Analysis of the EUR/HRK exchange rate and pricing options on the Croatian market: the NGARCH model as the alternative to the Black-Scholes model

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Outline

1. Croatian Market of Derivatives: a Challenge of forming one
   - An explosive increase in trading on the Croatian market
2. Non-linear in mean asymmetric GARCH model
   - The NGARCH model for the EUR/HRK time series
   - Risk management
3. GARCH option pricing model
   - The European call option on the foreign currency
   - Analysis of foreign currency options

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Trading on the Croatian market

- The interest of professional investors for financial derivatives on the Croatian market is steadily increasing
- The Croatian market of derivatives exists, but trading is of over-the-counter type only
- trading with nonlinear derivatives-options-does not exist *yet*
- trading is expected to start after the establishment of the legal framework

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Analysis of foreign exchange rates

- The interest of market participants has been always attracted by interventions of the Croatian national bank
- Croatia faces a possibility of changing the domestic currency
- We are motivated thus in exploring different kinds of foreign currency options
- We analyze in details the empirical distribution of the EUR/HRK currency time series and the *consequences* of such analysis for pricing foreign currency

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The model of Black and Scholes

Connection to the Black-Scholes model

- Prices on option markets are commonly quoted in terms of Black-Scholes implied volatility
- This *does not* mean that market participants believe in the hypothesis of the Black-Scholes model
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The goal of this study is:

- to estimate the parameters in the dynamics of the currency time series using the NGARCH model,
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Modelling the EUR/HRK time series

- P_t ...the EUR/HRK exchange rate price at time t, defined as the number of Croatian kunas required to purchase 1 euro
- The dynamics of returns R_t is described with

a non linear in mean, asymmetric GARCH (1,1) model:

$$R_{t+1} \equiv \ln\left(\frac{P_{t+1}}{P_t}\right) = r_d - r_f + \lambda\sigma_{t+1} - \frac{1}{2}\sigma_{t+1}^2 + \sigma_{t+1}Z_{t+1}, \quad (1)$$

$$\sigma_{t+1}^2 = \omega + \alpha(\sigma_t Z_t - \rho\sigma_t)^2 + \beta\sigma_t^2, \quad (2)$$

where Z_t are i.i.d. $N(0,1)$ and

$$\omega > 0, \quad \alpha \geq 0, \quad \beta \geq 0 \quad \text{and} \quad \alpha(1+\rho^2) + \beta < 1. \quad (3)$$

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Interpretation of parameters

- r_d and r_f denote the one-period continuously compounded domestic and foreign interest rate
- λ is the unit risk premium for the exchange rate
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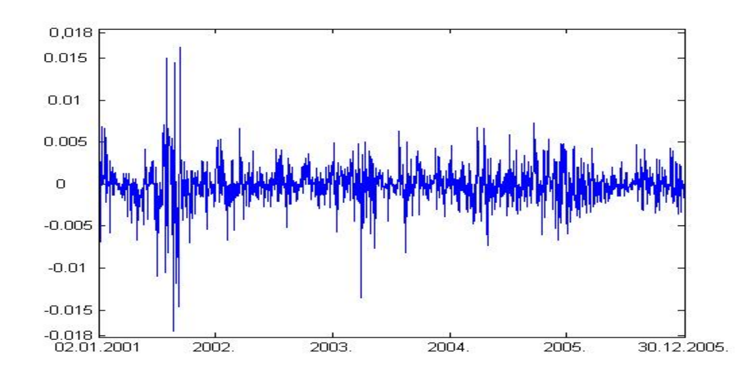
EUR/HRK 2001-2005



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$$L_T = \frac{1}{T}\sum_{t=1}^{T}\left[-\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{\left(P_t - (r_d - r_f + \lambda\sigma_t - \frac{1}{2}\sigma_t^2)\right)^2}{\sigma_t^2}\right], \quad (4)$$

where $T = 1297$.

