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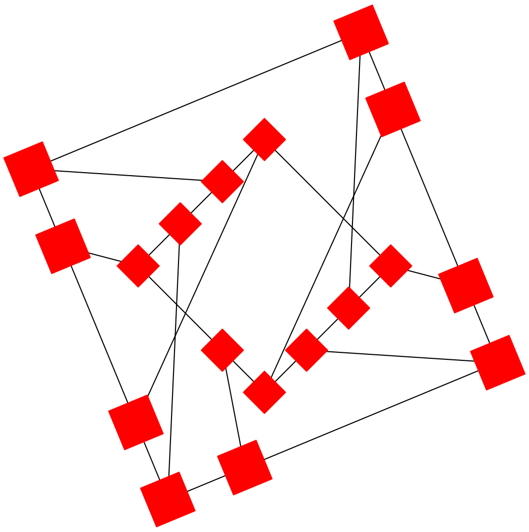
2nd CROATIAN CONGRESS OF MATHEMATICS



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PLENARY LECTURES

Comparisons of Biological Sequences: Some Tools from Linear Algebra

Alex Grossmann

Abstract. Not available.

Recent Progress on the Langlands Conjectures for $\mathrm{GL}(N)$

Guy Henniart

Abstract. It was recently proved that every elliptic curve over $\mathbb{Q}$ is modular. That result can be seen as a special case of an outstanding set of conjectures, due to R. P. Langlands. The most elementary result of this kind is the reciprocity law of Gauss. In general, the conjectures say roughly the following: if you consider a system of polynomial equations (in several variables) with integer coefficients, and you compute the number of solutions modulo a prime number p, then when p varies that number exhibits a regularity which can be expressed by harmonic analysis via devices named L-functions, which generalize Riemann's zeta function.

Gauss' reciprocity law concerns the equation $X^2 - n$ where n is an integer. Modulo a prime number p not dividing n, that equation has 0 or 2 solutions. The reciprocity law implies that the number of solutions only depends on congruences on p modulo $4n$, which is rather surprising. Since Gauss such a reciprocity result has been greatly generalized. If, for example, we consider a polynomial P irreducible in $\mathbb{Q}[X]$, and we compute the number of solutions of $P(X) \equiv 0 \bmod p$ for a large enough prime number p, then that number depends only on congruences on p, **provided that** the Galois group of P is abelian: this is a consequence of the celebrated class field theory (for the number field $\mathbb{Q}$).

What Langlands proposes is a generalization of class field theory, which would deal with equations in one variable with non abelian Galois group, or even with systems of equations in several variables. For example, we can consider an elliptic curve E with equation $y^2 = x^3 + ax + b$, where $x^3 + ax + b \in \mathbb{Q}[x]$ has no multiple roots in $\mathbb{C}$. For a prime number p not dividing the denominator of a or b, we can consider the number of solutions of $y^2 \equiv x^3 + ax + b$ modulo p; write it $p - a_p$ (to compare it to the affine line over $\mathbb{Z}/p\mathbb{Z}$). Then we can form an L-function $L(E, s)$ which is a product of L-factors $L_p(s)$, where for p large, $L_p(s)$ is the inverse of $(1 - a_p p^{-s} + p^{1-2s})$. Write $L(E, s)$ in the form $\sum_{n \geq 1} a_n n^{-s}$, $a_n \in \mathbb{Z}$. The Shimura–Taniyama–Weil conjecture, which can be seen as a special case of the Langlands conjectures, says that the function $f(z) = \sum_{n \geq 0} a_n e^{2\pi i n z}$, as a function of a complex

number z with $\mathrm{Im}(z) > 0$, is a (classical) modular form of weight 2 for a specific subgroup of $\mathrm{SL}_2(\mathbb{Z})$. In particular, there is a functional equation $f(-1/z) = \pm f(z)$. Such modular forms can often be constructed by other means, for example using theta-series, and that yields some regularity for the number a_p as a function of p (but in general it is much more subtle than saying that a_p depends only on congruences on p!).

The Shimura–Taniyama–Weil conjecture has now been proved completely by Breuil, Conrad, Diamond and Taylor. Previously, Wiles had settled the conjecture for many elliptic curves over $\mathbb{Q}$, thus proving the celebrated Fermat's "theorem".

In the general case, to a system of equations in several variables over $\mathbb{Q}$, or even over a number field F, one would attach, essentially by counting points modulo prime numbers p, L-functions $L(s)$ as above. It is expected (by Langlands) that such L-functions also appear in harmonic analysis, as L-functions attached to automorphic forms, which are representation–theoretic generalizations of classical modular forms: automorphic forms exist for GL_n over F, whereas classical modular forms are attached to GL_2 over $\mathbb{Q}$ only. Harris and Taylor have recently shown that some – rather special, but already numerous and very interesting – L-functions coming from automorphic forms are also L-functions coming from systems of equations. This has led to a complete proof of the counterpart of the Langlands conjectures for the p-adic field $\mathbb{Q}_p$ and its extensions.

Hopefully there is some more progress to come in the near future.

Vertex Algebras and Combinatorial Identities

Mirko Primc

Abstract. Rogers–Ramanujan identities, first discovered by Rogers in 1894, are two analytic identities expressing certain infinite products as infinite sums; the combinatorial Rogers–Ramanujan identities are obtained by equating the coefficients in the formal power series. These identities were a starting point of an analytic theory developed by Ramanujan, Watson, Bailey and Andrews, to mention just a few, with many connections with number theory and combinatorics. In 1980's Rogers–Ramanujan identities unexpectedly appeared in two seemingly unrelated contexts: in statistical mechanics through the work of Baxter and in representation theory of affine Lie algebras through the work of Lepowsky and Wilson. Further developments in mid 1980's showed a close relation of these results with quantum groups and vertex algebras associated with affine Lie algebras.

In this talk I'll describe some ideas and results related to Lepowsky–Wilson's approach to combinatorial identities of Rogers–Ramanujan type.

The starting point in 1978 was the observation of Lepowsky and Milne that, up to some factor, the principally specialized characters of standard $A_1^{(1)}$-modules of level 3 coincide with the product sides of Rogers–Ramanujan identities. Lepowsky and Wilson constructed bases of these modules, "up to a factor" parameterized by partitions satisfying difference 2 condition, and proved the Rogers–Ramanujan identities by applying the Weyl–Kac character formula. The main ingredient of the proof is the first vertex operator construction of representations of affine Lie algebras, which eventually led to other vertex operator constructions and to the theory of vertex operator algebras.

The ideas used in the Lepowsky–Wilson's proof of Rogers–Ramanujan identities were successfully applied to different standard modules for different affine Lie algebras, sometimes yielding new constructions of standard modules with underlying "classical" combinatorial identities (Lepowsky–Wilson, Misra, Mandia), sometimes giving new identities related to statistical physics (Lepowsky–Primc, Capparelli, Georgiev). If we forget some technical difficulties, the problem of constructing a basis of standard module is solved in two steps: (1) one should construct "enough" relations among elements of standard module to be able to reduce a PBW-spanning set to a smaller spanning set parameterized by partitions satisfying certain difference conditions and (2) one should prove linear independence of the obtained spanning set.

