Optimal Foreign Reserves: The Case of Croatia

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the optimal level of foreign reserves a central bank (CNB) should hold?

are CNB foreign reserves sufficient to mitigate negative effects of potential sudden-stop of capital inflows which comes along with banking crisis?

can we find rationale for huge CNB reserves accumulation during the last 10 years (quadrupled since 1998)

alternative measure of foreign reserves adequacy in dollarized economy

comparison of “optimal reserves” to standard indicators of foreign reserves adequacy
Why alternative measure?

- standard indicators of foreign reserves adequacy (Greenspan-Guidotti 100%-of-short-term debt rule and 3-months-of-imports rule)
  - based on arbitrary thresholds
  - taking into account only benefits (but not costs) of holding reserves

- **BENEFITS**: insurance against sudden stop/banking crisis episodes
- **COSTS**: opportunity costs of holding reserves - repaying external debt, investing in (riskier) assets yielding higher returns
Why alternative measure?

- standard indicators emphasize precautionary motives for holding reserves but are not taking into account potential vulnerabilities of the economy
- sources of vulnerabilities

in dollarized economies protection role of reserves is more important: insurance against sudden stop and key tool for managing domestic financial (in)stability by lessening credit crunch during bank run
The Plan

- provide a new measure based on optimality criterion and capturing all the vulnerabilities an economy (Croatian economy) is exposed to
- building a model (based on stylized facts) of precautionary demand for reserves in a dollarized economy that is subject to sudden stop which occurs in hand with banking crisis
- “simulate” the model and compare optimal and CNB accumulated reserves throughout the history (to see if the precautionary demand for reserves can explain huge increase of reserves during the last 10 years)
- see if today the CNB is holding enough reserves to fight possible crisis
- comparing a new measure of reserves adequacy with the standard ones
Stylized facts and the Model

- first period: tranquil period (before crisis)
- second period: crisis period
  - short-term debt of every sector is not rolled over
  - GDP falls
  - exchange rate depreciates (goes up)
  - part of domestic currency (kuna) deposits is exchanged into foreign currency (euro) deposits
  - bank run on kuna and euro deposits
  - banks and firms withdraw their liquid foreign assets to use them as a buffer against a crisis
maximize (expected: the only source of uncertainty comes from probability of crisis) lifetime utility subject to budget constraint (get deterministic exogenous endowment $y_t$ that is growing over time at the rate of $g$)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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</thead>
<tbody>
<tr>
<td>kuna deposits, $d_t^k$</td>
<td>short-term kuna loans, $l_t^k$</td>
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<tr>
<td>euro deposits, $d_t^e$</td>
<td>short-term euro loans, $S_t l_t^f$</td>
</tr>
<tr>
<td>foreign liquid assets,</td>
<td>short-term foreign borrowing, $S_t b_t$</td>
</tr>
<tr>
<td>$S_t F R B_t^n$</td>
<td></td>
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<tr>
<td>profits, $\Pi_t$</td>
<td></td>
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<tr>
<td>government transfers, $T_t$</td>
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</table>
kuna depreciates (by an absolute change of $\Delta S$), access to foreign loans market is canceled

- a fraction, $\eta$ of kuna deposits is exchanged into euro deposits and a fraction of overall deposits, $\phi$ is withdrawn from banks: kuna household deposits together with kuna and euro corporate deposits and foreign liquid assets will act as a buffer against sudden stop effects

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<td>kuna household deposits, $d_{kh}^h - \phi(1 - \eta)d_{kh}^h$</td>
<td>short-term kuna loans, $l_t^K$</td>
</tr>
<tr>
<td>euro household deposits, $(S_t + \Delta S)d_{fh}^h$</td>
<td>short-term euro loans, $(S_t + \Delta S)l_t^f$</td>
</tr>
<tr>
<td>kuna corporate deposits, $d_{kc}^h - \phi(1 - \eta)d_{kc}^h$</td>
<td>short-term foreign borrowing, $(S_t + \Delta S)b_t = 0$</td>
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<tr>
<td>euro corporate deposits, $(S_t + \Delta S)d_{cf}^h - (S_t + \Delta S)\phi(d_{cf}^h + \frac{\eta}{S + \Delta S}d_{kc}^h)$</td>
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Optimization problem - nonfinancial sector

