IDENTIFYING BUSINESS CYCLE TURNING POINTS IN CROATIA

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6 May 2011

WHAT: MAIN GOALS

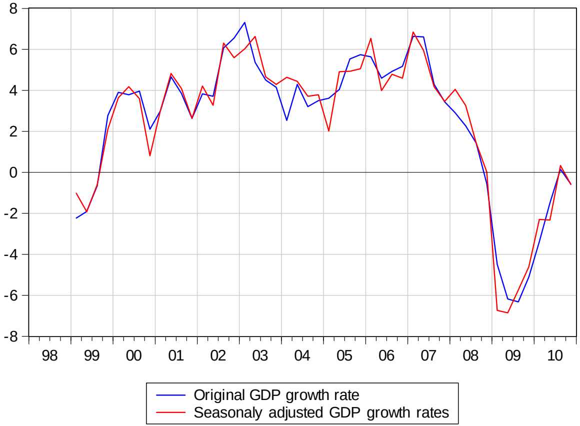
- Identify turning points of croatian economic activity for the period 1998-(end) 2010
- Provide clear and transparent methodology for doing that

Why: Motivation

- We do not know when (current) recession(s) began and if we are still in recession
- No institution (like NBER business cycles dating committee for US or CEPR Euro Area business cycles dating committee) that is responsible for dating turning points
- "A must have" for explaining business cycles
- Evidence that correlation between variables change over business cycle (important for forecasting)
- Measure of systemic risk: important for everyday running your business
- News on turning point attract large media and political attention

Why: Motivation (year-on-year GDP growth rates)

- We do not know when (current) recession(s) began and if we are still in recession

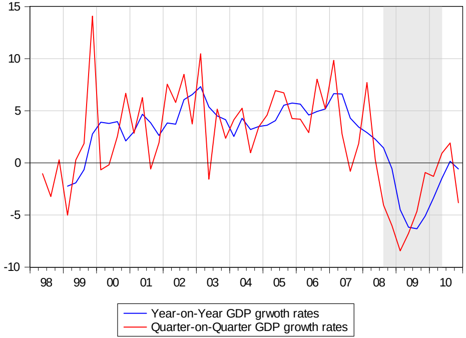


How: three approaches

- Simple Quarterly (q-o-q) GDP growth rates analysis
- Bry-Boschan algorithm on GDP data
- Markov switching model on GDP data and common component of a dynamic factor model

Q-O-Q GDP GROWTH RATE

- Economy is in recession after it reaches a peak: in the first *negative* quarter followed by one more quarter with negative growth rate: "two consequtive negative GDP growth rates" rule of thumb
  - 1st recession ended in 1999 q3 (trough in 1999 2q), 2nd recession starting in 2008 3q (peak 2008 2q)
  - Croatian economy was out of recession in 2q 2010 ?!

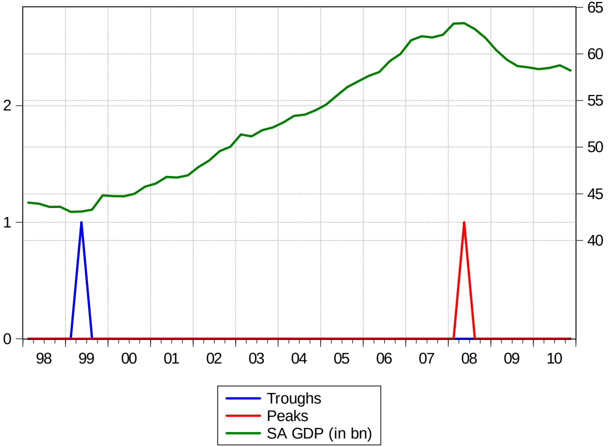


BRY-BOSCHAN ALGORITHM

- Nonparametric algorithm that identifies turning points as local minima/maxima of the (level of) GDP using *ad hoc* rules
  - potential trough/peak GDP at time t if its value is smaller/larger than GDP in $t+1$ and $t+2$
  - a trough must be followed by a peak and vice versa
  - a cycle (peak/trough to peak/trough) can not be shorter that 5 quarters; a phase (peak to trough and vice verca) must have a duration of at least 2 quarters
  - first and last turning point have to be larger/smaller than first and last observation