The proper setting for a description of relations among elements of standard module is the structure of (generalized) vertex operator algebras: "all possible" relations are the annihilating fields of the given representation and their structure is precisely described in terms of representation theory.

On the other hand, there is no universal recipe for a proof of linear independence; several methods were used so far, depending on a type of basis that is constructed. In a joint work with A. Meurman we developed an approach based on Verma modules and vertex algebras, in several aspects resembling the Groebner base theory. By using this approach we constructed bases of all standard $A_1^{(1)}$-modules, and a basis of the basic $A_2^{(1)}$-module, and as a result we obtained several infinite series of new Rogers–Ramanujan type identities. It seems that one part of this approach — the construction of relations among the annihilating fields of standard modules — will have a transparent description for all affine Lie algebras in terms of representation theory of vertex operator algebras. The combinatorial part of this approach gets to be very complicated even for the low rank affine Lie algebras $A_1^{(1)}$ and $A_2^{(1)}$, but some examples suggest that the combinatorial difficulties for affine Lie algebras of higher ranks might be circumvented by using the crystal base theory for quantum groups — a theory designed to generalize Baxter's results.

Toward Logical Foundations of Security Protocol Analysis

Andre Ščedrov

Abstract. Protocols based on cryptographic primitives are commonly used to protect access to computer systems and to protect transactions over the internet. Two well-known examples are the Kerberos authentication scheme, used to manage encrypted passwords on clusters of interconnected computers, and the Secure

Sockets Layer, used by internet browsers and servers to carry out secure internet transactions. A variety of methods have been developed for analyzing and reasoning about these and similar protocols. These approaches include specialized logics such as BAN logic, special-purpose tools designed for cryptographic protocol analysis, as well as theorem proving and model-checking methods using general purpose tools.

Although there are many differences among these approaches, most current formal approaches use the same basic model of adversary capabilities, which appears to have developed from positions taken by Needham and Schroeder [5] and a model presented by Dolev and Yao [3]. In this model, a protocol adversary is allowed to nondeterministically choose among possible actions. Messages are composed of indivisible abstract values, not sequences of bits, and encryption is modeled in an idealized way. Adversary may only send messages comprised of data it "knows" as the result of overhearing previous messages.

Perhaps the simplest interpretation of protocol transitions is to consider them as a form of rewriting. This observation may be sharpened to a rigorous, formal definition of the Dolev–Yao model by means of multiset rewriting with existential quantification, introduced in [1, 4]. We naturally gravitated toward some form of rewriting so that protocol execution could be carried out symbolically. In addition to rewriting to effect state transitions, we also needed a way to choose new values, such as nonces or keys. While this seems difficult to achieve directly in standard rewriting formalisms, the proof rules associated with existential quantification appear to be just what is required. Therefore, we have adopted a notation that may be regarded as an extension of multiset rewriting with existential quantification. This framework provides a very precise way of specifying security protocols and has been incorporated into a high-level specification language for authentication protocols, CAPSL, developed by Denker and Millen [2]. As presented in [1, 4], a protocol theory consists of three parts: a bounded phase describing protocol initialization that distributes keys or establishes other shared information, a role generation theory that designates possibly multiple roles that each principal may play in a protocol (such as initiator, responder, client, or server), and a disjoint union of bounded subtheories that each characterize a possible role. Furthermore, the multiset rewriting formalism allows us to formulate one standard intruder theory that describes any adversary for any protocol. Using this form of protocol theory, it is shown show that secrecy is an undecidable property even if data constructors, message depth, message width, number of distinct roles, role length, and depth of encryption are bounded by constants. DEXPTIME–completeness is shown for a class further restricted to have no new data (nonces) [1, 4].

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Conjugacy Classes in Unitriangular Groups

John G. Thompson

Abstract. I am trying to prove that for each positive integer n, there is a polynomial $f_n(x)$ with integer coefficients, such that for every finite field F, the number of conjugacy classes in $U_n(F)$ is $f_n(\mathrm{card}(F))$. Here $U_n(F)$ is the group of unitriangular n by n matrices with coefficients in F. The problem leads to some combinatorial questions.

Nonlinear Potentials in Functions Spaces

Zoran Vondraček

Abstract. The goal of this research is to develop basic concepts of a nonlinear potential theory in a fairly general setting. This setting provides a unified approach to several aspects of nonlinear potential theory with kernel (as described in D. R. Adams and L. I. Hedberg, *Function Spaces and Potential Theory*, Springer, 1996), as well as some kernel free potential theory (in particular, nonlinear potential theory special to quasilinear second order PDE as in J. Heinonen, T. Kilpeläinen and O. Martio, *Nonlinear Potential Theory of Degenerate Elliptic Equations*, Oxford UP, 1993). Our formulation of nonlinear potential theory is in the spirit of potential theory of Dirichlet spaces. The concepts we discuss include capacities of sets and functions, nonlinear potentials, equilibrium potentials, reduced functions, balayage, and measures of finite energy.

Our framework is as follows: Let X be a nonempty set, and let $\mathcal{N}$ be an ideal of subsets (which plays the role of "small" sets). Let $(\mathcal{F}, \|\ \|)$ be a normed space of (equivalence classes of) $\mathcal{N}$-a.e. defined extended real valued functions on X, with the dual space $\mathcal{F}^*$. Corresponding cones of positive elements are denoted by $\mathcal{F}_+$ and $\mathcal{F}^*_+$. We assume that $(\mathcal{F}, \|\ \|)$ is a reflexive Banach space, which is strictly convex and smooth. The main potential theoretic hypothesis (which, in a way, replaces the contraction principle) is

Hypothesis $(\mathbf{D}^+)$: For every $u \in \mathcal{F}$ there exists $v \in \mathcal{F}$ such that $u^+ \leq v$ and $\|v\| \leq \|u\|$.

We also need a stronger hypothesis ($\mathbf{D}$) in which u^+ is replaced by $|u|$. Examples of spaces which satisfy these hypotheses include various Sobolev spaces, weighted Sobolev spaces $H_0^{1,p}(\Omega, \mu)$, Besov spaces $B_\alpha^{p,q}$ and Lizorkin–Triebel spaces $F_\alpha^{p,q}$ for $1 < p < \infty$, $1 < q < \infty$, as well as spaces that appear in L^p-potential theories and potential theories for Orlicz spaces.

The cornerstone of our approach to nonlinear potential theory is the following definition of a nonlinear potential which includes all other definitions known to us. Let $U : \mathcal{F}^* \to \mathcal{F}$ be the duality mapping (single valued, since $\mathcal{F}^*$ is smooth) with the weight function γ^*. We say that $u \in \mathcal{F}$ is a *nonlinear potential* if $u = U\phi$ for $\phi \in \mathcal{F}_+^*$, and we call u the nonlinear potential of ϕ. It is an interesting fact that hypothesis ($\mathbf{D}^+$) is equivalent to nonnegativity of U. Thus all potentials are nonnegative functions. We give several equivalent characterizations of potentials.

