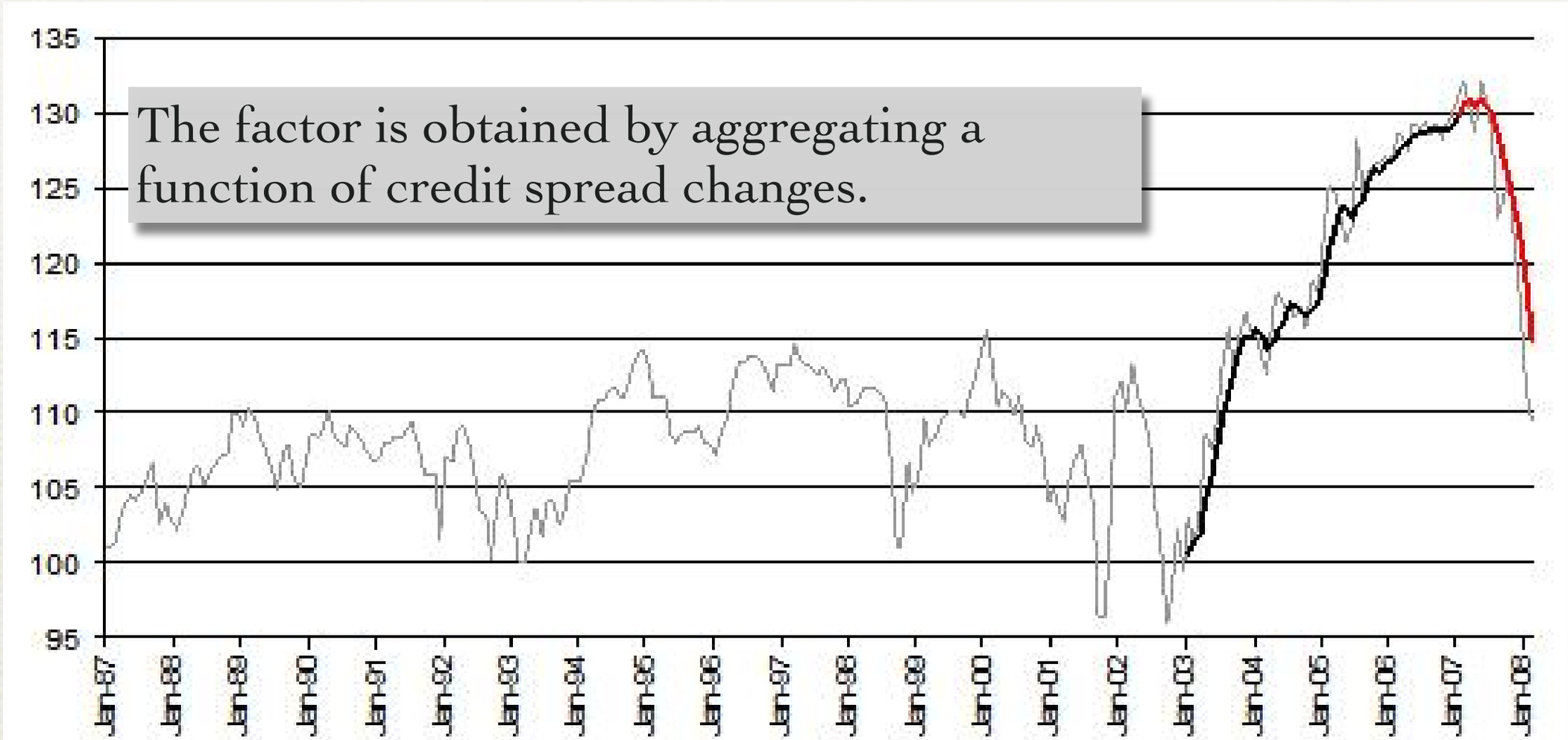


- Credit driven fund overlaid on top of AAA spread over T-Bonds
- This fund was just surfing the good wave during the analysis period

Factor analysis



The factor is obtained by aggregating a function of credit spread changes.

