Modelling electricity day–ahead prices by multivariate Lévy semistationary processes

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The problem

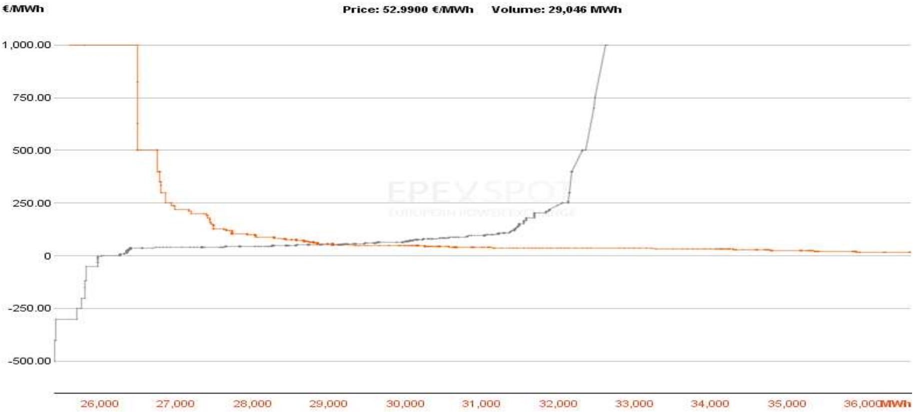
- How can one model electricity day–ahead (“spot”) prices?
- What is special about electricity (prices)?
    - Electricity is difficult to store.
    - Different sources for electricity generation: coal, nuclear, natural gas, hydroelectric, petroleum, solar, wind etc.

  Price impact!
- Why is it important?
    - Forward contracts, futures and option prices.
    - Risk management.

The structure of the market

- Focus on the European Energy Exchange (EEX) market.
- Two types of trading activities: Auctions and continuous trading. Focus on auction.
- Day–ahead prices determined by a daily auction at 12:00 noon, 7 days a week all year.
- Underlying quantity to be traded is the electricity for delivery the following day in 24 hour intervals.
- Two types of orders: Orders for individual hours and block orders.

Aggregated supply & demand curve: 1st March 2012, Hour 10-11, Phelix



Grey curve: Volume Sale, Orange curve: Volume Purchase

Stylized facts of electricity spot prices

- Equilibrium prices: Supply and demand determine the spot price (results in some form of mean–reversion)
- Non–Gaussian returns
- (Semi-) heavy–tailed distributions
- Strong seasonality (over short and long time horizons)
- Extreme spikes
- Negative spot prices: Permitted in EEX spot auctions since September 2008. First occurrence: October 2008.

Main features of the new modelling framework

- Panel approach: Model hourly time series as vector of daily observations.
- Continuous–time set–up.
- Model in stationarity (equilibrium prices).
- Flexible and analytically tractable.
- Arithmetic model (negative prices!)

Multivariate Lévy semistationary ($\mathcal{MLSS}$) processes

$\mathcal{MLSS}$ process $\mathbf{Y} = \{\mathbf{Y}(t)\}_{t \in \mathbb{R}}$ on $\mathbb{R}^m$, $m \in N$

$$\mathbf{Y}(t) = \int_{-\infty}^{t} \mathbf{g}(t-s)\boldsymbol{\sigma}(s-)d\mathbf{L}(s),$$

- $\mathbf{L}$ two-sided d–dimensional Lévy process.
- $\mathbf{g} = (g_{ij}) : \mathbb{R} \to \mathbb{R}^{m \times \delta}$ deterministic, nonnegative kernel function with $\mathbf{g}(s) = \mathbf{0}\ \forall s < 0$,
- $\boldsymbol{\sigma} = \left(\sigma_{ij}\right)$ $\delta \times d$–dim., càdlàg, adapted stochastic volatility matrix.
- Assume independence of $\boldsymbol{\sigma}$ and $\mathbf{L}$.

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Assumptions

- Some regularity assumptions needed to guarantee that integral is well defined.
- $\exists$ sufficient conditions such that $\mathbf{Y}$ semimartingale.