The concept of a reduced function is fundamental in linear potential theories, but has not been much studied for nonlinear theories. Our definition of a reduced function is similar to the one from the Dirichlet space theory: For a function $u \in \mathcal{F}$ and a subset $B \subset X$ the reduced function of u onto B is a unique function $u_B \in \mathcal{F}$ satisfying $\|u_B\| = \min\{\|v\| : v \geq u \text{ on } B\}$. We show that u_B is a potential of a unique functional $\phi_B \in \mathcal{F}_{B,+}^*$. Functionals in the set $\mathcal{F}_{B,+}^*$ can be loosely described as "supported by the set B" (which is a desirable property that holds in linear theories). There is also the dual representation of u_B: $\|u_B\| = \sup\{\langle \phi, u \rangle : \phi \in \mathcal{F}_{B,+}^*, \|\phi\| \leq 1\}$. In proving these results, instead of the usual minimax theorem approach, we use a variant of the Gauss–Frostman quadratic functional: Set $\Gamma^*(t) = \int_0^t \gamma^*(s)\, ds$ and define $Q = Q_u : \mathcal{F}^* \to \mathbf{R}$ by $Q(\phi) = \Gamma^*(\|\phi\|) - \langle \phi, u \rangle$. Minimization of Q over various sets of functionals is our main tool in studying potentials and reduced functions.

Capacities of sets and functions are developed in the usual way after imposing some topological assumptions on the underlying set X. Quasieverywhere notions are introduced and the existence of quasicontinuous modifications of functions from $\mathcal{F}$ is established in a standard way. As a consequence of previous results we show the existence and uniqueness of the equilibrium potential for an arbitrary subset $E \subset X$.

With additional assumptions on both X and the function space $\mathcal{F}$, we identify nonnegative functionals with measure of finite energy. This identification makes it possible to show some standard results, e.g. the dual representation of the capacity of a compact set.

Joint work with Murali Rao.

INVITED LECTURES

Methods of Partially Hedging Risks in Financial Markets

Jakša Cvitanić

Abstract. In this talk I survey methods of dealing with the following problem: A financial agent is trying to hedge a claim C, without having enough initial capital to perform a perfect (super)replication. In particular, we describe results for minimizing the expected loss of hedging the claim C both in complete and incomplete continuous-time financial market models, and for maximizing the probability of perfect hedge in complete markets and markets with partial information. In these cases, the optimal strategy is in the form of a binary option on C, depending on the Radon–Nikodym derivative of the equivalent martingale measure which is optimal for a corresponding dual problem. We also present results on dynamic measures for the risk associated with the liability C, defined as the supremum over different scenarios of the minimal expected loss of hedging C.

Nonlinear Conservation Laws

Sunčica Čanić

Abstract. In this talk I will present some recent developments in the theory of systems of conservation laws with the focus on two applications. The first is in the area of multi-dimensional systems of conservation laws. It addresses a benchmark problem in nonlinear wave interactions: the problem of shock reflection by a wedge (also related to the stability of transonic shocks arising around a wing of an aircraft traveling at supersonic speeds). A novel theoretical approach toward proving existence and structure of solutions will be presented. The second application is in the area of one-dimensional systems of conservation laws. This application is related to a problem in hemodynamics: modeling of treatment of aortic abdominal aneurysm. A simple model will be presented and several open questions will be discussed. Movies showing numerical simulations will be presented.

Perturbation Theory and Algorithmic Development for the Symmetric Eigenvalue Problem

Zlatko Drmač

Abstract. We study the development of accurate algorithms for matrix (generalized) symmetric (Hermitian) eigenvalue problems. A desirable property of such an algorithm is that it approximates well determined eigenvalues with high relative accuracy independent of their magnitudes. This is an important issue because the smallest eigenvalues are in applications usually the most interesting ones and, unfortunately, the most sensitive ones in presence of numerical errors. To design such an algorithm, we need detailed knowledge of the structure of errors produced by finite precision implementation of the algorithm, as well as deep understanding of the sensitivity to perturbations of the original problem.

The focus of our attention is the partial symmetric (Hermitian) eigenvalue problem. To find only a few eigenpairs of a large symmetric matrix one usually constructs a sequence of low-dimensional subspaces and hopes to approximate the target eigenpairs from these subspaces, using the Rayleigh–Ritz procedure. The convergence is assured e.g., via trace minimization over a suitably chosen Stiefel manifold.

To be successful in designing efficient algorithms, one needs to know

(i) how good is the current subspace, e.g., how accurate are the Ritz pairs obtained from that subspace;

(ii) how to enrich the current subspace with directions close to the target vectors, thus ensuring fast convergence;

(iii) how to compute accurate Rayleigh quotient matrix and the Ritz pairs in finite precision (floating-point) arithmetic.

So, for instance, if the task is to approximate the lowest eigenvalue λ of a positive definite matrix, we wish to know how many correct digits are in our current (floating-point) approximation $\lambda + \delta\lambda$. Sharp computable error bound assures a good stopping criterion and accurate approximations. These issues clearly belong to the perturbation theory. We present computable bounds for $|\delta\lambda/\lambda|$ and show how perturbation theory sheds a new light on some well-known eigensolvers. We also discuss how to use these results to construct better subspaces, which could lead to substantial improvement of some known algorithms.

What is an Algorithm?

Yuri Gurevich

Abstract. One may think that the title problem was solved long ago by Church and Turing. It wasn't; there is more to an algorithm than the function it computes. (Besides, what function does an operating system compute?) The interest to the problem is not only theoretical. Applications include modeling, specification, verification and design of software and hardware systems. We will explain the sequential Abstract State Machine thesis (see `http://www.acm.org/tocl/accepted.html`). If time permits, we will mention parallel, distributed and real-time ASMs.

Sequential Homology

Luis Javier Hernández Paricio

Abstract. An exterior space consists of a topological space together with a certain nonempty family of open subsets that is thought of as a "system of open neighborhoods at infinity". We use sequences of cycles converging to infinity to introduce sequential homology and cohomology theories in the category of exterior spaces. It is interesting to note that the singular homology, the locally finite homology and the end homology can be obtained as sequential homology with coefficients in adequate modules. Similarly, the compact support cohomology, the singular cohomology and the end cohomology are particular cases of sequential cohomology with adequate coefficients. We also analyze the relation with Steenrod homology and give a version of Poincaré duality for sequential theories.

Joint work with J. Gracía Calcines (University of La Laguna, Spain).

Polynomial Invariants, Differential Operators and Eigenspace Representations

Jing–Song Huang

Abstract. The aim of this talk is to describe the eigenspace representations on semisimple symmetric spaces, which is a fundamental problem in harmonic analysis on homogeneous spaces. This branch of mathematics has its root in the classical theory of Fourier series and Fourier integrals. It is closely connected with group representations theory in infinite-dimensional spaces that plays an important role in many recent developments of mathematics and in the interaction of mathematics

and physics. The main content of the talk includes the description of G-invariant differential operators on a semisimple symmetric space G/H in terms of polynomial algebras and the corresponding space of eigenfunctions in terms of well understood infinite-dimensional representations.