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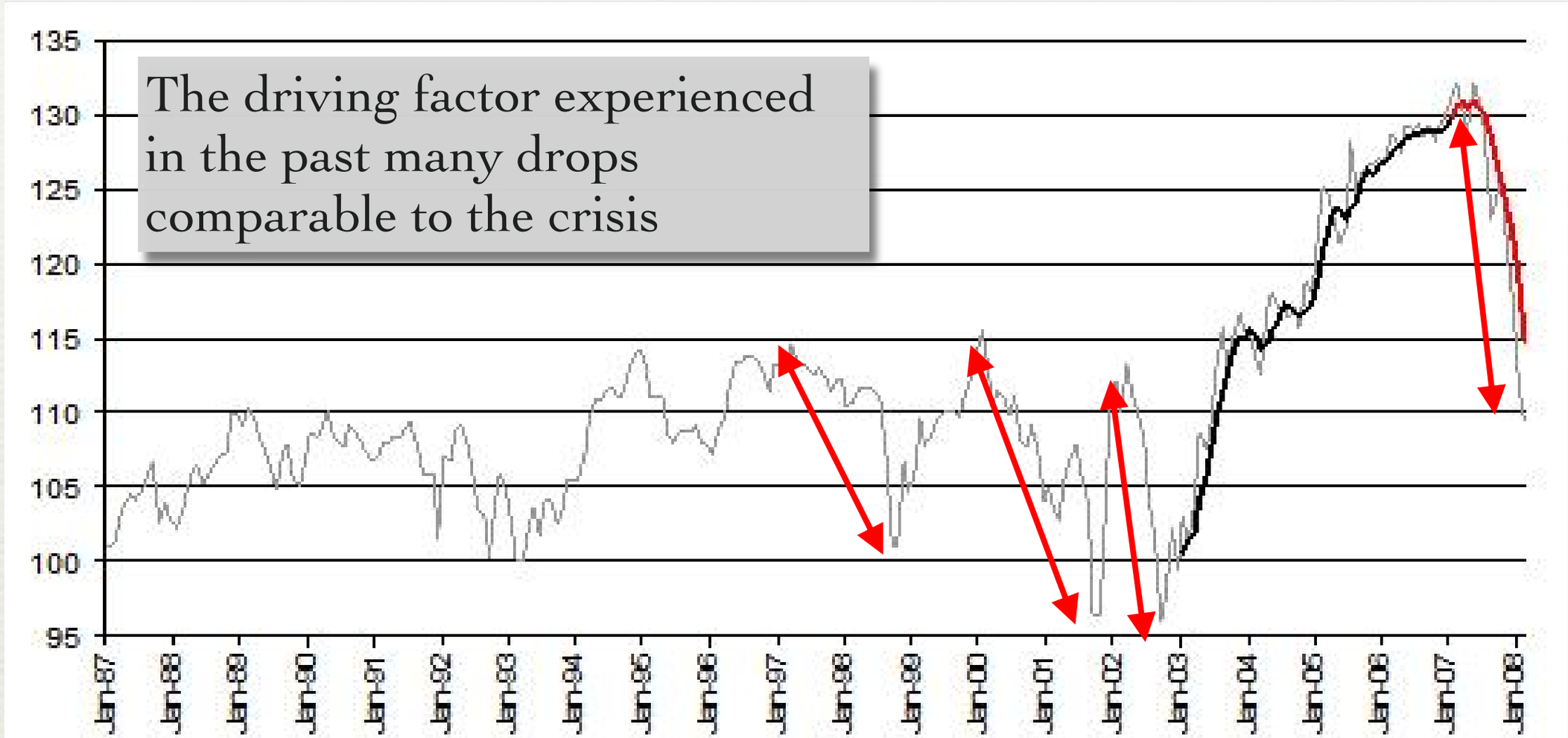
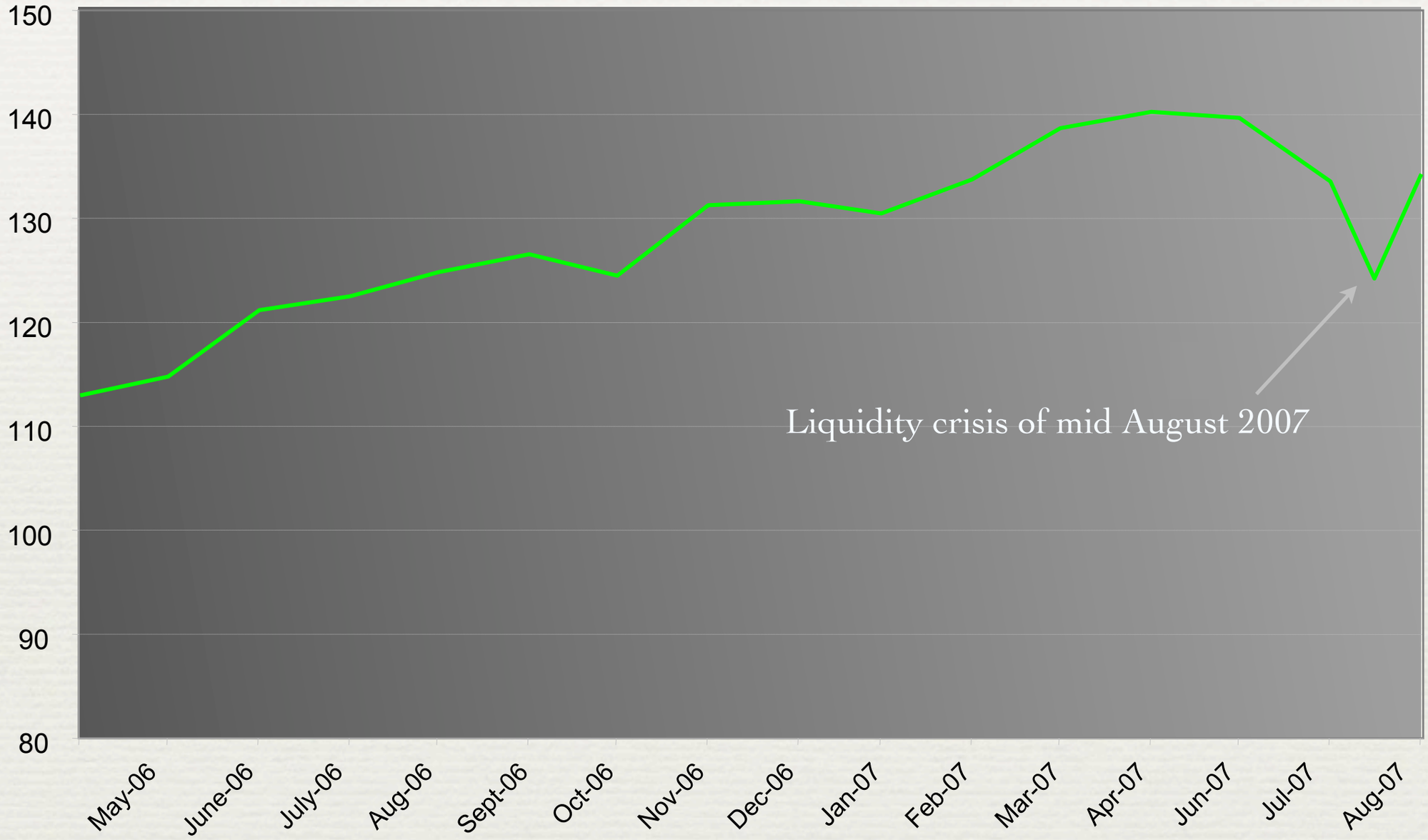
Factor analysis