Maximizing the L_T function we obtain

Parameter	Value	Sample standard error
$\hat{\omega}$	$1.7339 \cdot 10^{-7}$	$2.92 \cdot 10^{-8}$
$\hat{\lambda}$	-0.0301153	0.11311
$\hat{\alpha}$	0.095345	0.012028
$\hat{\beta}$	0.86840994	0.014289
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- A key advantage of GARCH models for risk management is that the one-day ahead forecast of variance, $E_t[\sigma^2_{t+1}]$, is given directly by the model as σ^2_{t+1}
- The imposed conditions on parameters enable to define the unconditional variance as

$$\sigma^2 \equiv E[\sigma^2_{t+1}] = \frac{\omega}{1 - \alpha(1+\rho^2) - \beta}$$

- Forecasting the variance of the daily return k days ahead, using only information available at the end of today:

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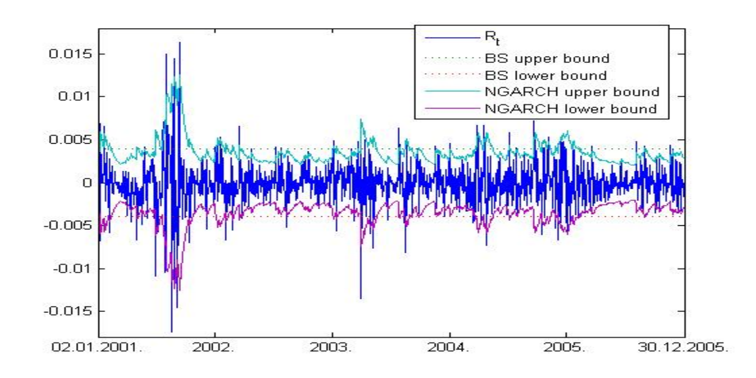
The NGARCH model gives a more accurate estimation of risk since it incorporates the *heteroscedasticity*.



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We use the GARCH option pricing model of Duan (1995).

Days to maturity are counted in calendar days (365) not in business days per year (256)

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K . . . strike
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The equilibrium price measure satisfies the local risk neutral valuation relationship (LRNVR) if every asset value X_t measured in domestic currency satisfies

1. X_{t+1}/X_t is conditionally log-normal distributed w.r.t the equilibrium measure *
2. $$E_t^*\left[X_{t+1}/X_t\right] = e^{r_d}, \quad (5)$$
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where $Z_{t+1}^* = Z_{t+1} + \lambda \sim N(0,1)$, i.e. Z_t^* are i.i.d. with respect to the measure *, satisfies properties 1, 2 and 3.

Relation (7) enables pricing foreign currency options!

We have even more: from relation (8) it follows that the risk premium λ has global influence on the conditional variance even the risk was locally neutralized with respect to *.

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The price of the European call option on foreign currency

Definition

The *fair* price of the European call option, in the risk neutral world, in time t with strike K and maturity date $\tau + t$, $\tau > 0$ is given by

$$co_t = \exp(-r_d \tau) E_t^* \left[\max(P_{t+\tau} - K, 0) \right]. \tag{9}$$

$P_{t+\tau}$ is not known explicitly in analytical form!

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We use Monte Carlo simulations

$$c^{GH} \approx \exp(-r_d\tau)\frac{1}{MC}\sum_{i=1}^{MC}\max\left\{P_{i,t+\tau}-K,0\right\}, \tag{10}$$

where $MC = 50000$, and for the $i-th$ simulation we have

$$P_{i,t+\tau} = P_t \exp\left(\sum_{j=1}^{\tau} R_{i,t+j}\right), \qquad i = 1, 2, \ldots, MC. \tag{11}$$

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Illustration: a simulation study

Option prices are calculated for:

- different days to maturity: $\tau = 30, 60$ and 90 days
- different moneyness $m = 0.97, 0.985, 1, 1.015$ and 1.03 which corresponds respectively to strikes $K = 7.11495, 7.224975, 7.335, 7.445$ and 7.555 for **currency spot price** $P_t = 7.335$.
- the obtained prices are then compared to their Black-Scholes counterparts with the average annual volatility 0.036128
- in order to analyze the effect of the asymmetry parameter ρ, we repeat the simulation procedure for $\rho = -0.461$ and $\rho = 0$