Model specification for EEX market

Seasonality and trend

- $\mathbf{D} : \mathbb{R} \to \mathbb{R}^{24}$ deterministic seasonality and trend function.
- $\mathbf{Y}$ $\mathcal{MLSS}$ process.
- Daily observations of the 24 hourly electricity prices in the EEX market modelled by arithmetic model

$$\mathbf{S}(t) = \mathbf{D}(t) + \mathbf{Y}(t).$$

Spike and base component

Assumption:

$$\mathbf{Y}(t) = \underbrace{\boldsymbol{Z}(t)}_{\text{base component}} + \underbrace{\xi(t)}_{\text{spike component}} .$$

The spike component

- The spike component is sum of a two stochastic processes

$$\xi_i(t) := \xi_i^{\mathrm{up}}(t) + \xi_i^{\mathrm{down}}(t).$$

  - ξ^{up} can only jump upwards and then decreases exponentially until next jump.
  - ξ^{down} can only jump downwards and then increases exponentially until next jump.
- For $t \geq 0$

$$\xi^{\mathrm{up}}(t)_i := \int_{-\infty}^{t} e^{-\eta_i^{\mathrm{up}}(t-s)} dL_i^{\mathrm{up}}(s),$$

$$\xi^{\mathrm{down}}(t)_i := \int_{-\infty}^{t} e^{-\eta_i^{\mathrm{down}}(t-s)}(-1) dL_i^{\mathrm{down}}(s),$$

$\eta_i^{\mathrm{up}}, \eta_i^{\mathrm{down}} \geq 0$, $\mathbf{L}^{\mathrm{up}} = (L_1^{\mathrm{up}}, \ldots, L_{24}^{\mathrm{up}})$ and $\mathbf{L}^{\mathrm{down}} = (L_1^{\mathrm{down}}, \ldots, L_{24}^{\mathrm{down}})$ independent pure jump Lévy subordinators.

The base component

- Each base component Z_i is a univariate continuous–time autoregressive moving average (CARMA) process:

$$Z_i(t) := \int_{-\infty}^{t} \tilde{g}_i(t-s) d\widetilde{L}_i(s),$$

  where
  - $\tilde{g}_i$ univariate CARMA kernel, $i = \{1, \ldots, 24\}$,
  - $\widetilde{\mathbf{L}} = (\widetilde{L}_1, \ldots, \widetilde{L}_{24})$ two–sided Lévy process.
- For now, we do not allow for stochastic volatility here.

CARMA(p_i, q_i) process

Let $p_i > q_i$. Consider CARMA(p_i, q_i) process Z_i:

$$Z_i(t) = \mathbf{b}_i^\top \mathbf{V}_i(t)\,,$$

where $\mathbf{V}_i(t)$ is a p_i–dimensional Ornstein–Uhlenbeck

$$d\mathbf{V}_i(t) = \mathbf{A}_i\mathbf{V}_i(t)dt + \zeta d\widetilde{L}_i(t), \qquad (1)$$

where $p_i \times p_i$–matrix $\mathbf{A}_i$ and p_i–dimensional vectors $\mathbf{b}_i$ and ζ are

$$\mathbf{A}_i := \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ -a_i^{(p_i)} & -a_i^{(p_i-1)} & \cdots & \cdots & -a_i^{(1)} \end{pmatrix} \zeta := \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \mathbf{b}_i := \begin{pmatrix} b_i^{(0)} \\ b_i^{(1)} \\ \vdots \\ b_i^{(p_i-1)} \end{pmatrix}$$

Note that $b_{q_i} = 1$ and $b_j = 0$ for all $q_i < j < p_i$.

CARMA(p_i, q_i) process

- If all eigenvalues of $\mathbf{A}_i$ have negative real parts, then $\mathbf{V}_i(t)$ defined as

$$\mathbf{V}_i(t) = \int_{-\infty}^{t} e^{\mathbf{A}_i(t-s)} \zeta \, d\widetilde{L}_i(s) \, ,$$

  is the (strictly) stationary solution of (1).
- Moreover,

$$Z_i(t) = \mathbf{b}_i^\top V_i(t) = \int_{-\infty}^{t} \mathbf{b}_i^\top e^{\mathbf{A}_i(t-s)} \zeta \, d\widetilde{L}_i(s) \, ,$$

  is a CARMA(p_i, q_i) process.
- CARMA process can be derived from a $\mathcal{LSS}$ process by choosing

$$\tilde{g}_i(t-s) = \mathbf{b}_i^\top e^{\mathbf{A}_i(t-s)} \zeta.$$

Splitting the data into spikes and base component

- Klüppelberg et al. (2010) proposed method to split data in spike (upwards spikes only) and base components.
- Extended their method to split into upwards and downwards spikes and base component.
- Used tools from extreme value statistics to determine an upper and a lower threshold.
  If price is above upper threshold or below lower threshold it is considered to be a spike.
- Generalized Pareto distribution for spike jump distribution.

