Capital requirements, market, credit, and liquidity risk

Ernst Eberlein

Department of Mathematical Stochastics
and
Center for Data Analysis and Modeling (FDM)
University of Freiburg

Joint work with Dilip Madan and Wim Schoutens.

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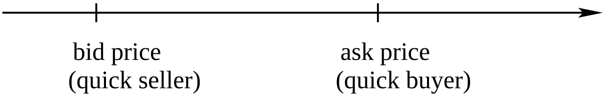
Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Law of one Price

In complete markets and for liquid assets

$$E^Q[X]$$

Reality however is incomplete: no perfect hedges



Acceptability of Cashflows

X random variable: outcome (cashflow) of a risky position

In complete markets: unique pricing kernel given by a probability measure Q

value of the position:	$E^Q[X]$
position is acceptable if:	$E^Q[X] \geq 0$
company's objective is:	maximize $E^Q[X]$

Real markets: incomplete

Instead of a unique probability measure Q we have to consider a set of probability measures $Q \in \mathcal{M}$

$$E^Q[X] \geq 0 \quad \text{for all } Q \in \mathcal{M} \qquad \text{or} \qquad \inf_{Q \in \mathcal{M}} E^Q[X] \geq 0$$

Coherent Risk Measures

Specification of $\mathcal{M}$ (test measures, generalized scenarios)

Axiomatic theory of risk measures: desirable properties

Monotonicity: $X \geq Y \Longrightarrow \varrho(X) \leq \varrho(Y)$

Cash invariance: $\varrho(X + c) = \varrho(X) - c$

Scale invariance: $\varrho(\lambda X) = \lambda \varrho(X), \quad \lambda \geq 0$

Subadditivity: $\varrho(X + Y) \leq \varrho(X) + \varrho(Y)$

Examples: Value at Risk (VaR)

Tail-VaR (expected shortfall)

General risk measure: $\varrho_m(X) = -\int_0^1 q_u(X) m(du)$

Any coherent risk measure has a representation

$$\varrho(X) = -\inf_{Q \in \mathcal{M}} E^Q[X]$$

Operationalization

Link between acceptability and concave distortions
(Cherny and Madan (2009))

$\rightarrow$ Concave distortions

Assume acceptability is completely defined by the distribution function of the risk

$\Psi(u)$: concave distribution function on $[0,1]$

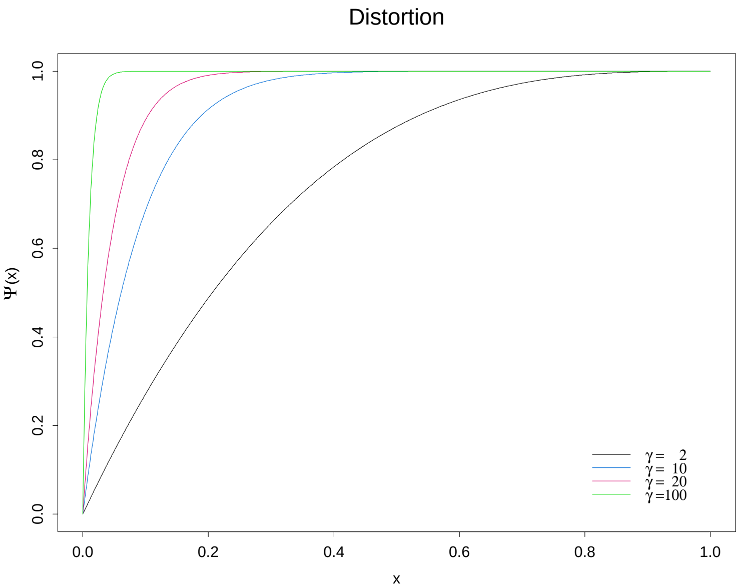
$\Rightarrow$ $\mathcal{M}$ the set of supporting measures is given by all measures Q with density $Z = \frac{dQ}{dP}$ s.t.

$$E^P[(Z-a)^+] \leq \sup_{u \in [0,1]} (\Psi(u) - ua) \quad \text{for all } a \geq 0$$

Acceptability of X with distribution function $F(x)$

$$\int_{-\infty}^{+\infty} x d\Psi(F(x)) \geq 0$$

Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References


Distortion
Ψ(x)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
γ = 2
γ = 10
γ = 20
γ =100

Families of Distortions (1)

Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Consider families of distortions $(\Psi^\gamma)_{\gamma \geq 0}$

γ stress level

Example: MIN VaR

$$\Psi^\gamma(x) = 1 - (1 - x)^{1+\gamma} \qquad (0 \leq x \leq 1, \gamma \geq 0)$$

