Regular variations for functions, random variables and stationary processes

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Outline

2. The Markowitz approach
3. Regularly varying functions
4. Regularly varying random variables
5. Regularly varying random vectors
6. Regularly varying processes
Solvency II and the SCR

**Definition (Solvency Capital Requirement)**

"The SCR is the capital required to ensure that the (re)insurance company will be able to meet its obligations over the next 12 months with a probability of at least 99.5%." (Wikipedia)

Extrapolation of the magnitude of an event that occurs once per 200 years, i.e. that is not observed!
Motivation: insurance companies as Hedge Fund investors

Standard formula for "other equities"

The capital requirement is 48% of the investment whatever is the HF strategy.

Annualized returns Lyxor indices
SCR as a VaR of the log ratios

**Definition**

Log ratios \( r_t = \log(P_{t+1}/P_t) \) where \((P_t)\) are weakly prices of HF indices.

As \( \log(x) \approx x - 1 \) and \( \log(xy) = \log(x) + \log(y) \), we have

\[
\mathbb{P}\left( \frac{P_{T+1} - P_T}{P_T} \leq -SCR \right) = 0.005 \quad \approx \quad \mathbb{P}\left( \log\left( \frac{P_{T+1}}{P_T} \right) \leq -SCR \right) = 0.005
\]

\[
\approx \quad \mathbb{P}\left( \sum_{t=T}^{T+52} r_t \leq -SCR \right) = 0.005.
\]

**Definition**

The Value at Risk (VaR, quantile) at the confidence level \( \alpha \) of the r.v. \( X \) is

\[
\text{VaR}_\alpha = \inf\{x \mid \mathbb{P}(X \leq x) \geq \alpha\}
\]

\[
SCR \approx -\text{VaR}_{0.005} \quad \text{with} \quad X = \sum_{t=T}^{T+52} r_t
\]
The need for Quantitative Risk Management

3 different strategies, same SCRs under standard formula

Solvency II: a challenge for the mathematician

Use a standard formula or find a more realistic calculation of the SCR (quantiles, VaR) using an internal model.
Outline

1 Motivations: Empirical Risk Management

2 The Markowitz approach

3 Regularly varying functions

4 Regularly varying random variables

5 Regularly varying random vectors

6 Regularly varying processes
Definition (Standard gaussian distribution)

A r.v. $X$ is standard gaussian $\mathcal{N}(0, 1)$ if

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2), \quad x \in \mathbb{R}.$$ 

Properties

1. If $Y = \mu + \sigma X$ then $Y \sim \mathcal{N}(\mu, \sigma^2)$ with $\mu = \mathbb{E}[Y]$ and $\sigma^2 = \text{Var}(Y) = \mathbb{E}[(Y - \mu)^2]$.

2. If $X_i \sim \mathcal{N}(0, 1)$ iid and $X = (X_1, \ldots, X_d)'$ then $Y = A + \Sigma X \sim \mathcal{N}_d(A, \Sigma^2)$ with $\Sigma^2 = \Sigma \Sigma'$.

$$\text{qnorm}(0.005) [1] \approx -2.575829 \implies \text{SCR} \approx -2.6 \times \text{var} (\sum_{t=T}^{T+52} r_t).$$
Mean-VaR Portfolio Optimization: Markowitz (1952)

Remark

Work for other elliptical distribution where $SCR \approx -\beta \times \text{var}(\sum_{t=T}^{T+52} r_t)$, $\beta$ being the risk aversion coefficient, see McNeil et al. (2006).
Advantages

1. Easy calculations: VaR reduces to \( \text{Var} \),
2. Any point of the convex hull corresponds to a feasible strategy: stability of the gaussian law.
Gaussian modeling works on average

Dark: marginal density of the log-ratio of an index
Red: Gaussian model
Blue: Gaussian model excluding the 40 worst days
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Exceedances

”Let the tails speak by themselves”! (Embrechts et al., 1997)

**Definition (Exceedance)**

For a threshold $u$, exceedance $E$ is the excess of the r.v. $X$ above $u$:

$$E_u = X - u \quad \text{conditionally to} \quad X \geq u.$$
Feasible calculations when $u \to \infty$

The threshold $u$ is large and varies. Let $U$ be a monotone function on $(0, \infty)$ (for instance $U(u) = \mathbb{E}[|E_u|^k]$).