Bry-Boschan algorithm

- First recession ended in 1999 q3 (trough 1999 q2), current recession started in 2008 3q (peak 2008 q2) and is not over yet



MARKOV SWITCHING MODEL

- Simple idea for *q-o-q* GDP growth rates

$$\begin{aligned} E(y_t) &= c_1 \text{ if the economy is in recession} \\ E(y_t) &= c_2 \text{ if the economy is in expansion} \end{aligned} \tag{1}$$

or

$$E(y_t) = c_{s_t} \tag{2}$$

where $c_{s_t} = \{c_1, c_2\}$ and $s_t = \{1, 2\}$ is unobserved state variable of business cycle regime

$$\begin{aligned} s_t &= 1 \text{ if economy is in recession} \\ s_t &= 2 \text{ if economy is in expansion} \end{aligned}$$

- Incorporated into *AR(1)* process for (*q-o-q*) *GDP* growth rate

$$\begin{aligned} y_t &= \mu_{s_t} + \rho y_{t-1} + u_t \\ u_t &\sim IIN(0, \sigma^2) \end{aligned} \tag{3}$$

Markov switching model

- We have 2 states of business cycles (in the paper I have effectively 4 states...)

$$y_t = \mu_1 + \rho y_{t-1} + u_t \text{ if } s_t = 1 \quad (4)$$
$$y_t = \mu_2 + \rho y_{t-1} + u_t \text{ if } s_t = 2 \quad (5)$$

- Unobserved state, s_t evolves as a first order (2 state) Markov chain with (constant) transition probabilities

$$p_{ij} = \Pr(s_t = j | s_{t-1} = i) = \Pr(s_t = j | s_{t-1} = i, s_{t-2} = k, ..., y_{t-1}, y_{t-2}, ...)$$

- How do we estimate *AR(1)* process with 2 regimes: $\rho, \sigma^2, \mu_1, \mu_2$, transition probabilities?

MARKOV SWITCHING MODEL

- Write a *Likelihood* of model as a function of unknown parameters assuming that u_t has a normal distribution (then y_t is normal)

$$
\begin{aligned}
l_t &= \sum_{t=1}^{T} \ln \left[f\left(y_t \middle| \Omega_{t-1} \right) \right] = \\
&= \sum_{i=1}^{2} f\left(y_t \middle| s_t = i, \Omega_{t-1} \right) \Pr\left(s_t = i \middle| \Omega_{t-1} \right) \qquad (6)
\end{aligned}
$$

by theorem of total probability, $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, ...y_0\}$

- $f\left(y_t \middle| s_t = i, \Omega_{t-1} \right)$ is normal

$$
f\left(y_t \middle| s_t = i, \Omega_{t-1} \right) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(y_t - \mu_i - \rho y_{t-1})^2}{2\sigma}}
$$

- Compute recursively conditional probability of being in state i at time t given all the information at $t-1$, $\Pr\left(s_t = i \middle| \Omega_{t-1} \right)$ as a function of $\Pr\left(s_{t-1} = i \middle| \Omega_{t-2} \right)$

MARKOV SWITCHING MODEL

- With *natural* restrictions on transitional probabilities

$$\begin{pmatrix} \Pr(s_{t+1}) = 1) \\ \Pr(s_{t+1}) = 2) \end{pmatrix} = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix} \times \begin{pmatrix} \Pr(s_t = 1) \\ \Pr(s_t = 2) \end{pmatrix}$$

  the probabilities become function of 2 parameters only

$$\begin{pmatrix} \Pr(s_{t+1}) = 1) \\ \Pr(s_{t+1}) = 2) \end{pmatrix} = \begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix} \times \begin{pmatrix} \Pr(s_t = 1) \\ \Pr(s_t = 2) \end{pmatrix}$$

  where p is probability of recession (given last period was recession), a q is probability of expansion (given last period was expansion)

