Analysis of the EUR/HRK exchange rate and pricing options on the Croatian market: the NGARCH model as the alternative to the Black-Scholes model

Petra Posedel
Faculty of Economics and Business
University of Zagreb

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Department of Mathematics, University of Zagreb
1. Croatian Market of Derivatives: a Challenge of forming one
   - An explosive increase in trading on the Croatian market
   - The goal of the study

2. Non-linear in mean asymmetric GARCH model
   - The NGARCH model for the EUR/HRK time series
   - Risk estimation

3. GARCH option pricing model
   - The European call option on the foreign currency
   - Analysis of foreign currency options
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Croatian Market of Derivatives: a Challenge of forming one

Trading on the Croatian market

- The interest of professional investor for financial derivatives on the Croatian market has been increasing a lot recently.
- The Croatian market of derivatives does not exist yet!
- Some law conventions...trading with derivatives is expected to follow.
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The interest of market participants has been attracted lately by interventions of the Croatian national bank on the foreign capital market.

Croatia faces a possibility of changing the domestic currency.

We are motivated thus in exploring different kinds of foreign currency options.

We analyze in details the empirical distribution of the EUR/HRK currency time series and the consequences of such analysis for pricing foreign currency.
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The model of Black and Scholes

Connection to the Black-Scholes model

- Prices on option markets are commonly quoted in terms of Black-Scholes implied volatility.
- This *does not* mean that market participants believe in the hypothesis of the Black-Scholes model.
- The Black-Scholes formula is not used as a pricing model for vanilla options but as a tool for *translating* market prices into a representation in terms of implied volatility.
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- to estimate the parameters in the dynamics of the currency time series using the NGARCH model,
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*Modelling the EUR/HRK time series*

- $P_t$ ... the EUR/HRK exchange rate price at time $t$, defined as the number of Croatian kunas required to purchase 1 euro

- The dynamics of returns $R_t$ is described with

\[ R_{t+1} = \ln \left( \frac{P_{t+1}}{P_t} \right) = r_d - r_f + \lambda \sigma_{t+1} - \frac{1}{2} \sigma_{t+1}^2 + \sigma_{t+1} Z_{t+1}, \tag{1} \]

\[ \sigma_{t+1}^2 = \omega + \alpha (\sigma_t Z_t - \rho \sigma_t)^2 + \beta \sigma_t^2, \tag{2} \]

where $Z_t$ are i.i.d. $N(0, 1)$ and

\[ \omega > 0, \quad \alpha \geq 0, \quad \beta \geq 0 \quad \text{and} \quad \alpha (1 + \rho^2) + \beta < 1. \tag{3} \]
Modelling the EUR/HRK time series

- $P_t$ . . . the EUR/HRK exchange rate price at time $t$, defined as the number of Croatian kunas required to purchase 1 euro
- The dynamics of returns $R_t$ is described with a non linear in mean, asymmetric GARCH (1,1) model:

$$R_{t+1} \equiv \ln \left( \frac{P_{t+1}}{P_t} \right) = r_d - r_f + \lambda \sigma_{t+1} - \frac{1}{2} \sigma^2_{t+1} + \sigma_{t+1} Z_{t+1},$$

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Introduction
NGARCH
Summary
The NGARCH model for the EUR/HRK time series

EUR/HRK 2001-2005

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Option pricing on the Croatian market
We use the MLE where the log-likelihood function is

\[
L_T = \frac{1}{T} \sum_{t=1}^{T} \left[ -\frac{1}{2} \ln(2\pi) - \frac{1}{2} \ln(\sigma_t^2) - \frac{1}{2} \frac{(P_t - (r_d - r_f + \lambda \sigma_t - \frac{1}{2} \sigma_t^2))^2}{\sigma_t^2} \right], \quad (4)
\]

where \( T = 1297 \).

Maximizing the \( L_T \) function we obtain

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The NGARCH model gives a more accurate estimation of risk since it incorporates the *heteroscedasticity*. 

![Graph showing price movements over time with different bounds.](image-url)
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We use the GARCH option pricing model of Duan (1995).