**Lemma (Feller, 1971)**

It exists a function $\Psi$ such that for any $x > 0$

$$\frac{U(xu)}{U(u)} \to \Psi(x) \quad u \to \infty$$

iff $\Psi(x) = x^\rho$, $\rho \in \mathbb{R}$, 

iff $U(u) = u^\rho L(u)$ for some slowly varying function $L$ satisfying

$$\frac{L(xu)}{L(u)} \to 1, \quad u \to \infty.$$

Proof: for any $x, y > 0$ then $\frac{U(xyu)}{U(u)} = \frac{U(xyu)}{U(yu)} \frac{U(yu)}{U(u)}$ and then $\Psi(xy) = \Psi(x)\Psi(y)$. 
Regularly varying functions

**Definition (Karamata, 1930)**

The function $U$ on $(0, \infty)$ is regularly varying with index $\rho$ iff $U(u) = u^\rho L(u)$.

**Theorem (Karamata, 1930, Feller, 1971)**

If $U \in RV_\rho$ and if $U_p(x) = \int_x^\infty u^p U(u) \, du$ exists then

$$\frac{x^{p+1} U(x)}{U_p(x)} \to \lambda = -(p + \gamma + 1) \geq 0.$$

Conversely, if $\lambda > 0$, then $U \in RV_\gamma$ with $\gamma = -\lambda - p - 1$ and $U_p \in RV_{-\lambda}$.

**Remark**

The regularly varying functions $U$ and $U_p$ have slowly varying functions that are equivalent up to a constant at $+\infty$. 
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Definition (Feller, 1971)
The r.v. \( X \geq 0 \) is regularly varying of index \( \alpha > 0 \), \( X \in RV_\alpha \) iff

\[
1 - \mathbb{P}(X \leq u) = \mathbb{P}(X > u) = u^{-\alpha}L(u), \quad u > 0.
\]

Definition (Pareto distribution)
The r.v. \( X \) follows a Pareto distribution \((\alpha, \lambda)\) if

\[
\mathbb{P}(X > x) = \left(\frac{\lambda}{x}\right)^\alpha, \quad x > \lambda.
\]
Maxima of random regularly varying variables

Let $X_i$ iid and $M_n = \max\{X_i; 1 \leq i \leq n\}$.

**Theorem (Fisher, 1927)**

There exists a non-decreasing sequence $(a_n)$ such that $M_n/a_n$ converges to a non-degenerate limit iff $X_i$'s are regularly varying of index $\alpha > 0$. For some $c > 0$,

$$\mathbb{P}(M_n \leq xa_n) \to \exp(-cx^{-\alpha}), \quad n \to \infty.$$ 

Proof: Denote $F$ and $G$ the $X_i$'s and asymptotic distribution. Then

$$F^n(xa_n) \to G(x) \iff n(1 - F(xa_n)) \to -\log(G(x))$$

We conclude as $1 - F$ is non-increasing and for $a_n \leq t \leq a_{n+1}$

$$\frac{1 - F(xa_{n+1})}{1 - F(a_n)} \leq \frac{1 - F(xt)}{1 - F(t)} \leq \frac{1 - F(xa_{n})}{1 - F(a_{n+1})}.$$
Theorem (Pickands-Balkema-de Haan, 1975-1974)

Let \((X_t)\) iid regularly varying r.v. with index \(\alpha > 0\). Denote \(F_u\) the distribution of the exceedances \(E_u\) over \(u > 0\):

\[
F_u(x) = \mathbb{P}(X - u \leq x \mid X > u), \quad x \geq 0, \quad X \sim F.
\]

Then

\[
\lim_{u \to \infty} \sup_{0 < x < \infty} |F_u(x) - G_{\xi, \beta}(x)| = 0
\]

where \(G_{\xi, \beta}\) is the Generalized Pareto Distribution

\[
G_{\xi, \beta}(x) = 1 - (1 - \xi(x - u)/\beta)^{-1/\xi}, \quad \xi = \frac{1}{\alpha}, \beta > 0.
\]
Peak Over Threshold approach, Embrechts et al. (1997)

POT approach: fit by MLE a GPD on the exceedances $E_u$ for many $u \mapsto \hat{\xi}_u$:

![Graph](image-url)
Let $X_i$ iid and $M_n = \max\{X_i; 1 \leq i \leq n\}$.