MARKOV SWITCHING MODEL

- Or for any state i

$$\Pr(s_t = i | \Omega_{t-1}) = \sum_{j=1}^{2} \Pr(s_t = i | s_{t-1} = j, \Omega_{t-1}) \Pr(s_{t-1} = j | \Omega_{t-1}) \quad (7)$$

  where for example $\Pr(s_t = 1 | s_{t-1} = 1, \Omega_{t-1}) = p$ or $\Pr(s_t = 2 | s_{t-1} = 2, \Omega_{t-1}) = q$

- Using Bayes theorem write $\Pr(s_t = i | \Omega_{t-1})$ as a function of $\Pr(s_{t-1} = i | \Omega_{t-2})$ and iterate on recursively to derive state probabilities

$$\begin{aligned} \Pr(s_{t-1} = j | \Omega_{t-1}) &= \Pr(s_{t-1} = j | y_{t-1}, \Omega_{t-2}) = \\ &= \frac{f(y_{t-1} | s_{t-1} = j, \Omega_{t-2}) \Pr(s_{t-1} = j | \Omega_{t-2})}{\sum_{i=1}^{2} f(y_{t-1} | s_{t-1} = i, \Omega_{t-2}) \Pr(s_{t-1} = i | \Omega_{t-2})} \end{aligned}$$

- All together 6 parameters to estimate: $p, q, \rho, \sigma^2, \mu_1, \mu_2$
- Estimating parameters=Finding global maksimum (arg max) of Likelihood using numerical methods

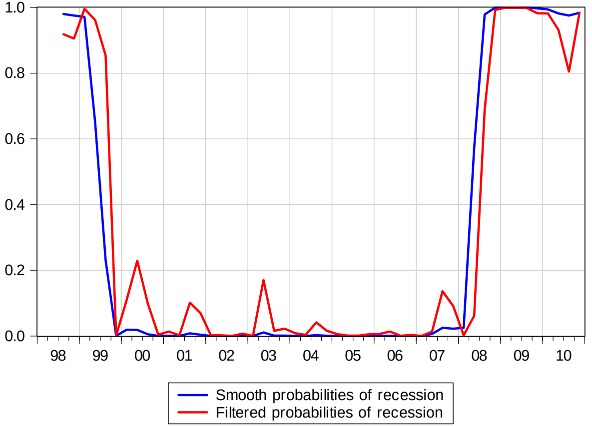
Markov switching model

Parameter	Estimated value (standard error)
μ_1	-0.70 (0.24)
μ_2	1.1 (0.15)
ρ	0.30 (0.06)
σ^2	0.71 (0.14)
p	0.96 (0.03)
q	0.95 (0.05)

- Interesting to calculate smooth state probabilities of recession $\Pr(s_t = 1 | \Omega_T, \widehat{p}, \widehat{q}, \widehat{\rho}, \widehat{\mu_1}, \widehat{\mu_2}, \widehat{\sigma^2})$ and filtered probabilities $\Pr(s_t = 1 | \Omega_{t-1}, \widehat{p}, \widehat{q}, \widehat{\rho}, \widehat{\mu_1}, \widehat{\mu_2}, \widehat{\sigma^2})$

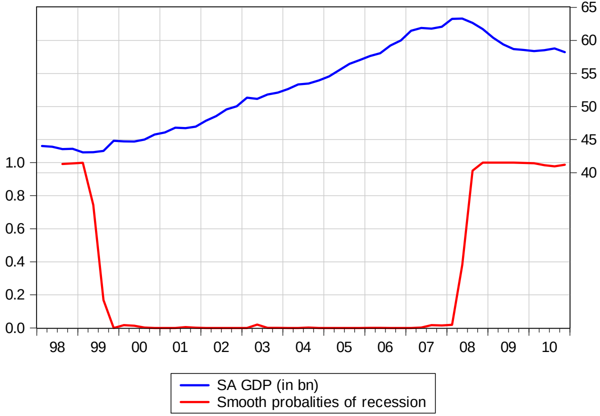
Markov switching model

- Smooth probabilities of recession used as an indicator of business cycle



MARKOV SWITCHING MODEL

- First recession ended in 1999 3q, current recession started in 2008 3q and still not over