Statistical interpretation:

Let γ be an integer, then $\varrho_\gamma(X) = -E(Y)$ where

$$Y \overset{\text{law}}{=} \min\{X_1, \ldots, X_{\gamma+1}\}$$

and $X_1, \ldots, X_{\gamma+1}$ are independent draws of X

Families of Distortions (2)

Further examples: MAX VaR

$$\Psi^{\gamma}(x) = x^{\frac{1}{1+\gamma}} \quad (0 \leq x \leq 1, \gamma \geq 0)$$

Statistical interpretation: $\varrho_{\gamma}(X) = -E[Y]$

where Y is a random variable s.t.

$$\max\{Y_1, \dots, Y_{\gamma+1}\} \overset{\text{law}}{=} X$$

and $Y_1, \dots, Y_{\gamma+1}$ are independent draws of Y.

Combining MIN VaR and MAX VaR: MAX MIN VaR

$$\Psi^{\gamma}(x) = (1 - (1-x)^{1+\gamma})^{\frac{1}{1+\gamma}} \qquad (0 \leq x \leq 1, \gamma \geq 0)$$

Interpretation: $\varrho_{\gamma}(X) = -E[Y]$ with Y s.t.

$$\max\{Y_1, \dots, Y_{\gamma+1}\} \overset{\text{law}}{=} \min\{X_1, \dots, X_{\gamma+1}\}$$

Families of Distortions (3)

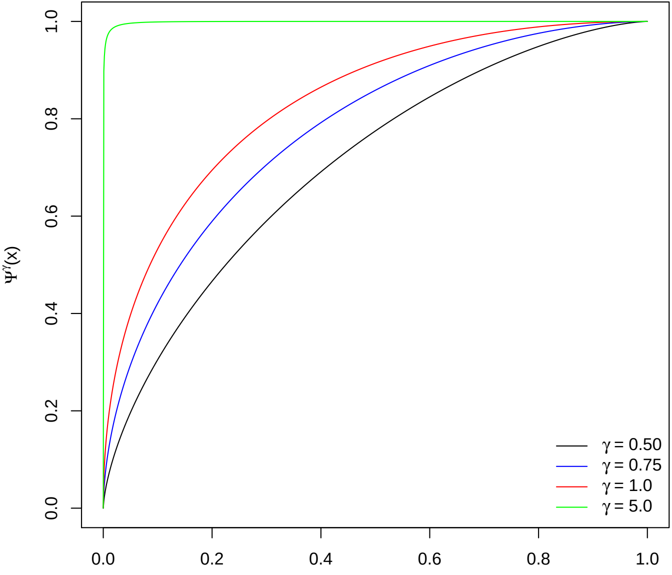
Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Distortion used: MIN MAX VaR

$$\Psi^{\gamma}(x) = 1 - \left(1 - x^{\frac{1}{1+\gamma}}\right)^{1+\gamma} \qquad (0 \leq x \leq 1, \gamma \geq 0)$$

$$\varrho_{\gamma}(X) = -E[Y] \quad \text{with } Y \text{ s.t.} \qquad \begin{aligned} & Y \overset{\text{law}}{=} \min\{Z_1, \ldots, Z_{\gamma+1}\}, \\ & \max\{Z_1, \ldots, Z_{\gamma+1}\} \overset{\text{law}}{=} X \end{aligned}$$

Families of Distortions (4)



Marking Assets and Liabilities

Assets: Cash flow to be received $\widetilde{A} \geq 0$

Largest value A s.t. $\widetilde{A} - A$ is acceptable

$$\Rightarrow \qquad A = \inf_{Q \in \mathcal{M}} E^Q[\widetilde{A}]$$

Bid Price

Liabilities: Cash flow to be paid out $\widetilde{L} \geq 0$

Smallest value L s.t. $L - \widetilde{L}$ is acceptable

$$\Rightarrow \qquad L = \sup_{Q \in \mathcal{M}} E^Q[\widetilde{L}]$$

Ask Price

Two Price Economics

Range of application: Markets which are not perfectly liquid

Bid and ask prices of a two price economy: not to be confused with bid and ask prices of relatively liquid markets like stock markets

Markets for OTC structured products or structured investments

Both parties typically hold a position out to contract maturity

Liquid markets: one price prevails

Nevertheless liquidity providers will need a bid-ask spread

Bid-ask spreads reflect

- the cost of inventory management
- transaction costs (commissions)
- asymmetric information cost, etc.