One can see that the loss experienced in 2007 had several similar precedents. The current loss of the fund is in line with the historical losses of the driving factor.
So, the fund loss was not a Black Swan, but a risk HIDDEN in the fund's return series.

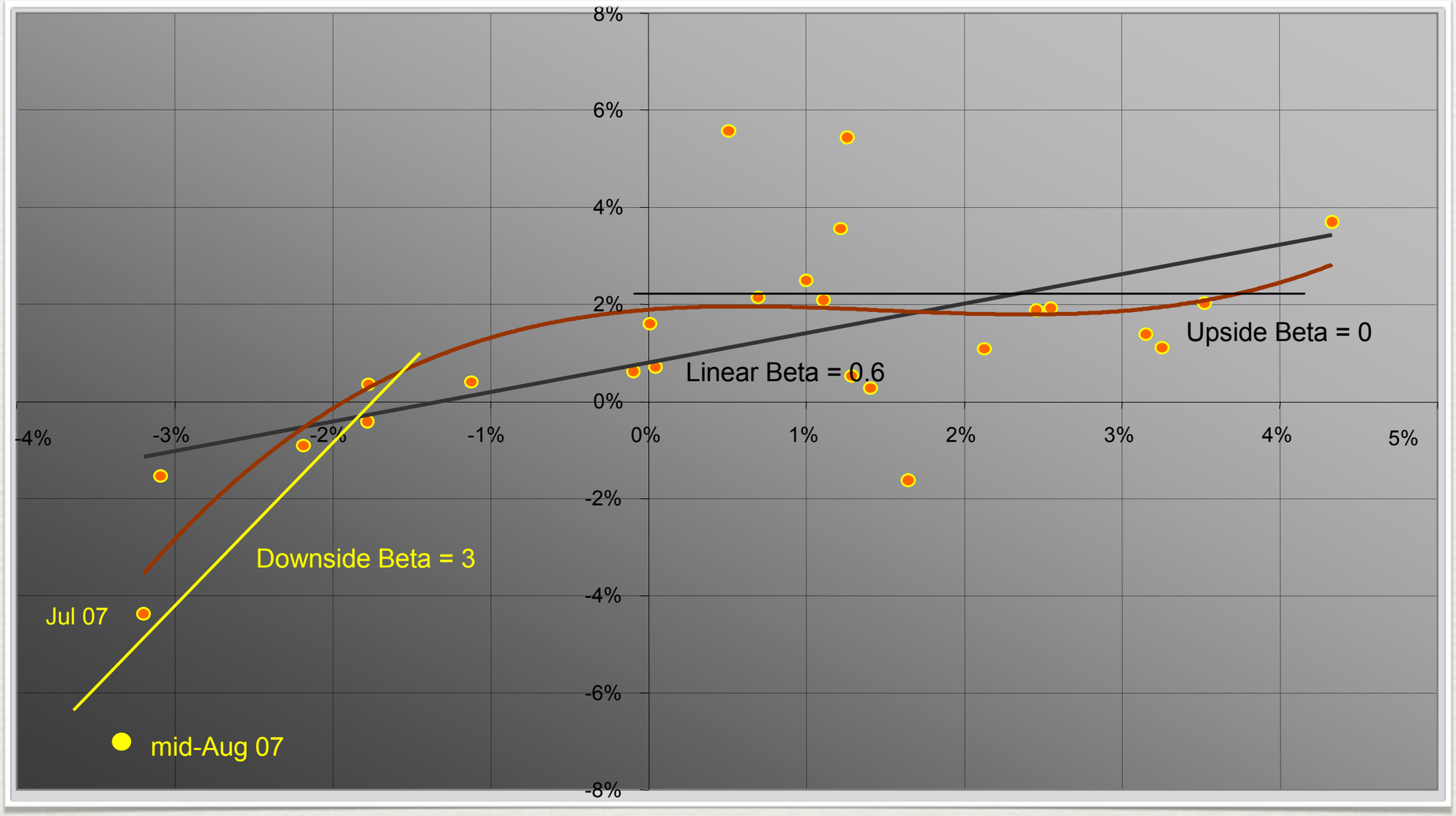
Nonlinearities: Major source of hidden risk