Estimating the kernel function

- Let V_i be a CARMA(p_i, q_i) process.
- Sample V_i only at time points nh where $h > 0$, $n \in \mathbf{N}$.
- Then $(V_i(nh))_{n \in \mathbf{N}}$ is a weak ARMA$(p_i, p_i - 1)$ process.
- "weak" means that noise is not necessarily i.i.d..
- Can transform the parameters from ARMA(2,1) to CARMA(2,1) and vice versa. Useful for estimation!
- Fit ARMA(2, 1) model parameters and compute the corresponding parameters for the CARMA(2, 1) kernel function.

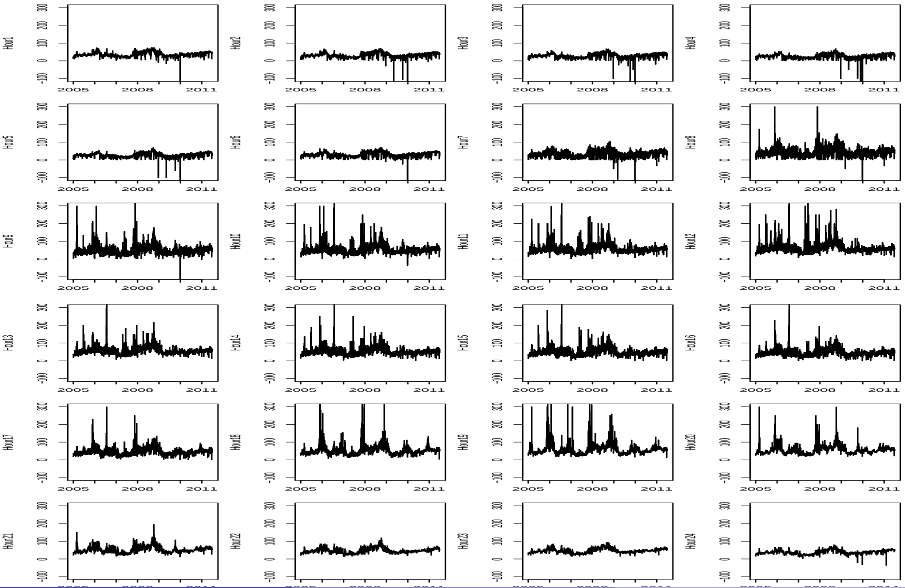
Recovering the Lévy increments

- Brockwell et al. (2011) and Brockwell and Schlemm (2011) proposed method for recovering the increments of the driving Levy process of a CARMA(p, q) process.
- Method is based on state space representation of CARMA process and initially uses continuous observations.
- Results for discrete time observations can be derived from there.

The data

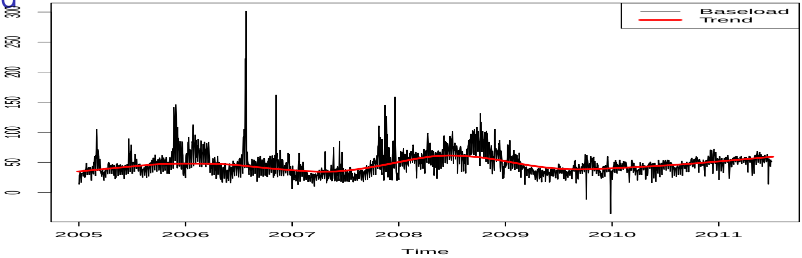
- EEX data: Daily day–ahead prices for 24 hours.
- Data from 01/01/2005 to 30/06/2011 (2372 daily data of the 24–dimensional vector).
- Analysis of the whole data set including weekends.
- Use the $\mathcal{MLSS}$ processes and fit them to deseasonalised and detrended data.
- Particular focus on the cross–correlation structure of the daily observations of the prices for each hour.