WHY GDP? DYNAMIC FACTOR MODEL

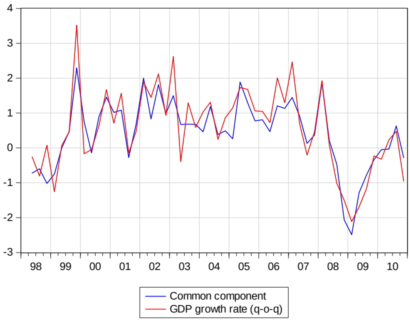
- Economic activity described by a numer of variables, not only GDP
- Estimate an indicator of economic activity by a dynamic factor model

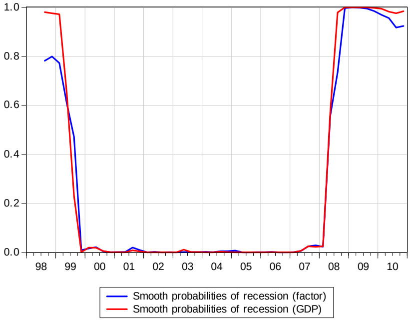
$$
\begin{aligned}
X_t &= \lambda(L) f_t + e_t \\
\Psi(L) f_t &= \eta_t \\
\Gamma(L) e_t &= \upsilon_t
\end{aligned}
$$

  where X_t is a matrix of variables that measure economic activity (GDP, exports, imports, industrial production, retail sales, empoyment, interest rates, loans ...), f_t is a factor that drives joint comovement of variables in X_t, and follows AR(p) process, e_t are errors that follow VAR(p) process
- $\lambda_i(L)$ is loading vector (L is lag operator), $\lambda(L)f_t$ is called common component
- Parameters to estimated: $f_t, \lambda(L), \Psi(L), \Gamma(L)$, variances of η_t and υ_t
- Write dynamic factor model in state space, write a Likelihood function of a state space representation using Kalman filter
- Estimate Markov swiching model on estimated common component and run BB algorithm

Why GDP? Dynamic factor model

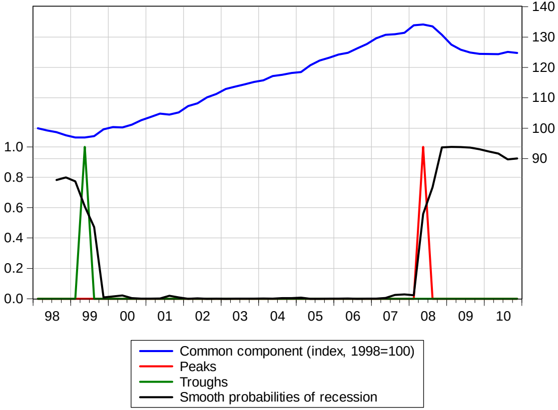
- GDP measures economic activity quite well





DYNAMIC FACTOR MODEL

- 1998/1999 recession ended in 1999 3q, current recession started in 2008 3q and not over yet



COMPARISON OF TURNING POINTS

- Presented clear and transparent methodology od dating turning points
- All methods identifyed 2 recession and 1 long expansion
- First recession ended in 1999 3q, second started in 2008 3q and not over yet

Trough/Peak	Q-o-Q gr. rates	BBQ (GDP)	MS (GDP)	BBQ (factor)	MS (factor)
Trough	1999:2 & 2010:1	1999:2	1999:2	1999:2	1999:2
Peak	2008:2 & 2010:3(?)	2008:2	2008:2	2008:2	2008:2