Quantitative Long-Short Equity US: As with most of its peers, this fund experienced a strong drop on Aug 13 2007

Scatter plot of returns vs. S&P 500

Beta changes in a systematic way



The traditional linear Beta of the fund is 0.6. But by carefully examining these returns, one finds that the "upside Beta" is 0, while the "downside Beta" reaches the value 3.

What does this apply to?

- Risk of Hedge funds
  - A few years of history
  - Unregulated and secretive
  - Large variety of strategies and trading universe
  - Often uncorrelated to markets: need exotic factors
  - But correlated during crises: need nonlinear models
- Risk of a portfolio
  - Do you properly understand what you are investing in?
  - Do you trust your risk models?
  - Model risk?

Problem definition

- $X_1(t), ..., X_n(t)$ are market (risk) factors
  - Indices, rates, correlations, exotic factors, ...
  - 100's or 1000's of risk sources
  - Long term history
  - Daily data or high-frequency
  - Accurate, liquid
- $Y(t)$ are portfolio returns (daily, monthly, high-freq)
  - Or collection of portfolio positions returns
- Risk measure question: Find the distribution of $Y(t + 1)$ including possible extreme events, looking forward
- Difficulty: The forward distribution may strongly differ from that of past returns, due to hidden risks

Factor analysis and Stress VaR

- General idea
  - Write $Y = \phi(X_1,...,Xn) + Z$
  - Estimate ϕ and the distribution of Z
  - Estimate the joint distribution $P(X_1,...,X_n)$
  - Push forward $\phi \bullet P$ and merge with the distribution of Z to get the future distribution of Y
  - The VaR of this distribution is the Stress VaR (stress test + VaR)
- Difficulties:
  - ϕ is strongly **nonlinear** and potentially dynamic
  - **Impossible** to calibrate a multi-dimensional model due to the large potential number of risk sources and nonlinearities/ dynamic relations

Poly-models = collection of single factor models

- Model the nonlinear dependency of the fund one factor at a time (Edgeworth decomposition)

$$Y = \psi_i(X_i) + Z_i = \sum_k \beta_{ik} H_k(X_i) + Z_i$$

$$H_k = \text{Hermitte polynomials}$$

- The multi-factor model is

$$Y = \psi(X_1, ..., X_n) + Z = \sum_{i,k} \lambda_{ik} H_k(X_i)$$

- The solution is $\Lambda = \Gamma^{-1} C$

$$\Gamma = (\gamma_{ik,jl}) = \text{Cov}(H_k(X_i), H_l(X_j)) \qquad C = (c_{ik}) = (\beta_{ik}\gamma_{ik,ik})$$

- Works under *ellipticity condition*: $\left\| \sum_{i=1}^{n} \varphi_i(X_i) \right\|_{L^2}^2 \geq c \sum_{i=1}^{n} \left\| \varphi_i(X_i) \right\|_{L^2}^2$

  which fails when there is tail concentration of the joint distribution along non axis directions. Unfortunately very common in practice: diversification disappears when crises occur!

Model selection with information maximization

$$Y = \psi_i(X_i) + Z_i = \sum_k \beta_{ik} H_k(X_i) + Z_i$$

The inverse Hessian of the log-likelihood fn gives the uncertainty in the β estimate

$$S_i(x) = (D^2 \mathcal{L})^{-1}$$

The signal/noise ratio is

$$I_i(x) = \frac{(\psi_i(x) - E(Y))^2}{S_i(x)^2 + Var(Z_i)}$$

Given a stress test $\{x_i\}$ we compute all $I_i(x_i)$ and select the factor with maximum signal/noise ratio.

Through an iterative procedure we can select a set of factors that maximize the information ratio.

Practical (conservative) Stress VaR

- LARGE set of nonlinear single factor models

$$Y = \psi_i(X_i) + Z_i = \sum_k \beta_{ik} H_k(X_i) + Z_i$$

scored for significance in extreme conditions.

- During crisis, factors tend to be correlated and fund returns are mostly driven by one single factor. Taking the worst value across all single factor models is sufficient for a good risk estimate.