Plot of daily prices for each hour



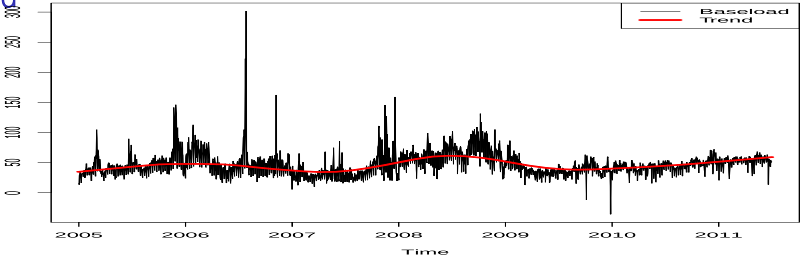
Trend and seasonalities

Trend



Trend and seasonalities

Trend



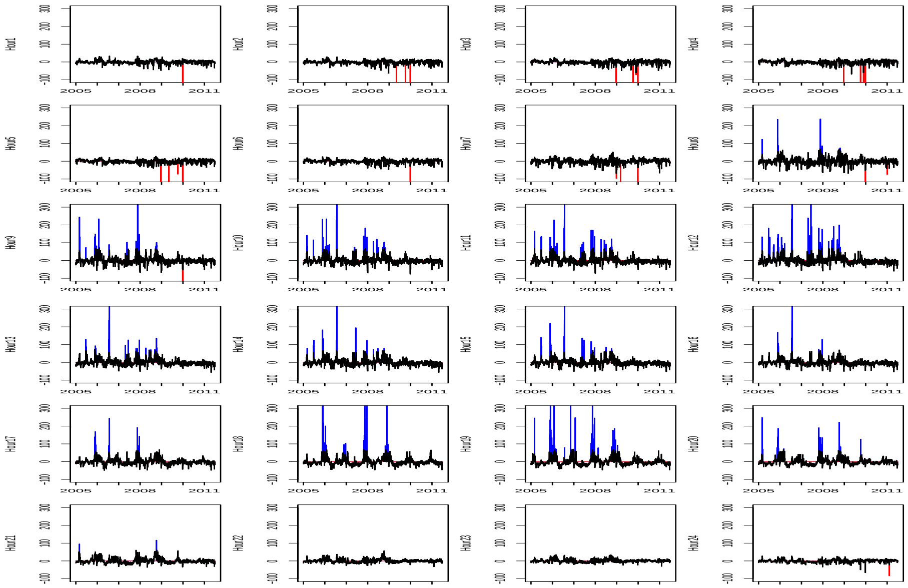
Seasonalities

Computed trimmed means (removing 5 % of data) of detrended data.

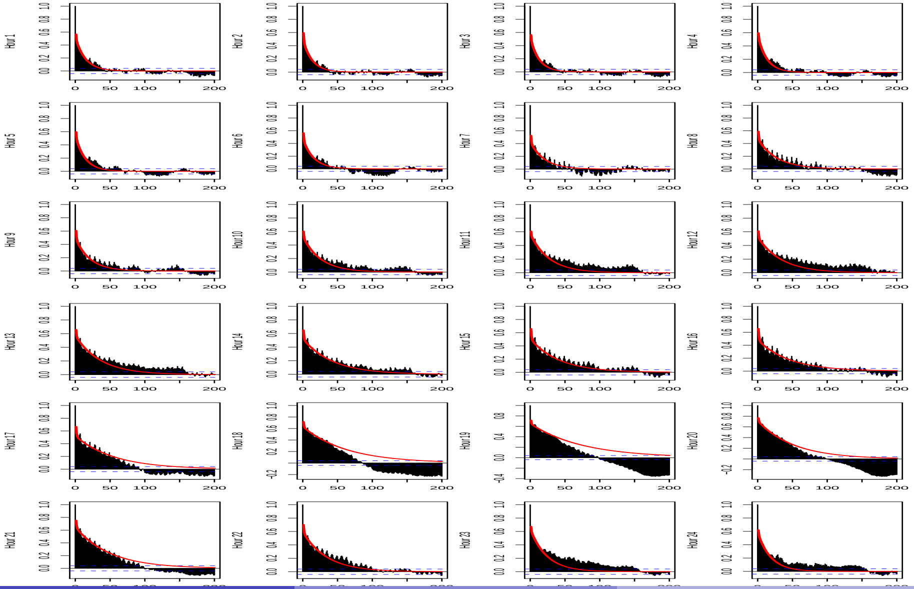
$$\mathbf{D}(t)_i - f(t) = \sum_{\text{weekday}=1}^{7} b_i^{\text{weekday}} \mathbb{I}_{\text{weekday}}(t),$$