- households maximize the expected discounted value of utility to make optimal decisions on how much to consume, how much to save and how much to borrow

\begin{align*}
\{c_t, l_t, l_t^k, b_t, d_{t}^{fh}, d_{t}^{fc}, d_{t}^{kh}, d_{t}^{kc}, FRB_{t}^{h}\} & \max_{t=0}^{\infty} E_{0}\left\{\sum_{t=0}^{\infty} \beta^{t} u(c_t)\right\} \\
\text{subject to budget constraints} \\
\text{before crisis:}
\begin{align*}
Pt c_t + S_t(1 + r)l_{t-1}^{f} + (1 + r)l_{t-1}^{k} + S_t(1 + r)b_{t-1} + S_t(d_{t}^{fh} + d_{t}^{fc}) + (d_{t}^{kh} + d_{t}^{kc}) + \\
S_t FRB_{t}^{h} = P_{t}y_{t} + S_t l_{t}^{f} + l_{t}^{k} + S_t b_{t} + S_t(1 + r)(d_{t-1}^{fh} + d_{t-1}^{fc}) + (1 + r)(d_{t-1}^{kh} + d_{t-1}^{kc}) + \\
+ S_t(1 + r)FRB_{t-1}^{h} + \Pi_t + T_t
\end{align*}
\end{align*}

\begin{align*}
\text{during crisis:}
\begin{align*}
Pt c_t + (S_t + \Delta S)(1 + r)l_{t-1}^{f} + (1 + r)l_{t-1}^{k} + (S_t + \Delta S)(1 + r)b_{t-1} + \\
+(S_t + \Delta S)(d_{t}^{fh} + d_{t}^{fc}) + (d_{t}^{kh} + d_{t}^{kc}) = \\
(1 - \gamma)P_{t}y_{t} + (S_t + \Delta S)l_{t}^{f} + l_{t}^{k} + (S_t + \Delta S)(1 + r)(d_{t-1}^{fh} + d_{t-1}^{fc}) + \\
+(1 + r)(d_{t-1}^{kh} + d_{t-1}^{kc}) + (S_t + \Delta S)\phi(d_{t}^{fc} + \frac{\eta}{S_t + \Delta S}d_{t}^{kc}) + \phi(1 - \eta)(d_{t}^{kh} + d_{t}^{kc}) + \\
+(S_t + \Delta S)(1 + r)FRB_{t-1}^{h} + \Pi_t + T_t
\end{align*}
\end{align*}
banks maximize profits (returned to households - owners of banks)

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<td>kuna deposits, $d^k_t (= d^{kh}_t + d^{kc}_t)$</td>
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<tr>
<td>short-term euro loans, $S_t l^f_t$</td>
<td>euro deposits, $S_t d^f_t = S_t (d^{fh}_t + d^{fc}_t)$</td>
</tr>
<tr>
<td>reserve requirement, $RB^k_t + S_t RB^f_t$</td>
<td>short-term foreign borrowing, $S_t FB_t$</td>
</tr>
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<td>foreign liquid assets, $S_t FRB^b_t$</td>
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Financial sector during crisis

- issuing new foreign debt is no longer possible
- loss of confidence in kuna + bank run on overall deposits
- using foreign liquid assets to fight sudden-stop and bank run

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<td>short-term euro loans, ((S_t + \Delta S)I_t^f)</td>
<td>euro household deposits, ((S_t + \Delta S)d_{fh}^t - (S_t + \Delta S)\phi(d_{fh}^t + \eta d_{kh}^t))</td>
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<tr>
<td>reserve requirement, ( RB_t^k + (S_t + \Delta S)RB_t^f )</td>
<td>kuna corporate deposits, ( d_{kc}^t - \phi(1 - \eta)d_{kc}^t )</td>
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<td>foreign liquid assets, ((S_t + \Delta S)FRB_t^b = 0)</td>
<td>euro corporate deposits, ((S_t + \Delta S)d_{fc}^t - (S_t + \Delta S)\phi(d_{fc}^t + \eta d_{kc}^t))</td>
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Optimization problem - financial sector