More precisely, let $\mathfrak{a}_{\mathfrak{q}}$ be the split Cartan subspace of G/H. If $\mathfrak{g}$ does not contain any simple ideal of type E, the we can choose $D_1, \ldots, D_r$ in the center $Z(\mathfrak{g})$ of universal enveloping algebra $U(\mathfrak{g})$ such that $D_1, \ldots, D_r$ are algebraically independent and $\mathbf{D} = \mathbb{C}[D_1, \ldots, D_r]$ is isomorphic to $S(\mathfrak{a}_{\mathfrak{q}})^{W(\mathfrak{a}_{\mathfrak{q}})}$. Denote by $\mathcal{E}_\lambda(G/H)$ the space of K-finite smooth functions on G/H such that $Zf = \chi_\lambda(z)f$. Let P be a σ-minimal parabolic subgroup and $\pi_{P,\xi,\nu}$ be the principal series for G/H. We show that $\mathcal{E}_\lambda(G/H)$ is an admissible $(\mathfrak{g}, K)$-module and for $\lambda \in \mathfrak{a}^*_{\mathfrak{q}\mathbb{C}}$ generic,

$$\mathcal{E}_\lambda(G/H) \cong \bigoplus_{w\in\mathcal{W}} \bigoplus_{\xi} \pi_{P,\xi,\lambda(\xi)},$$

where $\mathcal{W}$ is a set of representatives of $W(\mathfrak{a}_{\mathfrak{q}})/W_{K\cap H}$, and ξ runs over the discrete series of $M/w(M \cap H)w^{-1}$. For a general λ, we prove the following identity for distribution characters

$$\Theta(\mathcal{E}_\lambda(G/H)) = \sum_{w\in\mathcal{W}} \sum_{\xi} \Theta(\pi_{P,\xi,\lambda(\xi)}).$$

Application of these results to eigenspace representations on symmetric varieties will also be discussed.

Modular Groups and Ionic Subgroups of Coxeter Groups

Asia Ivić Weiss

Abstract. Matrices whose entries belong to certain rings of algebraic integers can be associated with discrete groups of transformations of inversive n-space or hyperbolic $(n+1)$-space H^{n+1}. For small n, these may be Coxeter groups, generated by reflections, or certain subgroups whose generators include direct isometries of H^{n+1}. We show how linear fractional transformations over rings of rational and quadratic integers are related to the symmetry groups of regular tilings of the hyperbolic plane or 3-space. New light is shed on the properties of the rational modular group $PSL_2(\mathbb{Z})$, the Gaussian modular (Picard) group $PSL_2(\mathbb{Z}[i])$, and the Eisenstein modular group $PSL_2(\mathbb{Z}[\omega])$.

Joint work with Norman W. Johnson.

Strong Shape with Compact Supports

Yury T. Lisica

Abstract. It is well-known that though the usual homotopy category is defined for arbitrary topological spaces, nevertheless, its homotopy theory is fruitful and meaningful only for spaces with good local behavior, like the ANR's, polyhedra or CW-complexes, i.e., the main theorems (Hurewicz, Whitehead, Hopf, Smale, etc.) of this homotopy theory are not valid in general, even for compact metric spaces which are not homotopy type of ANR's. Thus, one needs the *true homotopy category* without mentioned defects. The strong shape category is one of such true homotopy categories and is a projective version of ANR-approximation of topological spaces. Here we introduce another one, dual to the previous one, as an injective *Comp*-approximation of topological spaces, the so-called *strong shape category with compact supports*.

Direct systems as objects and coherent homotopy classes of coherent mappings as morphisms, form the coherent homotopy category $CH(inj\text{-}Top)$. The functor $C' : Ho(inj\text{-}Top) \to CH(inj\text{-}Top)$, induced by the coherence functor $C : inj\text{-}Top \to CP(inj\text{-}Top)$, is an isomorphism of categories, where $Ho(inj\text{-}Top)$ is a localization of the category *inj-Top* at the class Σ of level homotopy equivalences. We define the *strong coshape category* $ScoSh(Top)$ as a category whose objects are all topological Hausdorff spaces and morphisms $F : X \to Y$ are determined by triples $(\underline{i}, \underline{j}, [\underline{f}])$, where $\underline{i} : \underline{X} \to X$, and $\underline{j} : \underline{Y} \to Y$ are inclusions of direct systems of compact subspaces of X and Y, respectively, cofinal in the systems of all compact subspaces and $[\underline{f}] : \underline{X} \to \underline{Y}$ is a morphism of $CH(inj\text{-}Comp)$. Now we define the *strong shape category with compact supports* $SSh_c(Top)$ in the same way as above, but only $[\underline{f}] : \underline{X} \to \underline{Y}$ is a morphism of $SSh(inj\text{-}Comp)$, where the last is a localization of the category *inj-Comp* at the class Σ of level *strong shape* equivalences of direct systems of compact spaces.

There is a natural functor $S : ScoSh(Top) \to SSh_c(Top)$.

The main theorems mentioned above are valid in this category. Moreover, Steenrod–Sitnikov homology with compact supports and coherent cohomology of spaces are invariants of isomorphisms of this category. The last is the cohomology group of homotopy inverse limit of the inverse system formed by Massey cochain complexes of compact subspaces of topological space.

Quasigroup String Processing – Theory and Applications (in Cryptography, PRNG, Fractal Designs, ...)

Smile Markovski

Abstract. Quasigroup string processings can be defined in several ways and here we define the two simplest (and the most important) ones. Given a finite set A, we denote by A^+ the set of all finite strings on A. Let $*$ be a quasigroup operation on A. Then another quasigroup operation $\backslash$ on A is induced by $*$ as follows:

$$x * y = z \;\Leftrightarrow\; x \backslash z = y$$

and the following equations

$$x \backslash (x * y) = y, \quad x * (x \backslash y) = y$$

are identities in the algebra $(A, *, \backslash)$. Fix an element $l = a_0 = b_0 \in A$ and define two transformations E_l, $D_l : A^+ \to A^+$ as follows. Let $a_1 \cdots a_n \in A^+$. Then

$$\begin{aligned} E_l(a_1 \cdots a_n) = b_1 \cdots b_n \;&\Leftrightarrow\; b_{i+1} = b_i * a_{i+1}, \quad i = 0, 1, \ldots, n-1, \\ D_l(a_1 \cdots a_n) = c_1 \cdots c_n \;&\Leftrightarrow\; c_{i+1} = a_i \backslash a_{i+1}, \quad i = 0, 1, \ldots, n-1. \end{aligned}$$

We have that $ED = 1 = DE$, i.e., E and D are permutations, and this allows us to use E and D for defining suitable encryption and decryption functions.

Let $E_{l_1 \cdots l_k} = E_{l_1} \cdots E_{l_k}$ and $D_{l_1 \cdots l_k} = D_{l_1} \cdots D_{l_k}$. The most important properties of E and D are the following.