Directional Prices in a Two Price Economy

The goal is not to get a single risk neutral price which could be interpreted as a midpoint between bid and ask

Instead modeling two separate prices at which transactions occur
$\longrightarrow$ directional prices

Bid price: Minimal conservative valuation s.t. the expected outcome will safely exceed this price

Ask price: Maximal valuation s.t. the expected payout will fall below this price

$\longrightarrow$ specification of the set of valuation possibilities

Directional Prices in a Two Price Economy

Midquote in such two price markets is in general not the risk neutral price (Carr, Madan, Vicente Alvarez (2011))

Midquotes would generate arbitrage opportunities (Madan, Schoutens (2011))

Pricing of liquidity: nonlinear (infimum and supremum of a set of valuations)

Spread: capital reserve

No complete replication: spread is a charge for the need to hold residual risk

Relating Bid and Ask Prices

Consider real-valued cashflows X, e.g. swaps

$$X = X^+ - X^-$$

$\Rightarrow \qquad b(X) = b(X^+) - a(X^-)$

and $\qquad a(X) = a(X^+) - b(X^-)$

Valuation as asset: X^+ is an asset and priced at the bid
X^- is a liability and priced at the ask

Valuation as liability: X^- is an asset and priced at the bid
X^+ is a liability and priced at the ask

Explicit Bid and Ask Pricing

Bid Price of a cash flow X: Acceptability of $X - b(X)$

$$b(X) = \int_{-\infty}^{\infty} x d\Psi(F_X(x))$$

Ask Price of a cash flow X: Acceptability of $a(X) - X$

$$a(X) = -\int_{-\infty}^{\infty} x d\Psi(1 - F_X(-x))$$

Examples: Calls and Puts

$$\begin{aligned}
bC(K,t) &= \int_K^{\infty} \big(1 - \Psi(F_{S_t}(x))\big) dx \\
aC(K,t) &= \int_K^{\infty} \Psi(1 - F_{S_t}(x)) dx \\
bP(K,t) &= \int_0^{K} \big(1 - \Psi(1 - F_{S_t}(x))\big) dx \\
aP(K,t) &= \int_0^{K} \Psi(F_{S_t}(x)) dx
\end{aligned}$$

Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Modeling of Stock Prices

X self decomposable:
for every c, $0 < c < 1$: $\quad X = cX + X^{(c)} \quad X^{(c)}$ independent of X

subclass of infinitely divisible prob. distributions

Sato (1991): process $(X(t))_{t \geq 0}$ with independent increments

$$X(t) \stackrel{\mathcal{L}}{=} t^{\gamma} X \quad (t \geq 0)$$

Write $\quad E[\exp(X(t))] = \exp(-\omega(t))$

Define the stock price process

$$S(t) = S(0) \exp((r-q)t + X(t) + \omega(t))$$

with rate of return $r-q$ for interest rate r and dividend yield q

Discounted stock price: martingale

Choice of the Generating Distribution

Introduction

Acceptability

Bid and Ask

Modeling

Empirical Results

Accounting

References

Variance Gamma: difference of two Gamma distributions

$$f_{\text{Gamma}}(x; a, b) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp(-xb) \qquad (x > 0)$$

Take $X = \text{Gamma}(a = C, b = M)$
$Y = \text{Gamma}(a = C, b = G)$

X, Y independent $\quad \rightarrow \quad X - Y \sim VG$

Alternatively: $G = \text{Gamma}(a = \frac{1}{\nu}, b = \frac{1}{\nu})$
Define $X = \text{Normal}(\theta G, \sigma^2 G) \quad \rightarrow \quad X = VG(\sigma, \nu, \theta)$

Characteristic function

$$E[\exp(\mathrm{i}uX)] = \left(1 - \mathrm{i}u\theta\nu + \frac{\sigma^2 \nu u^2}{2}\right)^{-\frac{1}{\nu}}$$

$\rightarrow$ four parameter process

Accomodating Default in this Model

Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

$(\Delta(t))_{t\geq 0}$ process which starts at one and jumps to 0 at random time T

Survival probability given by a Weibull distribution

$$p(t) = \exp\left(-\left(\frac{t}{c}\right)^a\right)$$

$c =$ characteristic life time
$a =$ shape parameter

Define the defaultable stock price

$$\widetilde{S}(t) = S(t)\frac{\Delta(t)}{p(t)}$$

If $F_t(s) = P[S(t) \leq s]$, then $\widetilde{F}_t(s) = 1 - p(t) + p(t)F_t(sp(t))$