$i \in \{1, \ldots, 24\}$. b_i^{weekday} trimmed mean for particular weekday.

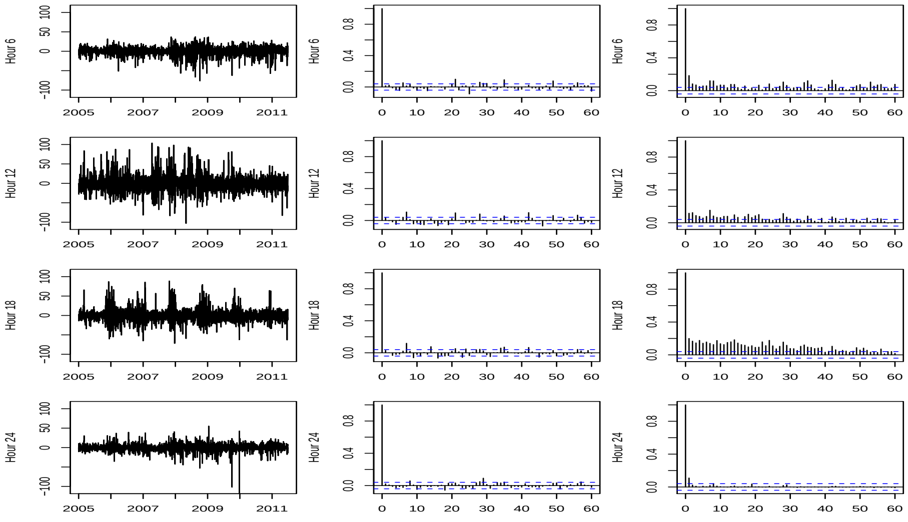
Detrended & deseas. data split into spikes & base



Empirical and estimated CARMA(2,1) ACF



Recovered increments of Lévy process driving CARMA(2,1)



Distributional properties of recovered Lévy increments

- Random vector **X** has m–dimensional multivariate generalized hyperbolic (GH) distribution $\mathbf{X} \sim GH_m(\lambda, \chi, \psi, \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{\gamma})$, if it is given by

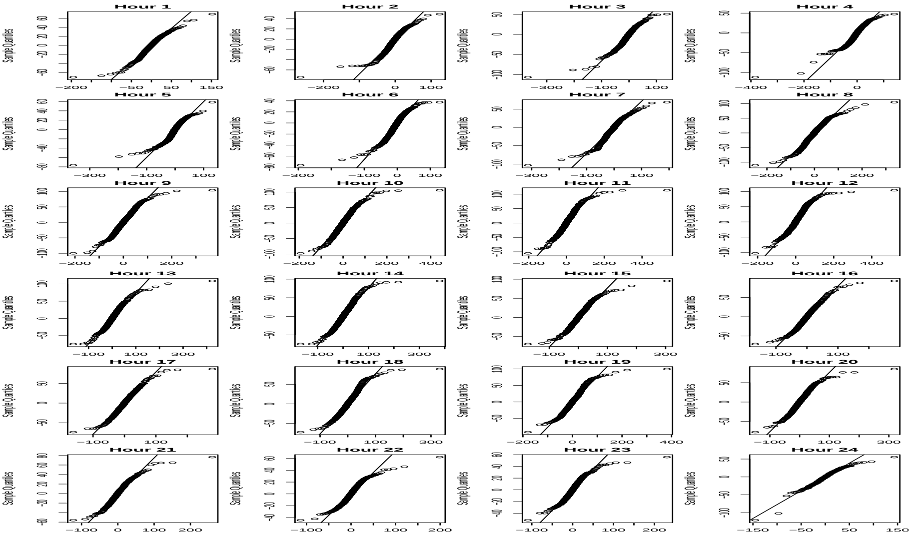
$$\mathbf{X} \overset{d}{=} \boldsymbol{\mu} + \Xi\boldsymbol{\gamma} + \sqrt{\Xi}\mathbf{C}\boldsymbol{\Psi},$$