- For each factor Xi define

$$\mathrm{StressVaR}_{1-\alpha}(Y, X_i) = \sqrt{\min_{E_\alpha(X_i)} \psi_i(x_i)^2 + \mathrm{VaR}_{1-\alpha}(Z_i)^2}$$

- Let *I* be the set of factors identified as having a significant relationship with the fund in extreme conditions. The Stress VaR is defined as:

$$\mathrm{StressVaR}_{1-\alpha}(Y) = \max_{i \in I} \mathrm{StressVaR}_{1-\alpha}(Y, X_i)$$

Practical Stress VaR

- **Large** set of potential factors
- **Nonlinear** single-factor models, ranked by statistical significance to the fund in extreme market conditions
- Largest loss predicted across relevant subset of factors is the potential loss of the fund

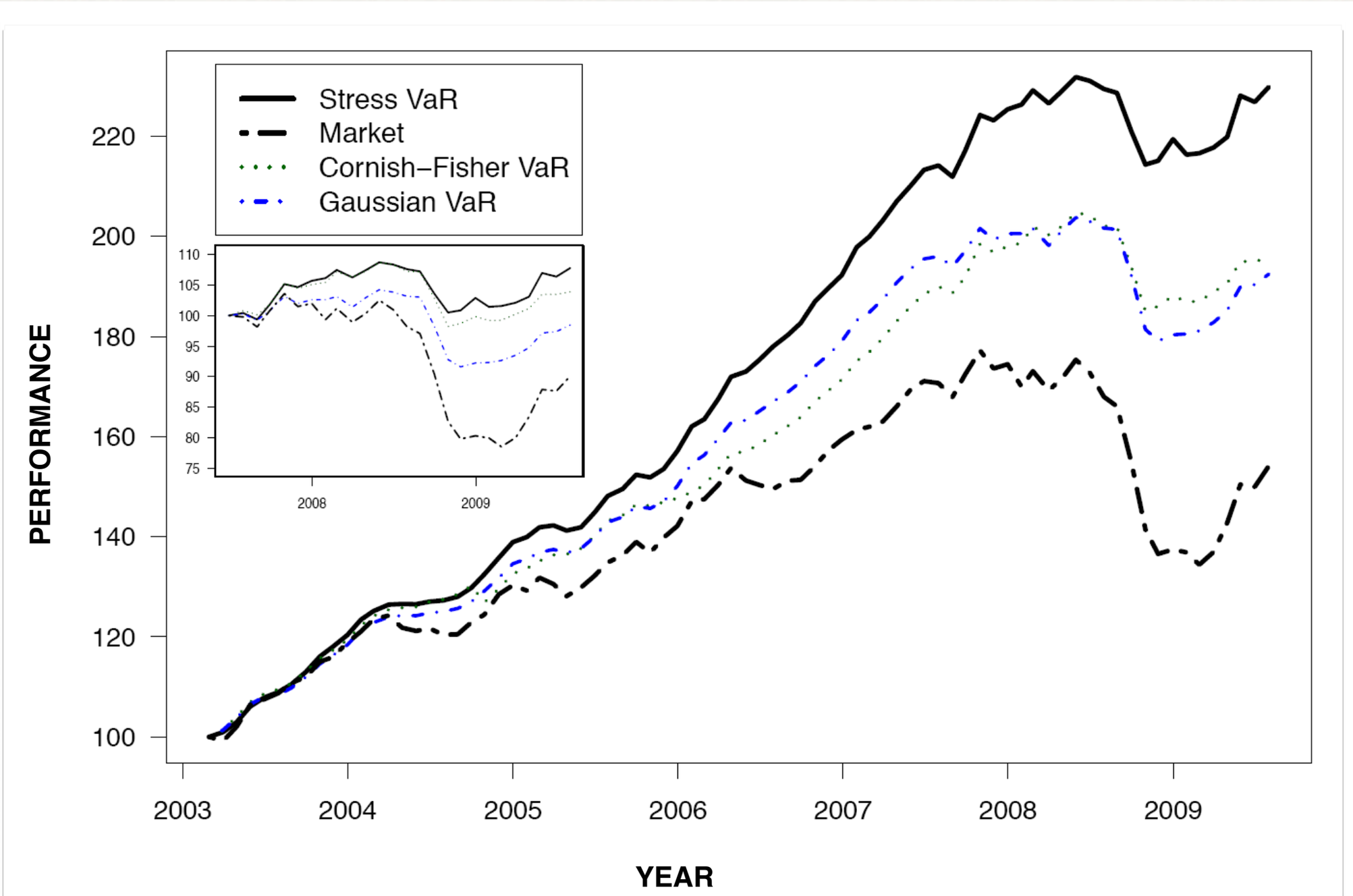
- Anticipative, because it relies on a very long-term factor history, including past crises when the portfolio did not exist.

$$\text{Hidden risk ratio} = \frac{\text{StressVaR - MaxDrawDown}}{\text{Volatility}}$$