$\rightarrow$ 6 parameter model so far

Liquidity

Refined distortion: MIN MAX VaR 2

$$\Psi(x) = 1 - \left(1 - x^{\frac{1}{1+\lambda}}\right)^{1+\eta}$$

λ:	rate at which Ψ' goes to infinity at 0 (coefficient of loss aversion)
η:	rate at which Ψ' goes to 0 at unity (degree of the absence of gain enticement)
λ, η	liquidity parameters
λ, η increased:	bid prices fall, ask prices rise (acceptable risks are reduced)

Capital Requirements (Risk)

Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Reserves expressed as difference between ask and bid prices

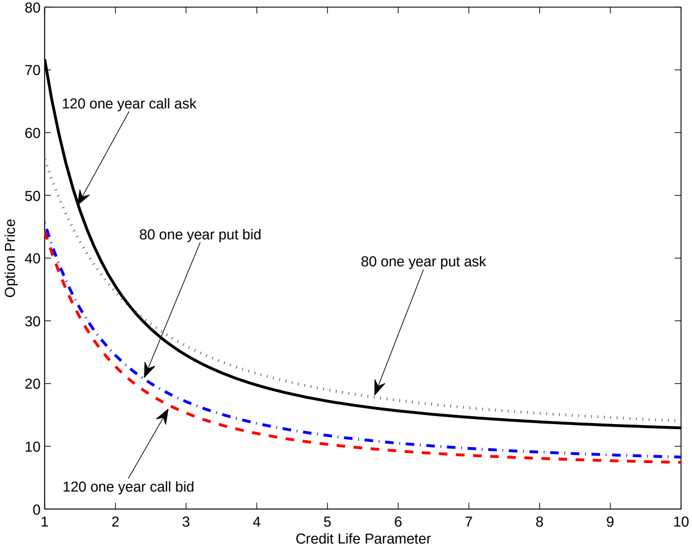
For a liability to be acceptable: ask price
    capital or cost of unwinding the position

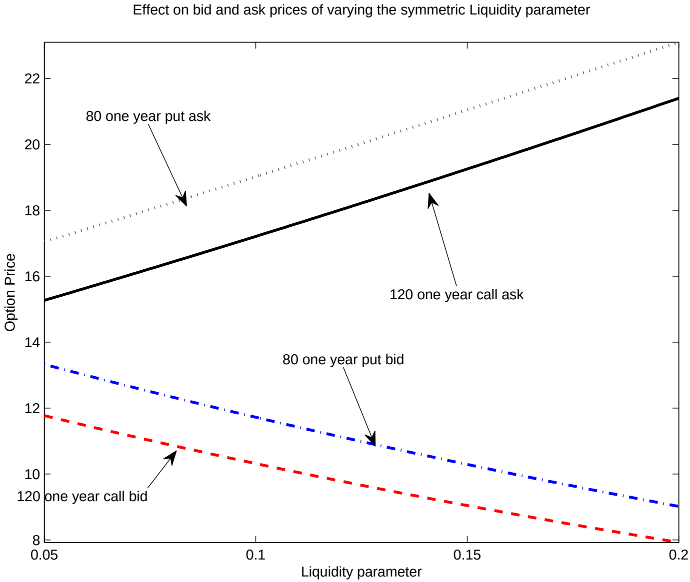
One gets credit for the bid price
    only excess needs to be held in reserve

Reserves are then responsive to movements of

option surface parameters:	$\sigma, \nu, \theta, \gamma$
credit parameters:	c, a
liquidity parameters:	λ, η

Effect of Credit Life Parameter on bid and ask prices of puts and calls





Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Empirical Results

Four banks: BAC, GS, JPM, WFC

Data from 3 years ending Sept. 22, 2010 $\rightarrow$ 237 calibrations

Movements around Lehman bankruptcy

Hypothetical options portfolio: spot 100; strikes 80, 90, 100, 110, 120; maturities 3 and 6 months

parameters: Aug. 26, 2008; Oct. 8, 2008

Sum over the spreads of the ten options

Pre and post Lehman capital needs on the hypothetical portfolio

	BAC	GS	JPM	WFC
Pre Lehman	2.3684	1.1851	2.0325	4.5648
Post Lehman	5.2694	3.8898	4.4995	8.3947
Percentage increase	122.48	228.22	121.38	83.89

Decomposition in Option, Credit and Liquidity Parameters

Reserve $c = g(\theta)$

$$\Delta c \approx \left(\frac{\partial g}{\partial \Theta} \Big|_{\Theta_0} \right) \Delta\Theta$$