The distributions of s-tuple in $E_{l_1 \cdots l_k}(a_1 \cdots a_n)$, $1 \le s \le k$, for large enough arbitrary string $a_1 \cdots a_n$ are uniform.

The distributions of s-tuple in $D_{l_1 \cdots l_k}(a_1 \cdots a_n)$, for large enough k and large enough arbitrary string $a_1 \ldots a_n$ are uniform.

There are quasigroup operations $*$ on A such that for an arbitrary string $a_1 \cdots a_n$ with period $p > 0$, i.e., such that $a_i = a_{i+p}$, $i = 1, 2, \ldots$, the period of $E_{l_1 \cdots l_k}(a_1 \cdots a_n)$ is $p2^k$. Hence, we can define a random sequence with a potentially infinite period.

There are quasigroup operations $*$ on A such that we will obtain a fractal pattern with the following procedure. Let $a_i = l \in A$, $i = 1, \ldots, n$, and define the strings $r_1 = a_1 \cdots a_n$, $r_{j+1} = (E_l)^j(r_1)$, $j = 1, \ldots, n-1$. Then, construct a matrix in which the j-th row is just r_j, $j = 1, 2, \ldots, n$.

Classifying Overlay Structures of Topological Spaces

Vlasta Matijević

Abstract. The well-known classification theorem for covering mappings establishes a bijection between classes of equivalent s-sheeted pointed covering mappings $f : X \to Y$ over a connected locally pathwise connected semi-locally 1-connected space Y and subgroups H of index s of the fundamental group $\pi_1(Y, *)$. In 1972 R. H. Fox [1] obtained a version of this result valid for arbitrary connected metric spaces Y. However, he had to replace covering mappings by newly introduced overlay structures and $\pi_1(Y, *)$ by its shape-theoretic analogue, the fundamental progroup $\underline{\pi}_1(Y, *)$. By definition, an s-sheeted overlay structure over a connected space Y is a fibre bundle, i.e., an equivalence class of coordinate bundles (X, Y, f, S, G), where the fibre S is discrete and of cardinality s, the transformation group G is the symmetric group $\Sigma(s)$ and the coordinate transformations are constant functions. If the base space Y is locally connected and paracompact or if s is finite, s-sheeted (indecomposable) overlay structures coincide with s-sheeted covering mappings (where the covering space X is connected).

Using ANR-resolutions of $(Y, *)$, the fact that overlays are pull-backs of covering mappings of ANR's, as well as the classical classification theorem, we generalize Fox's result to a classification theorem for overlay structures over arbitrary connected topological spaces [3]:

Theorem. *There exists a bijection between the set of all equivalent classes of s-sheeted indecomposable pointed overlay structures over a pointed connected topological space $(Y, *)$ and the set of all subprogroups of index s of the fundamental progroup $\underline{\pi}_1(Y, *)$.*

As in the classical case, the unpointed version of the theorem uses conjugacy classes of subprogroups of index s of the fundamental progroup $\underline{\pi}_1(Y, *)$. Both results can also be stated in terms of transitive representations of the fundamental progroup $\underline{\pi}_1(Y, *)$ in the symmetric group $\Sigma(s)$.

Using the Čech covering approach, L. J. Hernández Paricio [2] obtained a version of the classification theorem, which even allows non-connected base spaces and, therefore, it uses fundamental progrupoids.

References

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Three–Dimensional, Volume–Preserving Maps and Flows and Applications in Fluid Dynamics

Igor Mezić

Abstract. We present some developments in the theory of three-dimensional maps and flows. Symmetry considerations are used to discuss their integrability and the resulting structure is used to study the dynamics of maps and flows perturbatively. Two characteristic classes arise:

(1) action–action–angle and

(2) action–angle–angle maps and flows.

Class (2) has similar characteristics to two-dimensional symplectic systems, while class (1) is quite different. For example, an analogue of the Kolmogorov–Arnold–Moser theorem on persistence of tori upon perturbation is valid in class (2). In contrast, generically any invariant surface of an integrable map of class (1) will not persist upon perturbation. Applications to the theory of mixing in incompressible fluid flows are discussed.

On the Incompressible Viscous Flow over a Rough Surface and the Navier Slip Condition

Andro Mikelić

Abstract. We consider the laminar viscous incompressible flow over a boundary containing surface irregularities. Such surfaces cause boundary layers for the velocity gradient and the pressure field, and numerical solution of the Navier–Stokes equations requires very fine grids. In engineering practice, the grooved boundary effects are reduced to a coefficient in the effective interface law, posed on a smooth surface. These upscaled laws are called the wall laws and are of importance in applications.

In this talk we suppose the periodic irregularities of the same characteristic length and height. Characteristic size of the imperfections is small and we study the asymptotic behavior of the solution for the incompressible Navier–Stokes equation, when it tends to zero. After constructing appropriate boundary layers, we obtain the Navier slip condition as the corresponding wall law. We justify it for moderate Reynolds numbers by estimating the difference between the physical solution and the upscaled solution in appropriate norms. The norm of the difference behaves as a power of the characteristic roughness. The effective coefficient in the Navier's law is determined through an auxiliary boundary layer. Finally, we show that presence of the irregularities (riblets) diminishes the tangential drag force.

Geometry and Smooth Representations of Reductive Lie Groups

Dragan Miličić

Abstract. The standard way of studying representations of reductive Lie groups is to study the corresponding category of Harish–Chandra modules. This method reduces the problem to an essentially algebraic problem. In my lecture at the last Congress, I discussed how this problem can be transferred to a problem in D-module theory, where powerful methods from algebraic geometry can be used. A feature of this approach, which is sometimes unpleasant, is that Harish–Chandra modules do not admit an action of the reductive Lie group G itself (in fact, they are modules for the action of the enveloping algebra of the Lie algebra of the group G and the action of its maximal compact subgroup K).

Casselman and Wallach defined a category of "smooth representations of moderate growth and finite length" of G and constructed a "completion" functor from the category of Harish–Chandra modules into this category. They also established that this functor is an equivalence of categories. Its quasi-inverse is the functor which attaches to each smooth representation the dense Harish–Chandra module of its K-finite vectors. Therefore, smooth representations are just "Harish–Chandra modules in disguise", but with an action of the group G itself.

This allows us to apply to smooth representations some ideas inspired by the arguments used in the representation theory of p-adic reductive groups. In this lecture, I'll discuss the construction of the Bruhat filtration on smooth principal series representations. This is a natural filtration attached to the Bruhat cell stratification of the real flag variety. Using some powerful results from analytic geometry, one can describe completely the graded object attached to this filtration. Using standard techniques from homological algebra this leads to a number of interesting results on smooth representations. Among others, I'll discuss Kostant's results on existence and uniqueness of Whittaker vectors and Wallach's results on holomorphic continuation of Jacquet integrals.

Joint work with Bill Casselman and Henryk Hecht.