  where
  - $\boldsymbol{\Psi} \sim N_k(0, \mathbf{I}_k)$ for $k \in N$,
  - $\mathbf{C} \in \mathbb{R}^{m \times k}$, $\boldsymbol{\mu}, \boldsymbol{\gamma} \in \mathbb{R}^m$,
  - $\sqrt{\Xi}$ 1-dim r.v. with generalized inverse Gaussian distribution $GIG(\lambda, \chi, \psi)$; independent of $\boldsymbol{\Psi}$.
- $\boldsymbol{\mu}$ location parameter, $\boldsymbol{\Sigma} = \mathbf{C}\mathbf{C}^\top$ dispersion matrix, $\boldsymbol{\gamma}$ skewness parameter (if $\boldsymbol{\gamma} = \mathbf{0}$, then symmetric distribution around $\boldsymbol{\mu}$).
- Class of GH distribution contains: Student-t distribution, the normal inverse Gaussian distribution (NIG), the hyperbolic distribution (HYP) and the variance gamma (VG) distribution.
- If $\mathbf{X} \sim GH_m(\lambda, \chi, \psi, \boldsymbol{\mu}, \boldsymbol{\Sigma}, \boldsymbol{\gamma})$,then $X_i \sim GH_1(\lambda, \chi, \psi, \boldsymbol{\mu}_i, \boldsymbol{\Sigma}_{ii}, \boldsymbol{\gamma}_i)$.

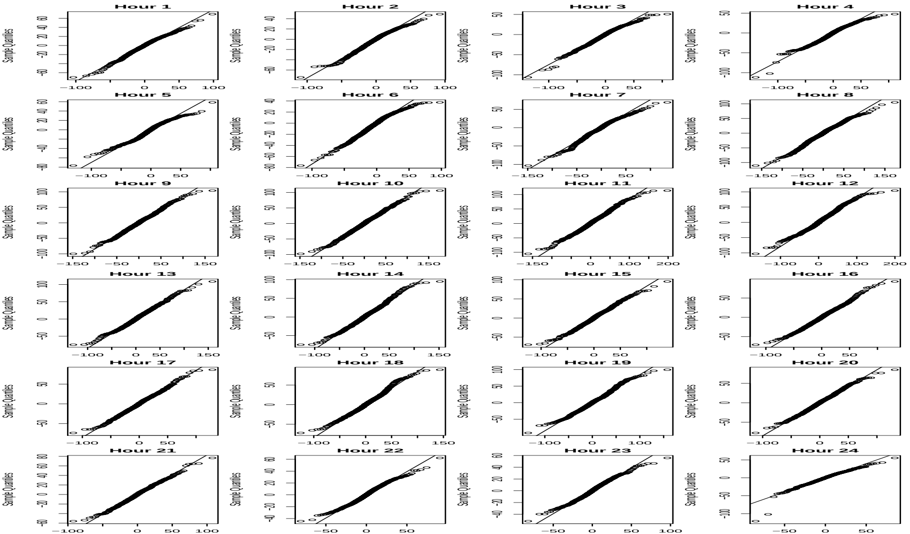
Model selection within the class of GH distributions using AIC

Model	Symmetric	$\widehat{\lambda}$	$\widehat{\overline{\alpha}}$	AIC	Log-Likel.	Converged
Student–t	FALSE	-1.381	0	378150.712	-188726.356	TRUE
GH	FALSE	-1.334	0.121	378151.520	-188725.760	TRUE
Student–t	TRUE	-1.380	0	378173.594	-188761.797	TRUE
GH	TRUE	-1.338	0.112	378174.655	-188761.327	TRUE
NIG	FALSE	-0.5	0.465	378352.788	-188827.394	TRUE
NIG	TRUE	-0.5	0.459	378383.696	-188866.848	TRUE
VG	TRUE	0.913	0	378853.575	-189101.787	TRUE
VG	FALSE	0.913	0	378899.689	-189100.844	TRUE
HYP	FALSE	12.5	0.000	390089.630	-194695.815	TRUE
HYP	TRUE	12.5	0.000	390197.786	-194773.893	TRUE
Gaussian	TRUE	NA	Inf	408684.766	-204018.383	TRUE