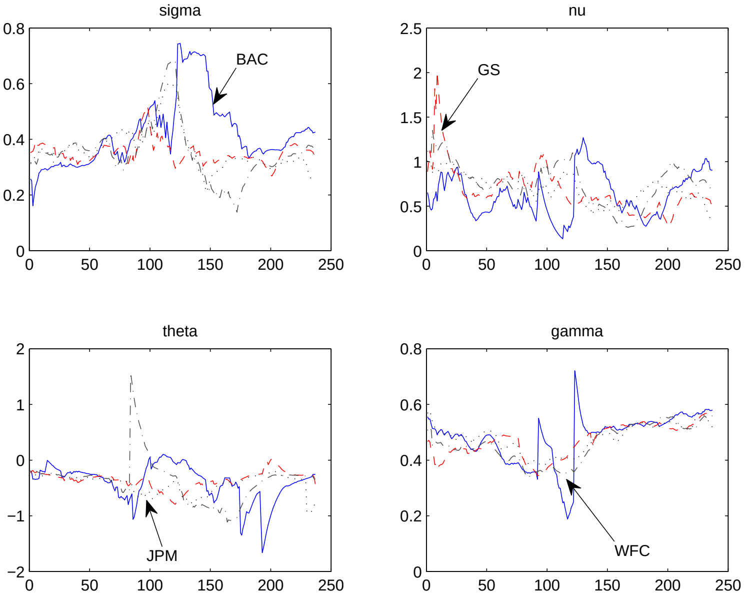
gradient vector at Θ_0 i.e. for Aug. 26, 2008

$\Delta\Theta$: change in parameter value from Aug. 26 to Oct. 8, 2008

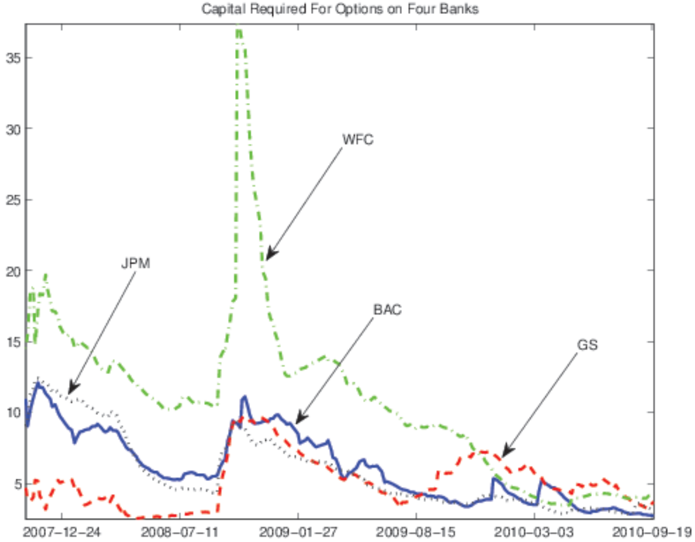
Relative parameter contributions to capital requirements (risk sources) from pre to post Lehman bankruptcy

	BAC	GS	JPM	WFC
σ	0.0406	-0.0657	0.0136	0.1003
ν	0.0254	-0.0026	0.0002	0.0192
θ	0.3476	0.0526	0.0307	-0.0264
γ	0.0409	0.0672	0.0750	0.1077
λ	0.0374	0.8513	0.3998	0.0673
η	0.4854	0.0972	0.4808	0.7318
c	-0.0073	0.0	0.0	0.0
a	0.0299	0.0	0.0	0.0

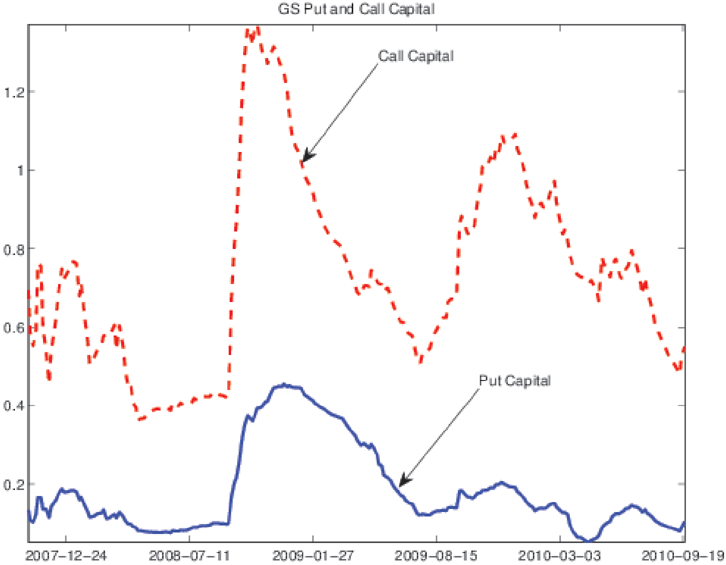
Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References