**Theorem (Feller, 1971)**

*If $X_1$ and $X_2$ are independent regularly varying r.v. with index $\alpha > 0$ and with respective slowly varying function $L_1$ and $L_2$ then

$$\mathbb{P}(X_1 + X_2 > x) \sim x^{-\alpha}(L_1(x) + L_2(x)).$$

Proof: We show that

$$\mathbb{P}(X_1 + X_2 > x) \sim \mathbb{P}(X_1 > x) + \mathbb{P}(X_2 > x).$$
Regularly varying distribution

**Definition (Feller, 1971)**

A r.v. $X \in RV_\alpha$ iff it exist $p, q \geq 0$ with $p + q = 1$ and a slowly varying function $L$ such that

$$P(X > x) \sim p \frac{L(x)}{x^\alpha} \quad \text{and} \quad P(X \leq -x) \sim q \frac{L(x)}{x^\alpha}, \quad x \to \infty.$$
Example: strictly stable r.v.

**Definition**

A r.v. $Y$ is strictly $\alpha$-stable distributed iff $\exists \ a > 0$, $Y_1$ and $Y_2$ independent, distributed as $Y$ such that $Y_1 + Y_2 = aY$ in distribution. Then $Y$ is strictly $\alpha$-stable with $0 < \alpha \leq 2$ and c.f. $\exp(-\sigma^\alpha |x|^\alpha \chi_\alpha(x, p, q))$,

$$\chi_\alpha(x, p, q) = \frac{\Gamma(2 - \alpha)}{1 - \alpha} (\cos(\pi \alpha/2) - i\text{sgn}(x)(p - q) \sin(\pi \alpha/2)).$$
Let $X_i$ iid and $S_n = \sum_{i=1}^{n} X_i$.

**Theorem (Central Limit Theorem with heavy tails, Feller 1971)**

Assume that $\mathbb{E}[X^2] = +\infty$ and $X$ is centered if $\mathbb{E}[X] < \infty$.

The sequence $(a_n^{-1}S_n)$ has a non degenerate limit for some $(a_n)$ iff $X \in RV_\alpha$.

The limit is distributed according to a (strictly) $\alpha$-stable law (for $\alpha \neq 1$).
Precise large deviations

Theorem (A.V. Nagaev, 1969)

\( (X_i) \) iid random variables with \( \alpha > 0 \) regularly varying (centered if \( \alpha > 1 \)) distribution then \( S_n = \sum_{i=1}^{n} X_i \) satisfies the precise large deviations relation

\[
\lim_{n \to \infty} \sup_{x \geq b_n} \left| \frac{\mathbb{P}(S_n > x)}{n \mathbb{P}(|X| > x)} - p \right| = 0 \text{ and } \lim_{n \to \infty} \sup_{x \geq b_n} \left| \frac{\mathbb{P}(S_n \leq -x)}{n \mathbb{P}(|X| > x)} - q \right| = 0
\]

with \( b_n = n^{\delta + 1/(\alpha \wedge 2)} \) for any \( \delta > 0 \).

Remark

\[ q n \mathbb{P}(|X| > x) \sim n \mathbb{P}(X \leq x) \]

If \( (r_t) \) iid \( \implies \mathbb{P} \left( \sum_{j=T}^{T+52} r_t \leq -SCR \right) = 0.005 \sim \mathbb{P} \left( r_t \leq -SCR \right) = 0.0001. \]

\[ SCR \approx -\text{VaR}_{0.005} \text{ with } X = r_t. \]
SCR calculation in the iid case

SCR calculation with POT approach, Smith (197)

\[
\widehat{SCR} = \max_{15 \leq m \leq 40} u_m + \frac{\hat{\beta}}{\hat{\xi}} \left( \frac{n \times 0.0001}{m} \right)^{-\hat{\xi}} - 1
\]

where \( m \) is the number of exceedances.
SCR extrapolation when extremes cluster

What is happening for dependent sequences for whom extremes cluster?
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Remark