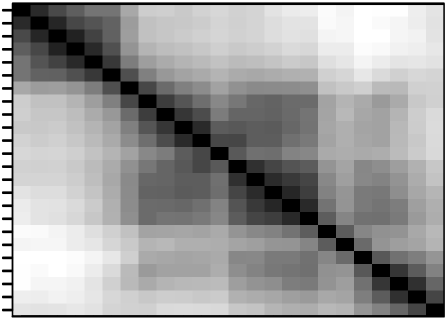
QQ-plots for components of fitted multivariate asymmetric Student–*t* distribution



QQ-plots for components of fitted multivariate asymmetric NIG distribution



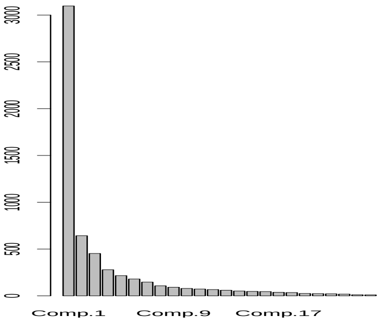
Cross-correlation of 24 Lévy processes (increments)

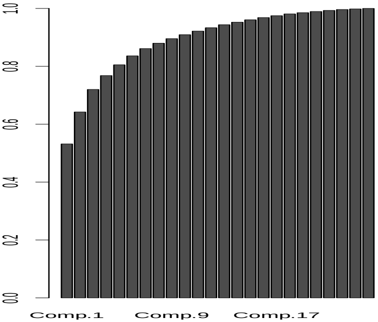


- Cross correlation structure of recovered increments of the driving process of the CARMA(2,1) process.
- Each square corresponds to an element in the sample cross–correlation matrix.
- Increasing shading intensity reflects stronger correlation (correlation 1 = black; correlation 0 = white).

Can we reduce the dimension of the model?

- Principal components analysis: The figure shows the individual variances explained by each component and also the cumulative explained variance depending on the number of components.
- 14 (!) components ensure that the cumulative proportion of the variance is greater than 95%.





Contributions

- Propose a continuous–time panel–framework to model day–ahead electricity prices.
- Main building block: multivariate Lévy semistationary processes .
- Derived integrability, semimartingale conditions, cumulant function, second order structure etc.
- New modelling framework accounts for mean–reversion/ stationarity, spikes, stochastic volatility, long memory, negative prices, cross correlations etc.
- Good empirical results for multivariate CARMA(2,1) process driven by generalized hyperbolic Lévy process.

Outlook

- General $\mathcal{MLSS}$ modelling framework allows for stochastic volatility.
- We found empirical evidence for stochastic volatility in peak hours.
- Estimation theory for this general model class (including stochastic volatility) not yet available.

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CARMA(2, 1) process

- If V_i is CARMA(2, 1) process, it has representation

$$V_i(t) = \int_{-\infty}^{t} \alpha_i^{(1)} e^{\lambda_i^{(1)}(t-s)} d\widetilde{L}_i(s) + \int_{-\infty}^{t} \alpha_i^{(2)} e^{\lambda_i^{(2)}(t-s)} d\widetilde{L}_i(s),$$

$$\alpha_i^{(1)} = \frac{b_i^0 + \lambda_i^{(1)}}{\lambda_i^{(1)} - \lambda_i^{(2)}}, \quad \alpha_i^{(2)} = \frac{b_i^{(0)} + \lambda_i^{(2)}}{\lambda_i^{(2)} - \lambda_i^{(1)}}.$$

- Hence, kernel function is

$$\tilde{g}_i(h) = \left(\alpha_i^{(1)} e^{\lambda^{(1)} h} + \alpha_i^{(2)} e^{\lambda_i^{(2)} h}\right) 1_{[0,\infty)}(h).$$

- $a(z) := z^2 + a_i^{(1)} z + a_i^{(2)} = (z - \lambda_i^{(1)})(z - \lambda_i^{(2)})$.
  $\lambda_i^{(1)}, \lambda_i^{(2)}$ are the eigenvalues of $\mathbf{A}_i$.