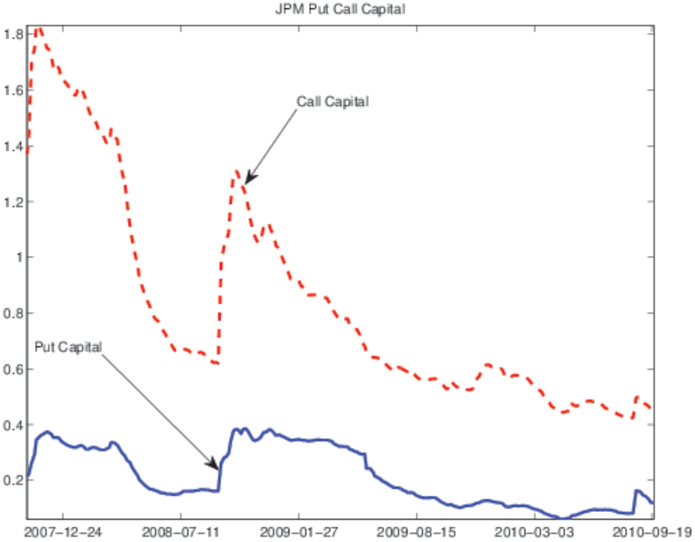
Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

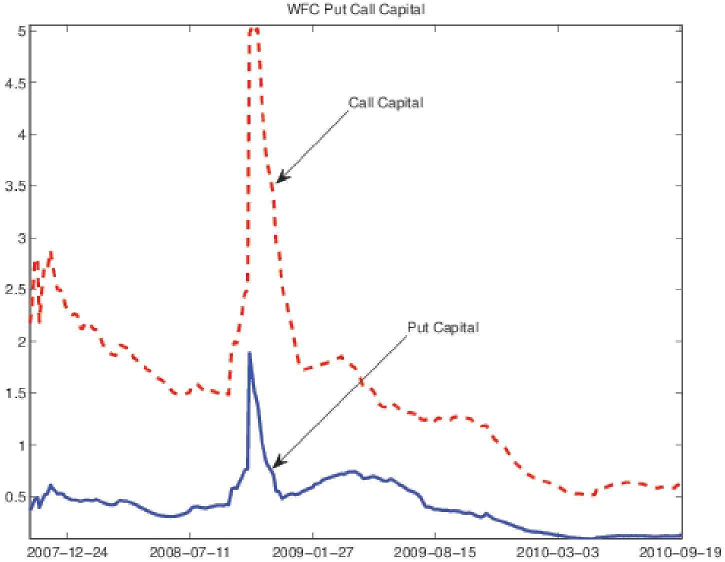


Capital Required For Options on Four Banks
35
30
25
20
15
10
5
2007−12−24
2008−07−11
2009−01−27
2009−08−15
2010−03−03
2010−09−19
WFC
JPM
BAC
GS