Some Remarkable Inequalities Connected to L_p–Spaces and Fractional Order Hardy Inequalities

Lars–Erik Persson

Abstract. I will give some examples of integral inequalities we have proved during the last two years. I will concentrate on Hardy type inequalities and some inequalities connected to L_p-spaces. More precisely, I will present some results and ideas

from papers [1]–[4], which I think are especially interesting and sometimes surprising. Also, some even newer results will be mentioned and some open questions posed.

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[2] N. Krugljak, L. Maligranda, L. E. Persson, *Failure of the Hardy inequality and interpolation of subspaces*, Ark. Mat., 37 (1999), pp. 323–344.

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On Standard Brownian Motion: The Einstein 1905 Arguments Revisited

Goran Peskir

Abstract. In the first part we shall present the arguments used by Einstein in 1905 to model Brownian motion. We shall discuss their relevance for the subsequent development of probability theory in the 20th century. In the second part we shall derive the equation of motion of a Brownian particle suspended in a heat pipe. We shall explain how this motion and equation relate to some key issues in stochastic mechanics and calculus.

Finding Exact Solutions of Linear Recurrences

Marko Petkovšek

Abstract. In the first part of my talk, I will review the algorithms for finding "nice" explicit solutions of linear recurrences with polynomial coefficients. These include solutions which are polynomials, rational functions, hypergeometric or q-hypergeometric terms, and Liouvillian functions.

In the second part, I will look at linear recurrences in the multivariate case. While in the univariate case solutions of linear recurrences with constant coefficients have rational generating functions, we show that the multivariate case is much richer: even though initial conditions have rational generating functions, the corresponding solutions can have generating functions which are algebraic but not rational, D-finite but not algebraic, and even non D-finite and non D-algebraic.

Besov Spaces, Martingales, and Brownian Motion Process

Zoran R. Pop–Stojanović / Murali Rao

Abstract. Since their introduction (*Integral Representations of Functions and Imbedding Theorems* by O. V. Besov, V. P. Ilin, and S. M. Nikolski, Wiley and Sons, 1978), Besov spaces are playing an ever increasing role in the treatment of stochastic processes. Here are some of the results obtained in this area:

– Various techniques have been developed enabling one to connect potentials of the Brownian motion process with Besov spaces (*Brownian Potentials and Besov Spaces*, Z. R. Pop–Stojanović, M. Rao, and H. Šikić, J. Math. Soc. Japan, 30, no. 2, 1998). The main result in the mentioned paper shows that the Brownian potentials of finite measures, given over bounded domains, belong to Besov spaces.

– In (*Approximation en Norme Besov de la Solution D'une Equation Differentielle Stochastique*, Stochastics and Stochastics Reports, 49, 1994), B. Roynette shows that a strong solution of an Ito stochastic differential equation in R^1 can be obtained as the limit in Besov norm $1/2, p, \infty$ of the Euler and Milshtein approximation schemes. The significance of the Besov norm $1/2, p, \infty$ in this setup is in the fact that it is larger than the L^2 norm customarily used.

– In the treatment of wavelet bases and thresholding, (*Wavelets and the Theory of Nonparametric Function Estimation*, I. M. Johnstone, Phil. R. Soc. Lond. A, 1999), the author uses Besov norm with smoothness index α (=number of derivatives), and the homogeneity index p, plotted as $1/p$ as is customary in harmonic analysis, where each point $(\alpha, 1/p)$ corresponds to a class of Besov spaces. The so-called Bump algebra (Meyer, 1990, Section 6.6) is an example of a Besov space.

– Two of the above papers show connections of Besov spaces with martingales. We aim to extend understanding of this connection further.

These and other results will be presented in our lectures.

New Overconstrained Models of Polyhedra

Otto Röschel

Abstract. We start with a set of congruent plane equiform motions in the faces of a polyhedron, each of them having a globally fixed point. Under the restriction of some combinatorial properties we link motions in neighbouring faces of the polyhedron. They give rise to some interesting new mechanisms with systems linked via 2R- and

1R-joints, respectively. Closure conditions are being formulated and used in order to generate one-sided or two-sided closed rectangular arrangements. We investigate the general degree of freedom of these mechanisms depending on their topological structure. Some examples (e.g., Möbius mechanisms and toroidal structures) are studied in detail.

Representing Integers by Quadratic Forms, and Its Generalizations

Gordan Savin

Abstract. Representing integers by quadratic forms is a classical theme in number theory. If we fix a positive definite quadratic form q defined over integers, let a_n be the number of ways a positive integer n is represented by q. A surprising fact is that a_n are coefficients of a modular form for SL_2. In this lecture we discuss how some different counting problems in number theory also give rise to automorphic representations.

Joint work with W. T. Gan and B. H. Gross.

On Hub Network Models

Darko Skorin–Kapov

Abstract. The study of hub location models involves designing communication networks where some of the nodes serve as focal points (i.e., hubs) and other nodes are connected to those hubs. The advantage of hub networks is the usage of relatively small number of links and exploitation of economies of scale by concentrating flows. Possible applications include telecommunications, distributed data bases, airline traffic flow, and parcel delivery networks. The presentation consists of two parts. The first part is an overview of recent results on solvability of some hub and hub-like location models, and the second part is an analysis of the associated cost allocation problem.

Part one includes a heuristic approach based on tabu search, tight linear programming relaxations, and a linkage between optimal and heuristic solutions. As a result of those studies the range of optimally solvable instances of NP-hard hub location problems was extended. In particular, well-known and heavily used benchmark data set of real world problems (Civil Aeronautics Board (CAB) data set), that has resisted efficient solutions for more than a decade, has been solved to optimality.

Part two is concerned with the formulation of the associated cost allocation problems and with the characterization of corresponding cost allocation solution concepts. The cost of services delivered through a hub network is distributed among

its users who may be individuals or organizations with possibly conflicting interests. The cooperation of these users is essential for the exploitation of economies of scale. Consequently, a need arises to find a fair distribution of the cost of providing the service among users of a hub network. In order to describe this cost allocation problem we formulate several associated cooperative games, to be referred to as the hub games, in which special attention is paid to users contribution to economies of scale. Then we investigate the structure and computation of the core and the nucleolus of hub games.

The presentation of each part concludes with the discussion of the respective recent research on hub-like networks.

Joint work with Morton O'Kelly (Ohio State University, USA), Hrvoje Podnar (State University of New York at Stony Brook, USA) and Jadranka Skorin–Kapov.