A r.v. $X$ is regularly varying with index $\alpha > 0$ iff there are r.v. $\Theta \in \{-1, +1\}$ and $Y \sim \text{Pareto}(\alpha, 1)$ independent satisfying

$$
P(|X|^{-1}X \in \cdot \mid |X| > u) \to P(\Theta \in \cdot), \quad u \to \infty,
$$

$$
P(|X| \leq xu \mid |X| > u) \to P(Y \leq x), \quad u \to \infty.
$$
Breiman’s Lemma

**Lemma (Breiman (1965))**

If $X_1 \geq 0$ and $X_2 \geq 0$ are independent, $X_1 \in RV_\alpha$ and $\mathbb{E}[X_2^{\alpha+\varepsilon}] < \infty$, then

$$
\mathbb{P}(X_1 X_2 > u) \sim \mathbb{E}[X_2^\alpha] \mathbb{P}(X_1 > u).
$$

**Proof:** (Jessen and Mikosch, 2006)

$$
\frac{\mathbb{P}(X_1 X_2 > u)}{\mathbb{P}(X_1 > u)} \approx \int \frac{\mathbb{P}(X_1 > u/y)}{\mathbb{P}(X_1 > u)} d\mathbb{P}(X_2 > y) \approx \int y^\alpha d\mathbb{P}(X_2 > y).
$$

**Remark**

Let $X \in RV_\alpha$ associated with $\Theta$. We also have

$$
\mathbb{P}(X > u) \approx \mathbb{P}(\Theta_+ |X| > u) \approx \mathbb{E}[\Theta_+^\alpha] \mathbb{P}(|X| > u)
$$

$$
= \mathbb{P}(\Theta = 1) \mathbb{P}(|X| > u) = p \mathbb{P}(|X| > u).
$$
Regularly varying multivariate distributions

Regular variations, Basrak et al. (2002)

A random vector \( X = (X_1, \ldots, X_d) \) is regularly varying if a non-null Radon measure \( \mu_d \) is such that

\[
n \mathbb{P}(a_n^{-1}(X_1, \ldots, X_d) \in \cdot) \xrightarrow{v} \mu_d(\cdot),
\]

where \( (a_n) \) satisfies \( n \mathbb{P}(|X| > a_n) \to 1 \) and \( \mu_d(tA) = t^{-\alpha} \mu_d(A), \ t > 0 \).

Definition (Resnick, 1987)

It is equivalent to the existence of the spectral tail vector \( \Theta = (\Theta_1, \ldots, \Theta_d) \) satisfying

\[
\mathbb{P}(\left|X\right|^{-1}(X_1, \ldots, X_d) \in \cdot \mid |X| > u) \to \mathbb{P}((\Theta_1, \ldots, \Theta_d) \in \cdot), \quad u \to \infty,
\]

\[
\mathbb{P}(\left|X\right| \leq xu \mid |X| > u) \to \mathbb{P}(Y \leq x), \quad u \to \infty.
\]
Choose $d = 2$, $X = (X_1, X_2)$ with $X_1 \in RV_\alpha$ independent of $X_2 \in RV_\alpha$ and $|X| = X_1 + X_2$. If $L_2(u) = o(L_1(u))$

$$\mathbb{P}(X_1 > u) \sim \mathbb{P}(|X| > u) \sim u^{-\alpha}(L_1(u) + L_2(u)),$$

$$\mathbb{P}(X_1/|X| = 1 \mid |X| > u) \to 1, \mathbb{P}(X_2/|X| = 1 \mid |X| > u) \to 1 \text{ and } \Theta = (1, 0).$$

To avoid degenerate $\Theta$, we assume that $X_i$ are identically distributed.

**Remark**

When $L_1(u) \sim L_2(u)$ then $\Theta = (1, 0)$ or $= (0, 1)$ w.p. 1/2.