Days to maturity are counted in calendar days (365) not in business days per year (256)

\[ T \ldots \text{maturity} \]
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We introduce some risk neutral criterium, namely the *equilibrium price measure*. 
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Definition

The equilibrium price measure satisfies the local risk neutral valuation relationship (LRNVR) if every asset value $X_t$ measured in domestic currency satisfies

1. $X_{t+1}/X_t$ is conditionally log-normal distributed w.r.t the equilibrium measure $\ast$

2. 
   
   \[ E_t^* \left[ X_{t+1}/X_t \right] = e^{r_d}, \quad (5) \]

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   \[ \text{Var}_t^* \left[ \ln(X_{t+1}/X_t) \right] = \text{Var}_t \left[ \ln(X_{t+1}/X_t) \right], \quad (6) \]
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Option pricing on the Croatian market
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Relation (7) enables pricing foreign currency options!

We have even more: from relation (8) it follows that the risk premium \( \lambda \) has global influence on the conditional variance even the risk was locally neutralized with respect to \(*\).

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The price of the European call option on foreign currency

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The *fair* price of the European call option, in the risk neutral world, in time $t$ with strike $K$ and maturity date $\tau + t$, $\tau > 0$ is given by

$$co_t = \exp(-r_d \tau) E_t^* \left[ \max(P_{t+\tau} - K, 0) \right].$$

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$\implies E_t^*$ cannot be computed explicitly!
We use Monte Carlo simulations

\[ c^{GH} \approx \exp(-r_d \tau) \frac{1}{MC} \sum_{i=1}^{MC} \max \{ P_{i,t+\tau} - K, 0 \}, \quad (10) \]

where \( MC = 50000 \), and for the \( i-th \) simulation we have

\[ P_{i,t+\tau} = P_t \exp \left( \sum_{j=1}^{\tau} R_{i,t+j} \right), \quad i = 1, 2, \ldots, MC. \quad (11) \]
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Illustration: a simulation study

Option prices are calculated for:

- different days to maturity: $\tau = 30, 60 \text{ and } 90 \text{ days}$
- different moneyness $m = 0.97, 0.985, 1, 1.015 \text{ and } 1.03$
  which corresponds respectively to strikes $K = 7.11495, 7.224975, 7.335, 7.445 \text{ and } 7.555$ for currency spot price $P_t = 7.335$.

- the obtained prices are then compared to their Black-Scholes counterparts with the average annual volatility $0.036128$

- in order to analyze the effect of the asymmetry parameter $\rho$, we repeat the simulation procedure for $\rho = -0.461$ and $\rho = 0$. 

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Option pricing on the Croatian market
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The comparison: \( \rho = -0.17074 \)

Mild negative asymmetry \( \implies \) options out of the money \( (K/P > 1) \) are underpriced in the Black-Scholes model with constant volatility.
The comparison: $\rho = -0.461$

- Moderate asymmetry $\implies$ options out of the money ($K/P > 1$) are underpriced in the Black-Scholes model with constant volatility and also some options ($\tau = 30$) in the money.
- The underpricing effect in the constant volatility model is now more pronounced for options in the money.
The comparison: $\rho = 0$

- The asymmetry is completely absent $\implies$ the graph of implied volatility becomes almost symmetric (centered in $K/P = 1$)
- Options deeply in the money and deeply out of the money are underpriced in the constant volatility model
For all the graphs:

- for options out of the money, independently of $\rho$, the IV is a **decreasing** function of the maturity.
- for options near the money ($K/P \approx 1$) if the asymmetry is absent or very mild the IV is an **increasing** function of the maturity.
For all the graphs:

- for options out of the money, independently of $\rho$, the IV is a **decreasing** function of the maturity
- for options near the money ($K/P \approx 1$) if the asymmetry is absent or very mild the IV is an **increasing** function of the maturity
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- the locally risk-neutral measure for the domestic economy is identified
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- Still, it is not a priori obvious what should be the risk premium for the volatility—using the time series data from the underlying we find it not significant
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