On Design Optimization in Optical Networks

Jadranka Skorin–Kapov

Abstract. New networking architectures (e.g., optical networks, ATM Virtual Path networking) and the requirements of nimbleness have created a large number of network design/configuration optimization problems, many of which fall in the category of NP-hard combinatorial optimization problems. Optimizing some design and usage characteristics of communication networks leads to considerable cost savings. Optical networks offer a huge bandwidth potential and are, therefore, suitable for architectures based on different types of multiplexing. By using wavelength division multiplexing, lightwave networks can provide logical connectivity, which is superimposed on the physical structure. (The assignment of wavelengths to receivers and transmitters defines the logical connectivity among nodes.) Because of changing traffic conditions, fast algorithms to rearrange logical connections are mandatory in order to optimize the use of network with respect to a given criterion. My talk will present the problem of finding the logical connection diagram and routing of flow which minimizes the maximal congestion on a link. A tabu search-based heuristic will be outlined. The approach of decomposing the problem into two subproblems (the connectivity and the routing problems) is used. The connectivity problem is a quadratic assignment problem (QAP) that seeks a 0/1 solution which maximizes the one hop path traffic, and the routing problem is a multicommodity flow problem. Comparisons will be made between arbitrary and regular connectivity topologies. Next, a genetic algorithm approach to a virtual topology design of a Wide–Area WDM optical network will be presented. The objective is to minimize the maximal throughput, which implies balancing the traffic loads on network links and accommodating on-growing traffic requirements in a timely fashion. Computational results on a benchmark data set will be presented.

Joint work with Jean–Francois Labourdette (ATT Laboratories, Holmdel, NJ, USA) and Hrvoje Podnar (State University of New York at Stony Brook, USA).

Quantum Mind Interactive Browser, QMIB

Branko Souček

Abstract. QMIB is based on a group of experiments in neural biology and behavior. QMIB explains the process of Verbal Understanding in human mind. Like the human mind, QMIB goes beyond the preprogrammed rules and creates always new associations. QMIB in an inspection network detects the faults which it has never seen before. Detection is based on a hierarchical decomposition of acquired experience, until a level of maintainable or replaceable items. When an unknown fault occurs, QMIB reuses the items, performs the discrete combinatorics and suggests the probable cause of fault. In a similar way, QMIB in knowledge mining and in problem solving reuses and combines the knowledge items and resolves some of the problems which it has never met before. QMIB carries many associative searches at the same time in such a way that it makes parallel search and computing feasible and efficient. QMIB offers its discrete combinatorics to the Distributed Object Computing, DOC. QMIB will introduce to DOC the selforganization and emerging association algorithms. This will lead to a new wave: a creative QMIB–DOC. QMIB will soon become the core of: decision making, browsing, research, communication, control, automation, and electronic systems. QMIB finds the applications in telecommunications, Internet, traffic, industrial production, financing, trade, tourism, creative EDI.

The brain performs an inexact compression of stories into a Verbal Quantum. The reconstructed story is different every time, though the meaning is preserved and new stories and thoughts are created. This compression emerges from the dynamic interactions between flocks of neural agents. The Reciprocal Adjustment, Self Organized Network, REASON, performs an iterative chaotic nested search: to recognize the story, it is necessary to know the words; to recognize the word, it is necessary to know the story. Which solution REASON finds, depends on the initial approximation. The massively parallel creative search bounces around before finally converging to a solution. This search supports the wandering between the stories and creation of new stories. Gradually, from the recursive, fractal search emerge the understanding, awareness and consciousness.

On Fock Representation of Braided Canonical Commutation Relations

Dragutin Svrtan

Abstract. In this lecture we shall explain recent results concerning the computation of Gram determinants (and inversion) on arbitrary weight spaces $X^{(\eta)}$ in the Fock representation of the multiparametric canonical commutation relations:

$$a_i a_j^+ - q_{ij} a_j^+ a_i = \delta_{ij}.$$

These types of problems are relevant to particle physics where one would like to study more general particles than bosons or fermions, which might violate the Pauli exclusion principle. The determinant evaluation is related to the Hilbert space realizability, and the inversion of Gram matrices of multiparticle states is related to the energy operator expansions.

The one-parameter case was studied by Greenberg, Zagier, Hanlon, Stanley and others. In the symmetric multiparametric case and multilinear weight case the determinants can be obtained as a special case of a formula of Varchenko applied to the arrangement of (diagonal) hyperplanes.

By a different method, which generalizes Zagier's approach, in a joint paper with Meljanac, we have obtained determinants for general multiparametric and multilinear weight case. In this case, we have also solved the inversion problem by enhancing a method of Božejko and Speicher. In the meantime, another approach emerged by Aguiar, who also treated weights $\eta = (n-1, 1)$ or $(n-2, 2)$. Finally, by using some properties of the Whitehouse modules we can handle the case of general weights.

Topology and Game Theory

Henryk Toruńczyk

Abstract. Topology has been present in game theory at least since von Neumann's use of Brouwer's fixed point theorem in his proof of the min-max theorem.

In my talk I'd like to concentrate on some recent results obtained jointly with Bob Simon (Göttingen) and Stanisław Spież (Warszawa), in which the existence of equilibria in games of incomplete information has been established by using topological results, resembling in some way Borsuk–Ulam theorem. In its simplest form one of the results states that if x_0 belongs to a compact set $C \subset \mathbb{R}^n$ and $f : C \to Y$ is a mapping such that the dimension of $f(\operatorname{int} C)$ is less than n, then in the boundary of C there exists a set C_0 mapped by f into a singleton and containing x_0 in its convex

hull. The setup actually needed is that of an acyclic set-valued mapping f, and then the existence of a finite set $C_0 \subset \partial C$ such that $x_0 \in \operatorname{conv} C_0$ and $\bigcap_{c \in C_0} f(c) \neq \emptyset$ is asserted.

Essential to our approach is the ability to establish handy homological conditions on a set $\Gamma \subset C \times Y$ implying that the level-wise convexification $c\Gamma$ of Γ projects onto a set containing C. In some special cases, sufficiently general from the point of view of game theory, homology can be removed completely and the conditions can be expressed in terms of the image of the projection of Γ separating a certain vertex of the cube Y from the union of the opposite faces of Y. The separation property, in turn, allows for establishing the existence of equilibria in a certain type of games by decomposing these games into more manageable pieces.

Secondary Structures

Darko Veljan

Abstract. In biological molecules such as RNA or DNA, certain combinatorial patterns often persist. Biologists call them secondary structures of given order $l \geq 1$. A secondary structure is a simple graph on vertices $1, 2, \ldots, n$ (say points on the x-axis) with two kinds of edges: all segments $[i, i+1]$, $1 \leq i \leq n-1$, and arcs (say in the upper half-plane). Any two arcs are disjoint and any two vertices joined by an arc must be separated by at least l consecutive vertices. Biologists call the vertices bases, segments p-bonds, arcs h-bonds and a structure without arcs trivial. Let $S^{(l)}(n)$ be the number of all order l structures on n vertices.

The main object of this work is to study secondary structures and, in particular, the behavior of the numbers $S^{(l)}(n)$. If we set $l = 0$ we get 2-cycles. This case is counted by Motzkin numbers which also count Motzkin paths. In fact, any l-structure corresponds to a Motzkin path (of the same size n) whose every plateau has length $\geq l$. The degenerate case is $l = -1$. These structures correspond to Motzkin paths without any plateau, that is Dyck paths, counted by Catalan numbers. The Narayana numbers count Dyck paths with given number of peaks.

We shall present explicit formulae for $S^{(l)}(n)$, recursions, generating functions, asymptotics, some log-convexity and unimodality questions, as well as some applications in geometry, some probabilistic estimates, etc.

Joint work with Tomislav Došlić.