If $X_i \sim F_i$ then we standardize via the transform $1/(1 - F_i(X_i)) \sim Pareto(1, 1)$. 
Identically distributed margins

Definition (Basrak and Segers, 2009)

Assume that $X_i$ are identically regularly varying distributed with index $\alpha > 0$ iff it exists the spectral tail vector $\Theta = (\Theta_1, \ldots, \Theta_d)$ satisfying

$$\mathbb{P}(|X_1|^{-1}(X_1, \ldots, X_d) \in \cdot \mid |X_1| > u) \rightarrow \mathbb{P}((\Theta_1, \ldots, \Theta_d) \in \cdot), \quad u \rightarrow \infty.$$  

Example

If the $X_i$ are independent then $\Theta = (1, 0, \ldots, 0)$.  

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Regularly varying processes

Definition (Basrak and Segers, 2009)
A stationary sequence \((X_t)\) is regularly varying of order \(\alpha > 0\) iff \(X_0 \in RV_\alpha\) and it exists the spectral tail process \((\Theta_t)\) defined for any \(k \geq 0\), any \(u > 0\) by the relation

\[
P(|X_0|^{-1}(X_0, \ldots, X_k) \in \cdot \mid |X_0| > u) \rightarrow P((\Theta_0, \ldots, \Theta_k) \in \cdot), \quad u \rightarrow \infty.
\]

Example
If the \((X_t)\) are iid then \(\Theta_t = 0, \ |t| \neq 0\).
The AR(1) model

**Definition (AR(1) model)**
The AR(1) model is the solution of $X_t = \phi X_{t-1} + Z_t$, $|\phi| < 1$ with $(Z_t)$ is an iid regularly varying sequence if order $\alpha > 0$.

**Proposition**
We have $X_0 \in RV_\alpha$ and $\Theta_t = \phi^t$, $t > 0$. 
The GARCH(1,1) model

Definition (Bollerslev, 1986)

The GARCH(1,1) model \((X_t)\) is the solution of \(X_t = \sigma_t Z_t, \ t \in \mathbb{Z}\) with \((Z_t)\) is an iid mean zero and unit variance sequence of random variables and \((\sigma_t^2)\) satisfies the stochastic recurrence equation

\[
\sigma_t^2 = \alpha_0 + (\alpha_1 Z_{t-1}^2 + \beta_1) \sigma_{t-1}^2, \quad t \in \mathbb{Z}.
\]

Proposition

If \(X_0 \in RV_\alpha\) then we have

\[
P(|X_0|^{-1}(X_0, \ldots, X_t) \in \cdot \mid |X_0| > x) \rightarrow \frac{1}{\mathbb{E}|Z_0|^{\alpha}} \mathbb{E}\left[|Z_0|^{\alpha} 1_{(Z_0, Z_1 \Pi_1^{0.5}, \ldots, Z_t \Pi_t^{0.5}) \in |Z_0| \cdot} \right],
\]

where \(\Pi_t = A_1 \cdots A_t\) with \(A_t = \alpha_1 Z_{t-1}^2 + \beta_1\).
Alternative measures of dependance

**Definition (The extremal index)**

The stationary sequence \((X_t)\) has extremal index \(\theta\) if for any \(x > 0\) we have

\[
P(\max\{|X_1|, |X_2|, \ldots, |X_n|\} \geq a_n x) \sim P(\max\{|X'_1|, |X'_2|, \ldots, |X'_n|\} \geq a_n x)^\theta
\]

where \(X'_t\) are iid copies.

**Proposition (Basrak and Segers, 2009)**

If \((X_t) \in RV_\alpha\) then

\[
\theta = \mathbb{E}[\max\{|\Theta_0|, |\Theta_1|, |\Theta_2|, \ldots\}^\alpha - \max\{|\Theta_1|, |\Theta_2|, |\Theta_3|, \ldots\}^\alpha].
\]
Alternative measures of dependance

Definition (Upper tail dependence coefficient)

For any vector \((X_0, X_h)\) the upper tail dependence coefficient \(\rho(h)\) satisfies

\[
\rho(h) = \lim_{u \to \infty} \mathbb{P}(X_h > u \mid X_0 > u).
\]

Proposition (Davis et al., 2013)

If \((X_t) \in RV_{\alpha}\) then

\[
\rho(h) = \frac{\mathbb{E}[\min\{\Theta_0, \Theta_t\}^{\alpha}]}{\mathbb{E}[(\Theta_0)^{\alpha}]}. 
\]

Proof: Applying Breiman’s Lemma, we have

\[
\mathbb{P}(\min\{X_h, X_0\} > u) \approx \mathbb{P}(\min\{\Theta_h, \Theta_0\} \mid X_0 > u) \approx \mathbb{E}[\min\{\Theta_h, \Theta_0\}^{\alpha}] \mathbb{P}(\mid X_0 \mid > u)
\]
Examples in the $m$-dependent case

Assume $(X_t, t \leq 0)$ is independent of $\sigma(X_t, t \geq m + 1)$ then $\Theta_t = 0$ for $|t| \geq m$.