The bank is choosing domestic deposit demand, domestic loan supply and international net borrowing so as to maximize its profit (returned to households) taking interest rates and exchange rate as given

$$\max_{\{d^f_t, d^{fc}_t, d^{kh}_t, d^{kc}_t, l^f_t, l^k_t, FB_t, RB^k_t, RB^f_t, FRB^b_t\}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} Q_t \Pi_t \right\}$$

subject to:

profits before crisis:

$$\Pi_t = S_t(d^f_t + d^{fc}_t) + (d^{kh}_t + d^{kc}_t) + S_t(1 + r)_t l^f_t - 1 + (1 + r)_t l^k_t -$$

profits during crisis:

$$\Pi_t = (S_t + \Delta S)(d^f_t + d^{fc}_t) + (d^{kh}_t + d^{kc}_t) + (S_t + \Delta S)(1 + r)_t l^f_t - 1 + (1 + r)_t l^k_t -$$
Government before crisis

- issuing long-term security (at a term premium) to finance a stock of international reserves and running a balanced budget

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<td>short-term foreign borrowing, $S_t FG_t$</td>
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<td>long-term foreign borrowing, $S_t PN_t - S_t PN_{t-1}$</td>
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<td>reserve requirement, $RB_t^k + S_t RB_f^f$</td>
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Government during crisis

- issuing new short-term foreign debt is no longer possible

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choosing international reserves to maximize the welfare of the economy

$$\max_{c_t, R_t} \beta E_t[u(c_{t+1})] = \max_{c_t, R_t} \beta \left[ (1 - \pi) u(c_{t+1}^b) + \pi u(c_{t+1}^d) \right]$$

subject to consolidated budget constraint:

before crisis

$$P_t c_t^b + S_t (1 + r) b_{t-1} + S_t FRB_t^h = P_t y_t + S_t b_t + S_t (1 + r) FRB_{t-1}^h + S_t FB_t +$$
$$+ S_t (1 + r) FRB_{t-1}^b - S_t (1 + r) FB_{t-1} - S_t FRB_t^b - S_t (1 + r) FG_{t-1} + S_t FG -$$
$$- S_t (\delta + \pi) R_{t-1}$$

during crisis

$$P_t c_t^d + (S_t + \Delta S)(1 + r) b_{t-1} = (1 - \gamma) P_t y_t + (S_t + \Delta S)(1 + r) FRB_{t-1}^h +$$
$$+ (S_t + \Delta S)(1 + r) FRB_{t-1}^b - (S_t + \Delta S) \phi (d_{t-1}^{fh} + \frac{\eta}{S_t + \Delta S} d_{t-1}^{kh}) -$$
$$- (S_t + \Delta S)(1 + r) FB_{t-1} - (S_t + \Delta S)(1 + r) FG_{t-1} +$$
$$+ (S_t + \Delta S)(1 - \delta - \pi) R_{t-1}$$
Optimality condition

- first order condition with respect to $R_t$:

$$S_{t+1}(1 - \pi)(\delta + \pi)u'(c_{t+1}^b) = (S_{t+1} + \Delta S)\pi(1 - \delta - \pi)u'(c_{t+1}^d)$$

- balances costs and benefits of holding reserves - expected marginal cost of holding reserves before sudden-stop (left hand side) equals expected marginal benefit of holding reserves during the crisis (right hand side)
Level of optimal foreign reserves

- Level of optimal foreign reserves

\[ R_t = \frac{1}{q_{t+1}} \left\{ (1 - \varepsilon^{\gamma}_{t+1})(1 + g)\frac{y_t P_{t+1}}{S_{t+1}} + [\lambda^*_t - (1 + r)(1 - \varepsilon^s_{t+1})\lambda^*_t] - \left[ \lambda^A_{t+1} - (1 + r)(1 - \varepsilon^s_{t+1})\lambda^A_t \right] + \phi \varepsilon^s_{t+1} \lambda^D_{t+1} \right\} \]

where \( \varepsilon^{\gamma}_{t+1}, \varepsilon^s_{t+1}, q_{t+1} \) are functions of \((\pi, \delta, \Delta S, \gamma)\) and

\[ \lambda^*_t = b_t + FB_t + FG_t, \quad \lambda^A_t = FRB^h_t + FRB^b_t, \quad \lambda^D_t = d^{fh}_t + \frac{\eta}{S_t + \Delta S} d^{kh}_t \]

- Optimal reserves increase with probability of crisis, depreciation of kuna and output loss; decrease with term premium.