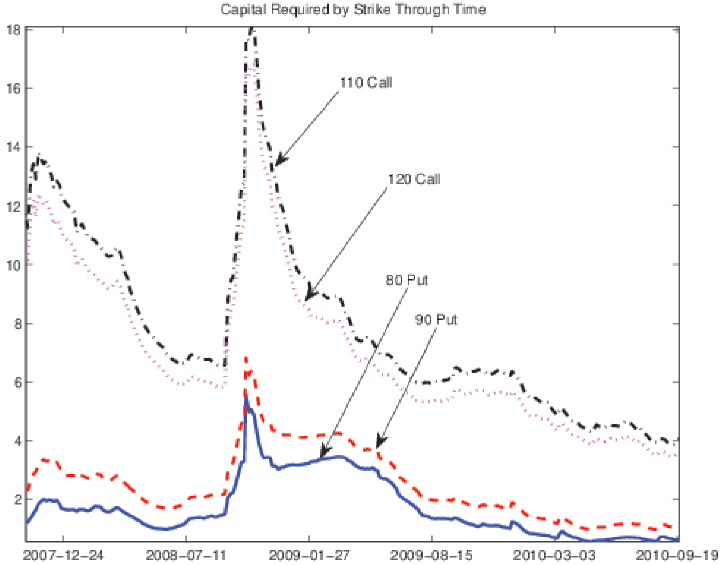


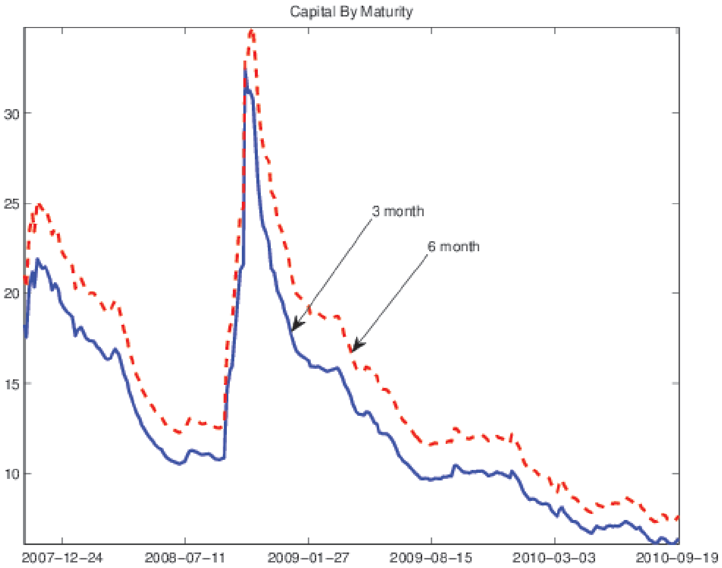
Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References


JPM Put Call Capital
Call Capital
Put Capital
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
2007−12−24
2008−07−11
2009−01−27
2009−08−15
2010−03−03
2010−09−19

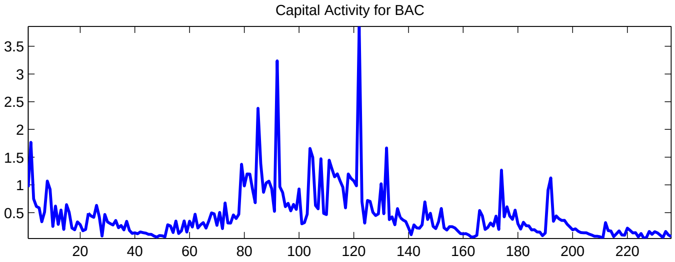


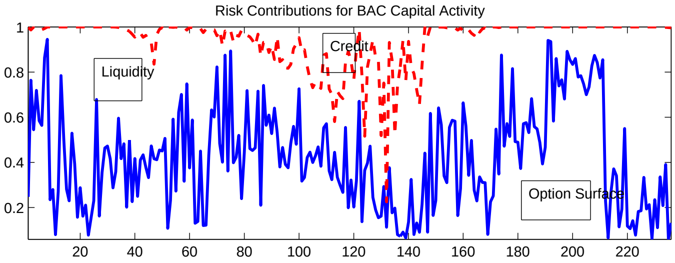
Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

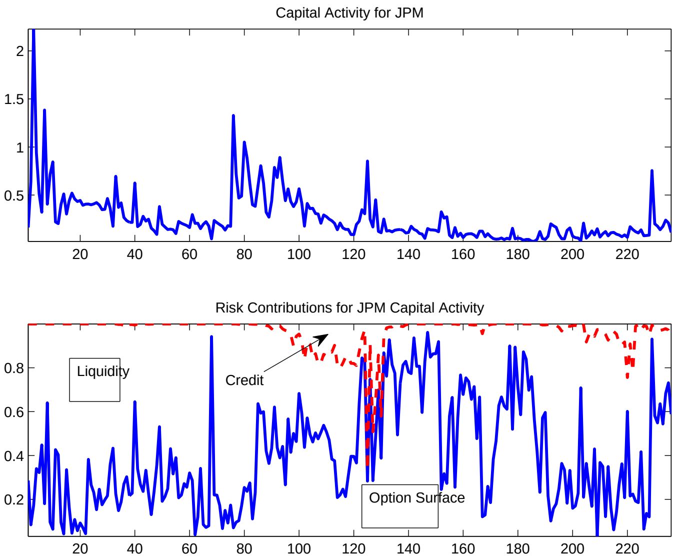

Capital Required by Strike Through Time
110 Call
120 Call
80 Put
90 Put
18
16
14
12
10
8
6
4
2
2007–12–24
2008–07–11
2009–01–27
2009–08–15
2010–03–03
2010–09–19



Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References


Capital Activity for BAC
3.5
3
2.5
2
1.5
1
0.5
20
40
60
80
100
120
140
160
180
200
220


Risk Contributions for BAC Capital Activity
1
0.8
0.6
0.4
0.2
Liquidity
Credit
Option Surface
20
40
60
80
100
120
140
160
180
200
220


Capital Activity for JPM
2
1.5
1
0.5
20
40
60
80
100
120
140
160
180
200
220
Risk Contributions for JPM Capital Activity
0.8
0.6
0.4
0.2
Liquidity
Credit
Option Surface
20
40
60
80
100
120
140
160
180
200
220

Bonds on a Balance Sheet

Balance sheet: investor

assets	liabilities
cash	equity
bonds	⋮
⋮	

Balance sheet: issuer (bank, corporate)

assets	liabilities
cash	equity
⋮	bonds
	⋮

Rating of the bonds deteriorates: $\rightarrow$ losses for investor, gains for issuer

Rating of the bonds improves: $\rightarrow$ gains for investor, losses for issuer

Financial Accounting Standards

FASB:	Financial Accounting Standard Board
US-GAAP:	United States General Accepted Accounting Principles
IFRS:	International Financial Reporting Standards
IASB:	International Accounting Standards Board

Question: What is the correct value of a position?