**Definition (Conditional spectral tail process)**

Define for $m$-dependent RV($\alpha$) processes $(\Theta'_0, \ldots, \Theta'_m) = (\Theta_0, \ldots, \Theta_m)$ conditionally to $\Theta_{-j} = 0, 0 < j \leq m$.

**Example**

1. $X_t = \max(Z_{t-1}, Z_t)$ then $(\Theta'_0, \Theta'_1) = (1, 1)$,
2. $X_t = Z_t + \frac{1}{2}Z_{t-1}$ then $(\Theta'_0, \Theta'_1) = (1, \frac{1}{2})$.  
Large deviations in the $m$-dependent case

Theorem (Mikosch and W., 2012)

Assume $(X_t)$ is $\alpha > 0$ regularly varying (centered if $\alpha > 1$) distribution then

$$\lim_{n \to \infty} \sup_{x \geq b_n} \left| \frac{\mathbb{P}(S_n > x)}{n \mathbb{P}(|X| > x)} - b_+ \right| = 0 \quad \text{and} \quad \lim_{n \to \infty} \sup_{x \geq b_n} \left| \frac{\mathbb{P}(S_n \leq -x)}{n \mathbb{P}(|X| > x)} - b_- \right| = 0,$$

with $b_n = n^{\delta + 1/(\alpha \wedge 2)}$ for any $\delta > 0$ and cluster indices

$$b_\pm = \mathbb{E} \left[ \left( \sum_{t=0}^{m} \Theta_t \right)^{\pm} \alpha - \left( \sum_{t=1}^{m} \Theta_t \right)^{\pm} \alpha \right] = \mathbb{P}(\Theta_{-j} = 0, 1 < j \leq m) \mathbb{E} \left[ \left( \sum_{t=0}^{m} \Theta'_t \right)^{\pm} \right].$$
Application to risk management

Definition (Empirical conditional spectral tail process)

Define \((\hat{\Theta}_j', \ldots, \hat{\Theta}_{j+k}') = (r_j/|r_j|, \ldots, r_{j+k}/|r_j|)\) if \(|r_t| > u\), \(t \in l = \{j, \ldots, j + k\}\), \(|r_{j-1}| < \varepsilon u\) and \(|r_{j+k+1}| < u\) are clusters of exceedances starting with a jump.
Approximation of the cluster index

\[ \mathbb{P}\left( \sum_{j=T}^{T+52} r_t \leq -SCR \right) = 0.005 \approx \mathbb{P}\left( |r_t| > SCR \right) = 0.0001 \big/ b_-, \]

with \( b_- = \gamma \mathbb{E}\left[ (\sum_{t=0}^{m} \Theta'_t)^\alpha \right] \) where \( \gamma = \mathbb{P}(\Theta_{-j} = 0, 1 < j \leq m) \) can be interpreted as the inverse of the average length of the clusters of exceedances starting with a jump.

Definition (Empirical cluster index)

\[ \hat{b}_- = \frac{1}{\ell} \sum_{i=1}^{\ell} \frac{1}{k_i} \left( \sum_{t=0}^{k_i} \hat{\Theta}'_{j_i+t} \right)^\hat{\alpha}. \]
Calculation of the SCR when extremes cluster

**Definition**

\[
\hat{SCR} = \max_{15 \leq m \leq 40} \max_{\ell} \mu_m + \frac{\hat{\gamma}}{\hat{\beta}} \left( \frac{n \times 0.0001}{m \times \hat{b}_-} \right)^{-\xi} - 1.
\]
Conclusion

- An alternative approach of risk management based on exceedances and not on variances,
- Regular variations ensure stability and feasible computations,
- The statistical inference remains to be done.
Conclusion

Thank you for your attention!