Performance of a portfolio of hedge funds

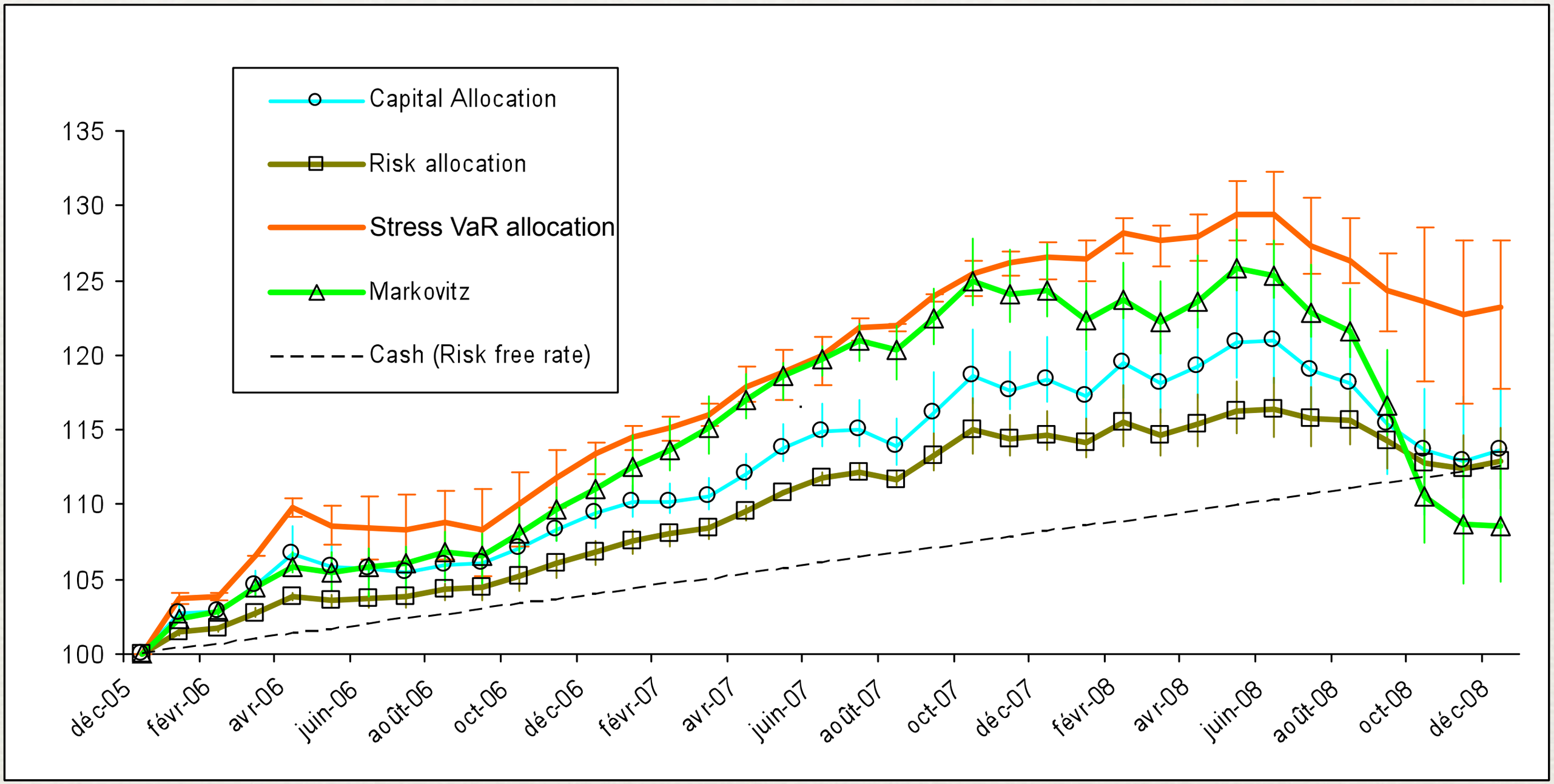
Investment strategy:

1. Select 250 less risky funds from 1000
2. Equal risk allocation 1/R regardless of correlations
3. Leverage up or down to maintain a constant total risk
4. Benchmark = Equal allocation on 1000 funds