- Comparing with Greenspan-Guidotti: \( R_t = \lambda^*_t \)
Calibration of the model and data

- calibrating model’s parameters to match the 1998/1999 sudden-stop/banking crisis
- short-term external debt data includes principal payments of its long term debt that are due
- differentiation between two models of parent banks’ behavior (two definitions of short-term debt)
  - *Lenders of Last Resort* - mother banks do not participate in sudden-stop  
    - they convert their short-term funding into long-term funding  
    - short-term debt of mother banks is excluded from the short-term external debt data
  - *Take the Money and Run* - mother banks participate in sudden-stop
Sudden-stop 1998/1999

- Croatia - highly dollarized (euroized) economy
- crisis 1998 – 1999 - double drain risk
  - external drain risk
    - sudden-stop of foreign capital inflows
    - foreign creditors stopped providing credits to Croatian economy
    - financial account reversal
  - internal drain risk
    - banking crisis
    - lost confidence in banking system
    - deposit run
    - credit crunch

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Sudden-stop 1998/1999

- short-term debt is not rolled over
- growth rate of GDP falls
Sudden-stop 1998/1999

- sudden-stop is triggering first a loss of confidence in domestic currency and then a bank run (left figure)
- banks are liquidating their buffer foreign reserves to mitigate bank run (right figure)
- exchange rate kuna/euro depreciates (right figure)
optimal foreign reserves - depend on assumed mother banks’ behavior

standard indicators of reserve adequacy do not take into account alternative sources of funding nor the parameters reflecting common features of sudden-stops with banking crisis (output loss, bank run)
Optimization problem

The government chooses the level of consumption in order to maximize expected discounted welfare of the economy:

$$\max_{c_t, R_t} \beta E_t(c_{t+1}) = \max_{c_t, R_t} \beta \left[ (1 - \pi) u(c_{t+1}^b) + \pi u(c_{t+1}^d) \right]$$

subject to consolidated budget constraint:

before crisis

$$P_t c_t^b + S_t (1 + r) b_{t-1} + S_t FRB^h_t = P_t y_t + S_t b_t + S_t (1 + r) FRB^h_{t-1} + S_t FB_t +$$
$$+ S_t (1 + r) FRB^b_{t-1} - S_t (1 + r) FB_{t-1} - S_t FRB^b_t - S_t (1 + r) FG_{t-1} + S_t FG_t -$$
$$- S_t (\delta + \pi) R_{t-1}$$

during crisis

$$P_t c_t^d + (S_t + \Delta S)(1 + r) b_{t-1} = (1 - \gamma) P_t y_t + (S_t + \Delta S)(1 + r) FRB^h_{t-1} +$$
$$+(S_t + \Delta S)(1 + r) FRB^b_{t-1} - (S_t + \Delta S) \phi(d_t^{kh} + \frac{\eta}{S_t + \Delta S} d_t^{kh}) -$$
$$-(S_t + \Delta S)(1 + r) FB_{t-1} - (S_t + \Delta S)(1 + r) FG_{t-1} +$$
$$+(S_t + \Delta S)(1 - \delta - \pi) R_{t-1}$$

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Endogenous probability

- Probability of crisis is endogenous $\pi_t = \pi(R_t)$, $\pi' < 0$
  - Government chooses reserves level which maximizes previous problem
  - Optimal reserves level depends on the probability of the crisis
  - Choosing certain level of reserves the government influences the probability of a crisis

- No analytical solution
- Endogenous probability model is solved numerically

$$V(R_t) = \max_{R_t} \left[ (1 - \pi(R_t))u^{bs}(R_t) + \pi(R_t)u^{ds}(R_t) \right]$$

Subject to (consolidated) budget constraint where

Welfare of the economy if no crisis occurs

$$u^{bs}(R_t) = u\left(c_t^b\right) + \beta V(R_t^*)$$

Welfare of the economy if crisis occurs

$$u^{ds}(R_t) = u\left(c_t^d\right) + \beta u^{bs}(R_t^*)$$
Optimal and actual foreign reserves

- probability of crisis - exogenous/endogenous
- differentiation between mother banks’ behavior
  - Lenders of Last Resort (left figure)
  - Take the Money and Run (right figure)
precautionary motives behind reserves accumulation since 2004.
optimaly of foreign reserves depends on assumed behavior of mother banks during crisis
standard indicators of reserve adequacy do not take into account many features of sudden-stop/banking crisis episode - might be misleading
CNB’s foreign reserves are (nearly) sufficient to fight the crisis of 1998/1999 magnitude if mother banks act as lenders of last resort