Answer according to the current standards

Mark to market

Consequences: Volatile behavior in times of crises

Introduction
Acceptability
Bid and Ask
Modeling
Empirical Results
Accounting
References

Marking Assets and Liabilities

Assets: Cash flow to be received $\widetilde{A} \geq 0$

Largest value A s.t. $\widetilde{A} - A$ is acceptable

$$\Rightarrow \qquad A = \inf_{Q \in \mathcal{M}} E^Q[\widetilde{A}]$$

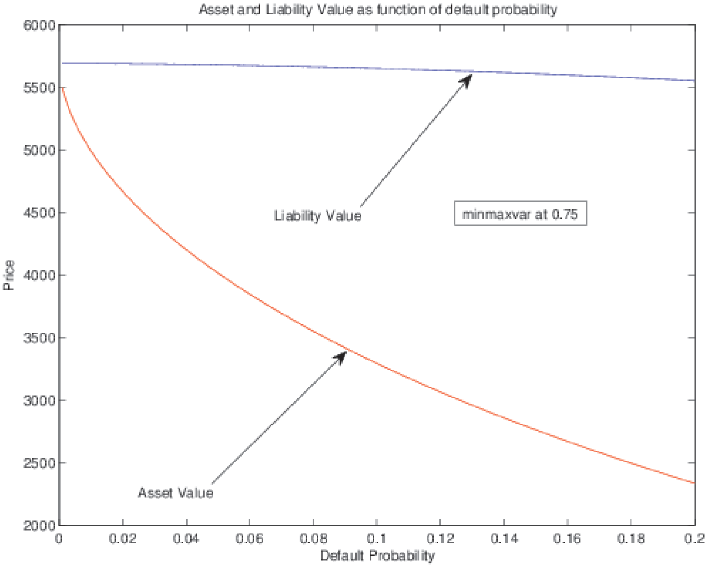
Bid Price

Liabilities: Cash flow to be paid out $\widetilde{L} \geq 0$

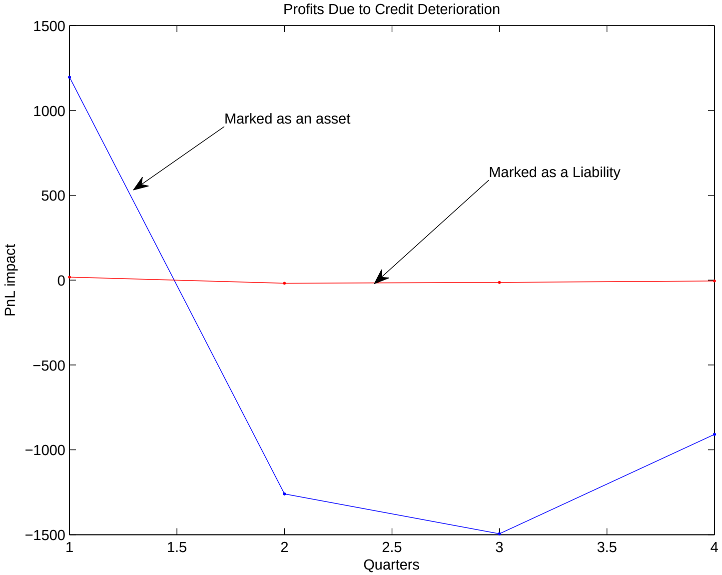
Smallest value L s.t. $L - \widetilde{L}$ is acceptable

$$\Rightarrow \qquad L = \sup_{Q \in \mathcal{M}} E^Q[\widetilde{L}]$$

Ask Price



Asset and Liability Value as function of default probability
6000
5500
5000
4500
4000
3500
3000
2500
2000
Price
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
Default Probability
Liability Value
minmaxvar at 0.75
Asset Value


Profits Due to Credit Deterioration
1500
1000
500
0
−500
−1000
−1500
1
1.5
2
2.5
3
3.5
4
PnL impact
Quarters
Marked as an asset
Marked as a Liability

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