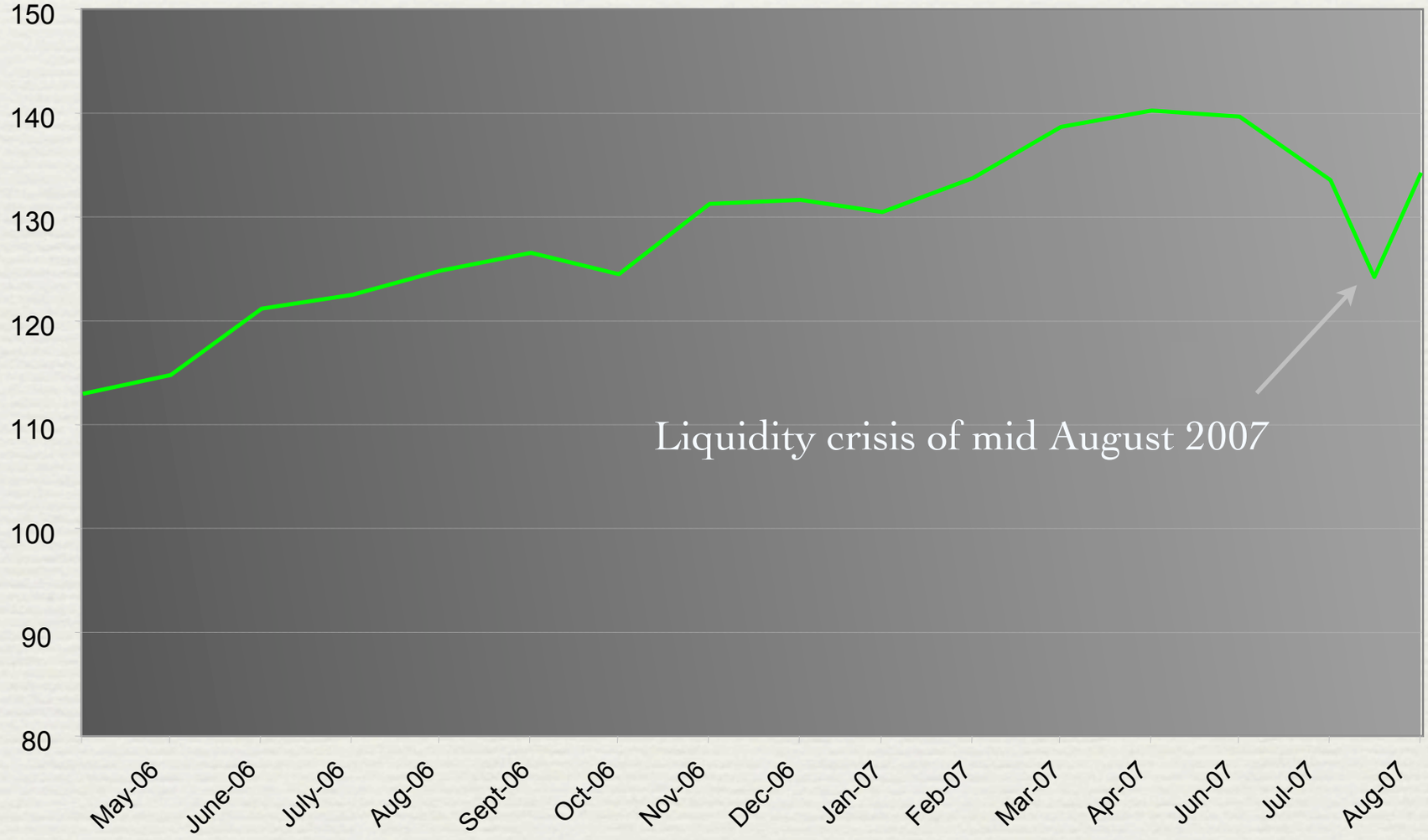
This graph shows the 7-year performance of a portfolio constructed using Stress VaR, compared to the same portfolio – as constructed using other risk measures purely based on performance. As can be seen, the Stress VaR protects the downside without altering performances.

Performance of an optimized portfolio



Pro-cyclicality of VaR

- Basel II: Capital requirements proportional to the VaR
- VaR is "reactive" to a market drop
- Selling of assets to meet capital requirements
- Pushes the market further



Importance of nonlinearities:
Did you write a put without knowing?

Sources of nonlinearities in order of importance:

1. Liquidity Gaps
   - They are SYSTEMATIC
   - Create CORRELATION BREAKS
2. Dynamic Trading
   - Positions change with market
   - Mimic OPTION REPLICATION
3. Nonlinear Relation Between Assets
   - BONDS vs. STOCKS (credit spreads increase when the stock price declines)
   - Options: options are commonly considered as being responsible for nonlinearities. However, this is only the least cause of nonlinearities.