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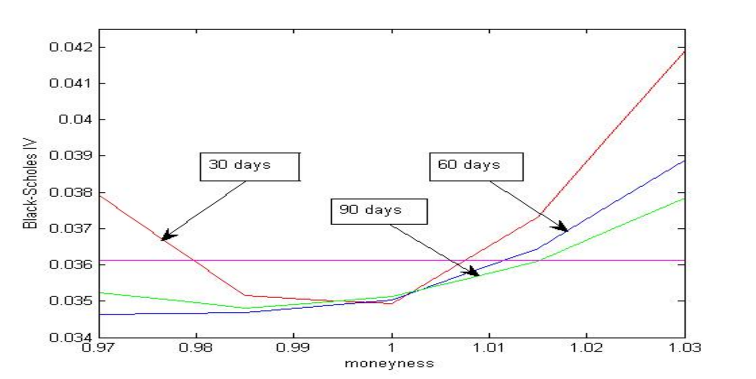
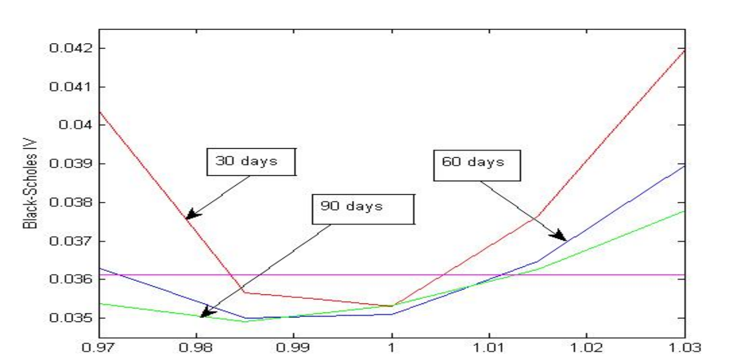
The comparison: $\rho = -0.17074$

Mild negative asymmetry $\Longrightarrow$ options out of the money ($K/P > 1$) are underpriced in the Black-Scholes model with constant volatility



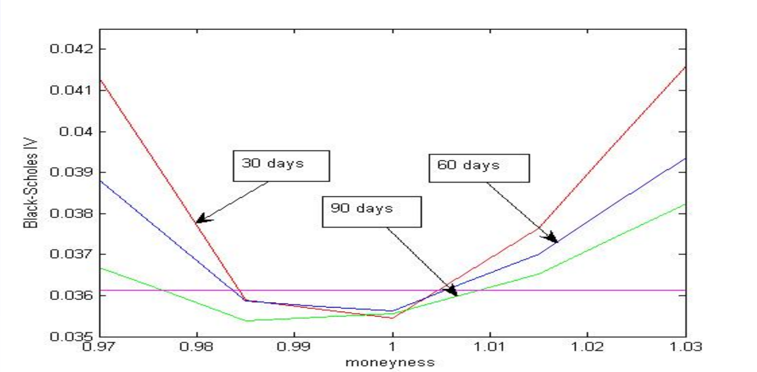
The comparison: $\rho = -0.461$

- Moderate asymmetry $\Longrightarrow$ options out of the money ($K/P > 1$) are underpriced in the Black-Scholes model with constant volatility and also some options ($\tau = 30$) in the money
- the underpricing effect in the constant volatility model is now more pronounced for options in the money



The comparison: $\rho = 0$

- the asymmetry is completely absent $\Longrightarrow$the graph of implied volatility becomes almost symmetric (centered in $K/P = 1$)
- options deeply in the money and deeply out of the money are underpriced in the constant volatility model



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- for options out of the money, independently of ρ, the IV is a **decreasing** function of the maturity
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Summary

- introducing heteroscedasticity results in better fitting of the empirical distribution of foreign currency
- the locally risk-neutral measure for the domestic economy is identified
- the GARCH model for option pricing in its formulation incorporates the risk premium
- Still, it is not a priori obvious what should be the risk premium for the volatility–using the time series data from the underlying we find it not significant
- once the market for derivatives will be formed, data from option markets!

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