Splitting the data into spikes and base component I

- Generalized method by Klüppelberg et al. (2010) to split data in spike (both upwards and downwards) and base components $\forall i \in \{1, \ldots, 24\}$.
- $Y_i(nh)$ is nth observation over a period of length h of a price for hour i after trend and seasonalities have been removed.

1. We consider an autoregressive transformation for known η_i^{up}, see Klüppelberg et al. (2010) (p. 969)

$$
\begin{aligned}
Y_i^{AR}(h) &:= Y_i(h), \\
Y_i^{AR}(nh) &:= Y_i(nh) - e^{-\eta_i^{\mathrm{up}} h} Y_i((n-1)h), \quad n = 2, \ldots, N.
\end{aligned}
$$

Splitting the data into spikes and base component II

2. We then consider the exceedances $(Y_i^{AR}(nh) - u_i)\mathbb{I}_{\{Y_i^{AR}(nh)>u_i\}}$ and determine the threshold $u_i > 0$ such that a (shifted) Generalized Pareto Distribution can be used to model the exceedances, see Klüppelberg et al. (2010) (p. 966) for details.
3. Let $\mathcal{J}_i := \{n \in \{1, \ldots, N\} \big| Y_i(nh) > u_i\}$. Then we estimate η_i^{up} by an estimator of Davis–McCormick–type, see Davis and McCormick (1989) :

$$\widehat{\eta^{\mathrm{up}}}_i = \frac{1}{h} \ln \left(\max_{n-1 \in \mathcal{J}_i} \frac{Y_i((n-1)h)}{Y_i(nh)} \right).$$

Splitting the data into spikes and base component III

4. The spike jumps are estimated as in Klüppelberg et al. (2010) (p. 969) by

$$\widehat{\epsilon}_i(nh) = \left(Y_i^{AR}(nh) - (1 - e^{-\widehat{\eta^{\mathrm{up}}}_i h} \mathcal{S}_i) \right) \mathbb{I}_{\{Y_i^{AR}(nh) > u_i\}},$$

where $\mathcal{S}_i$ depends on the estimate $\widehat{\eta^{\mathrm{up}}}_i$. In our data, we obtain estimates which suggest that the spike impact either vanishes essentially within one day in which case we use

$$\mathcal{S}_i = \frac{1}{\left|\{n \in \{1, \ldots, N\} \middle| Y_i^{AR}(nh) \leq u_i\}\right|} \sum_{n=1}^{N} Y_i(nh) \mathbb{I}_{\{Y_i^{AR}(nh) \leq u_i\}},$$

Splitting the data into spikes and base component IV

otherwise the spike impact in our data vanishes after essentially two days and then we use

$$\mathcal{S}_i = \frac{1}{\left|\{n \in \{1,\ldots,N\} \,\middle|\, Y_i^{AR}(nh) \leq u_i \text{ and } Y_i^{AR}((n-1)h) \leq u_i\}\right|} \cdot \sum_{n=2}^{N} Y_i(nh)\mathbb{I}_{\{Y_i^{AR}(nh)\leq u_i \text{ and } Y_i^{AR}((n-1)h)\leq u_i\}}.$$

5. Then the upwards spikes are recovered by setting

$$\xi_i^{\text{up}}(h) = \widehat{\epsilon}_i(h),$$
$$\xi_i^{\text{up}}(nh) = e^{-\widehat{\eta^{\text{up}}}_i h}\xi_i^{\text{up}}((n-1)h) + \widehat{\epsilon}_i(nh), \quad n \in \{2,\ldots,N\},$$

and the remainder is

$$Y_i^{\text{REM}}(nh) := Y_i(nh) - \xi_i^{\text{up}}(nh), \quad n \in \{1,\ldots,N\}.$$

Splitting the data into spikes and base component V

6. We then set $Y_i(nh) := -Y_i^{\mathrm{REM}}(nh)$ for all $n \in \{1, \ldots, N\}$ and go back to 1.) and repeat the analysis. Then, the downwards spikes are just (-1) times the new upwards spikes computed in 5.) and the base component Z_i is equal to (-1) times the remainder computed in 5.).