Conclusions

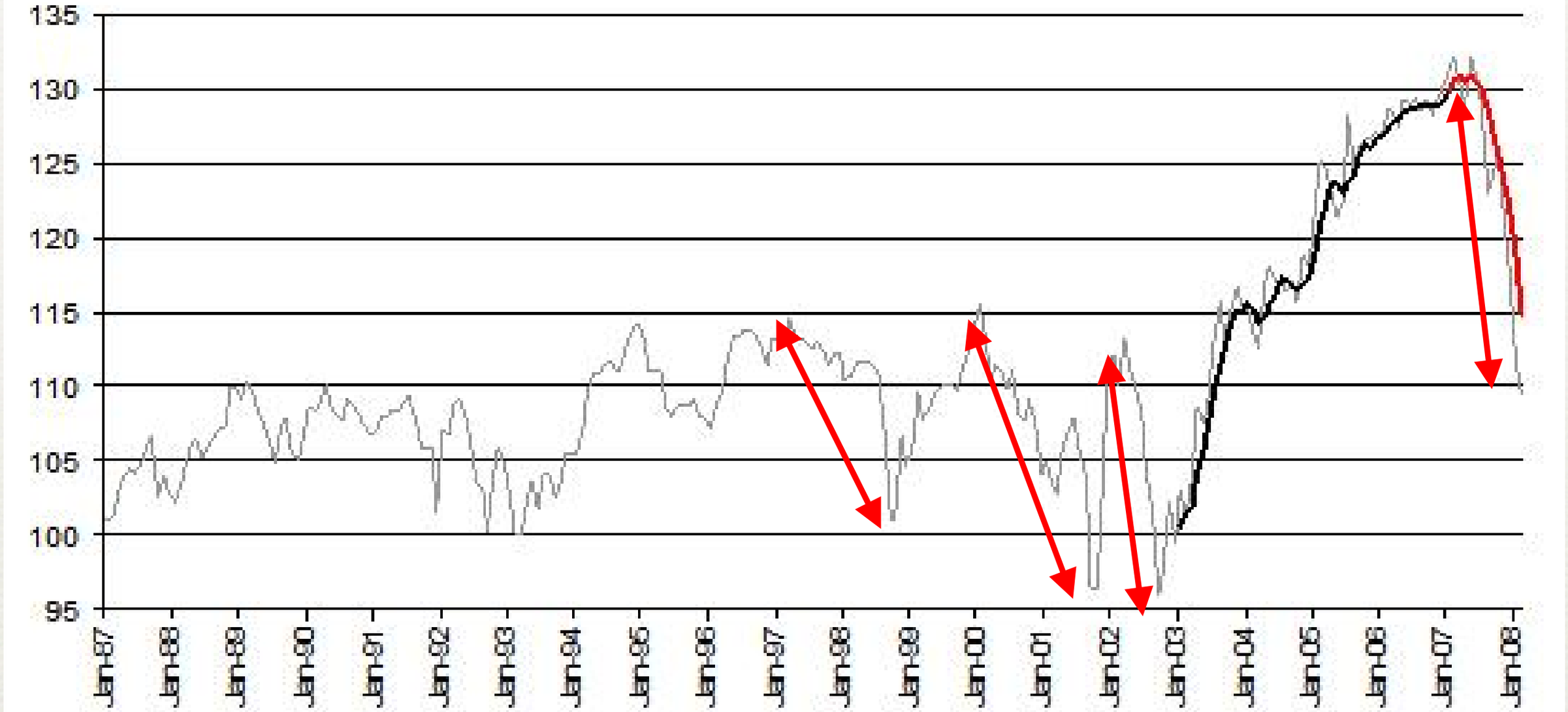
1. Risk of a portfolio: Dynamical relation of the portfolio with the underlying risk factors PLUS the distribution of risk factors

2. DO NOT confuse: PERFORMANCE ANALYSIS ≠ RISK ANALYSIS

3. Use LONG HISTORY of market factors to anticipate near-future moves and possible EXTREME SHIFTS

4. Distribution of risk factor can (and should) be conditional to current markets

5. DO NOT "sell a put" without noticing: OPTION PREMIUM ≠ TRUE ALPHA !



1. For a fund investor: Separate lucky winners from talented managers!

2. For a fund manager: Avoid using a risk measure leading to pro-cyclical behavior and liquidity traps

StressVaR and capital requirements: Basel III (??)

- Problems with the classical VaR:
  - The rationale for the economic capital is to avoid bankruptcy, hence the loss should never exceed k × VaR.
  - The risk measure is 1-dimensional and neither tells the exact risk source, nor the market scenario it corresponds to. As a consequence, the Regulator cannot realistically require that the loss never exceeds the declared risk.
  - The most serious problem is pro-cyclicality: in a market downturn, the risk measure increases, leading most market participants to sell out positions in order to meet capital adequacy, adding to the market turmoil.

StressVaR and capital requirements: Basel III (??)

We here propose to include stress tests in the measure of capital adequacy in such a way that we respect the following 3 golden rules:

- The Regulator defines which stress to apply to which indices. This will ensure that economic research is unbiased to anticipate potential market shifts.
- The Institution computes itself the impact of stress scenarios on its activity. It is free to add other scenarios that the Regulator didn't think of for its particular case, either by stressing other risk factors or by increasing the stress size given by the Regulator.
- The Regulator verifies that losses incurred by the Institution (if ever) do not exceed what could have been anticipated given the declared stress tests and the actual market moves. In other words, the Institution is responsible for correctly anticipating the impact of markets on its activity, but not for the moves of markets itself.

StressVaR and capital requirements: Basel III (??)

- The required operating capital is proportional to the worst declared stress test (as of 2). The initial multiplier value is 1. In case of a violation, the impact on the multiplier depends on the amount of the violation. Minor violations have a minor impact, while large violations severely impact the multiplier. If there is no violation, the multiplier is progressively brought down back to the value 1.
- We mean to exclude all notions of probability in this framework. Institutions should be responsible for the amounts they declare, not for the probability of such or such event.

StressVaR and capital requirements: Basel III (??)

Index	Stress++	Stress+	Stress0	Stress-	Stress--
S&P500	+20%	+10%	0%	–10%	–20%
TB Yield 10Y	+200bp	+100bp	0bp	–100bp	–200bp
BAA Credit Spread	+500bp	+200bp	–10bp	–100bp	–200bp
…					

Stress++ = 99% percentile up
Stress+ = 84% percentile up
Stress0 = Median
Stress– = 84% percentile down
Stress-- = 99% percentile down

- It is extremely important that the Regulator makes all efforts to anticipate the distribution of possible forward moves, and does not simply rely on the past volatility of each index.