Ruled Surfaces of E_n with Plane Orthogonal Trajectories of the Generators

Hans Vogler

Abstract. Not available.

CONTRIBUTED TALKS

Representations of Affine Lie Algebras Having Infinite Dimensional Weight Spaces

Dražen Adamović

Abstract. In this talk we will present our recent efforts in studying irreducible dense modules for affine Lie algebras with all infinite dimensional weight spaces based on the article D. Adamović, Mathematical Research Letters 1997, pp. 809–821. The connections with some aspects of representation theory of affine vertex operator algebras will also be discussed.

Extensions of Hilbert C^*–Modules

Damir Bakić

Abstract. The aim of the talk is to introduce the concept of an extension for Hilbert C^*-modules. Our approach depends on the properties of ideal submodules and morphisms of Hilbert C^*-modules over possibly different C^*-algebras. We show that there is a Hilbert C^*-module V_d over the multiplier algebra $M(A)$ which serves as a maximal essential extension of a given Hilbert A-module V. We also prove that V_d can be recognized as the strict completion of V with respect to a suitable strict topology defined on V.

Joint work with Boris Guljaš.

Young and H–Measures for Relaxation of Multi–Well Energies

Neven Balenović

Abstract. Non-convex energy functionals are known to be lacking minimisers in the usual sense, due to the fact that minimising sequences tend to oscillate rapidly aiming to satisfy imposed restrictions, thus exhibiting microstructures. Mathematical tools known to be able to give some information on microstructures are Young measures (introduced in the thirties by L. C. Young and applied in this context by L. Tartar in the seventies) and H-measures (L. Tartar 1990, and independently P. Gerard 1991). Particularly, in the case of the multi-well energy minimisation, we use the above tools to construct generalised minimisers and obtain good bounds, following the ideas of Kohn, Ball & James and Tartar. The relevance of these results for real materials will be discussed as well.

Joint work with Nenad Antonić and Marko Vrdoljak.

Isometric Invariants of Conics in Isotropic Plane. Classification of Conics.

Jelena Beban–Brkić

Abstract. A real affine plane A_2 is called an isotropic plane I_2, if a metric in A_2 is induced by an absolute $\{f, F\}$, consisting of the line at infinity f of A_2 and a point $F \in f$.

In this work the conditions for the canonical form of second order curve equation in an isotropic plane have been studied and all the exceptions have been discussed. The conic classification according to their invariants is made with regard to the group of motions on I_2. In this way 20 types of conics have been obtained.

The Baum–Welch Algorithm for Parameter Estimation of Gaussian Autoregressive Mixture Models

Thomas Benesch

Abstract. A *finite discrete Markov chain* consists of a finite set of states $\mathcal{S} = \{S_1, \ldots, S_N\}$ and is observed at discrete times $t = 1, \ldots, T$. We use the variable ξ_t as the state of the Markov chain at discrete time t.

$$a_{ij}(t, t+1) := \mathbb{P}(\xi_{t+1} = S_j \mid \xi_t = S_i)$$

is called the *transition probability* from state S_i to S_j at time t. If $a_{ij}(t, t+1) = a_{ij}$ for all t, i.e., the transition probabilities do not depend on t, we speak of a Markov chain with *stationary* transition probabilities.

In a *Hidden Markov Model*, the state ξ_t at time t cannot be observed directly, but there exists a set of possible observations $\mathcal{V} = \{V_1, \ldots, V_K\}$, $K < N$. Let ω_t denote the observation at time t. Being in state S_i at time t, an observation V_k is made with the probability

$$b_{ik} := \mathbb{P}(\omega_t = V_k \mid \xi_t = S_i).$$

In *Continuous Density Hidden Markov Models* the discrete probability function b_{ik} is replaced by the continuous density function $b_i(X)$, where X is an element of the continuous observation set that is d-dimensional Euclidean.

In *Autoregressive Hidden Markov Model* we assume a white noise source with unit variance, i.e., $\sigma^2 = 1$, followed by an all-pole filter $1/A(z)$, where

$$A(z) = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_p z^{-p}.$$

In this article we solve the problem of parameter estimation for Autoregressive Hidden Markov Model.

Collocation by Higher Order Tension Splines

Ivo Beroš

Abstract. Tension spline of order k is a function that, for a given partition $x_0 < x_1 < \cdots < x_n$, on each interval $[x_i, x_{i+1}]$ satisfies the differential equation

$$\left(D^k - \frac{p_i^2}{h_i^2} D^{k-2}\right) u = 0,$$

where p_i's are prescribed nonnegative real numbers.

Many articles deal with tension splines of order four applied to the collocation method for solving the singularly perturbed boundary value problem

$$(\mathcal{L}u)(x) = \varepsilon u''(x) + b(x)u'(x) + c(x)u(x) = f(x), \quad 0 \le x \le 1,$$

with boundary conditions $u(0) = u_0$, $u(1) = u_1$. The accuracy of considered approximations is $\mathcal{O}(h)$ or $\mathcal{O}(h^2)$ for small perturbation parameter ε, depending on the choice of collocation points.

Here we present an algorithm for the collocation method with high order tension splines.

Our objective is to obtain an approximation of higher order of accuracy for the solution of singularly perturbed differential equation. The proof of convergence for some classes of tension splines will be presented.

Joint work with Miljenko Marušić.

Projections of Inefficient DMU to Efficient Frontier

Valter Boljunčić

Abstract. Usually, when we apply DEA models, and the DMU under evaluation results as inefficient, we obtain a single efficient reference point which Pareto-dominates the DMU under evaluation. This point can also be interpreted as a target point on the frontier such that inefficient DMU should adopt its input/output mix to become efficient.

In this work we are interested in the part of the efficient frontier of production possibility set that can be used as a set of reference points for inefficient DMU. These points are obtained from DMU under evaluation by applying changes, which form a polyhedral cone, to its inputs and outputs.

We apply modified LP to obtain one projection point on the frontier of production possibility set along one of the edges of the cone of possible changes. Using the relationship between primal and dual program and applying parametric programming, where only RHS is changed, we obtain the desired subset of the efficient frontier.

On Efficient Solution of Linear Systems in Implicit Discretization of the Shallow Water Equation

Nela Bosner

Abstract. Numerical solution of partial differential equations via implicit discretization schemes often leads to linear systems of equations. The linear system to be solved is usually large and sparse and the methods of choice are the iterative methods. For an efficient computer solution one needs suitable solvers fitted to the special structure of the particular problem.

We study implicit discretization of the two-dimensional shallow water equation and numerical solution of the corresponding system of equations. Our goal is to design an efficient preconditioner for the system and develop a new solution procedure.

Our interest is both theoretical and practical. We will report the progress in theoretical analysis as well as the results of extensive numerical testings with recommendation which method is the optimal choice.

Some Mappings Associated with Hadamard's Inequalities for Convex Functions

Ilko Brnetić

Abstract. The following inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(s)\,ds \leq \frac{f(a)+f(b)}{2}$$

for a convex function f defined on the real interval is known as Hermite–Hadamard inequality. Some mappings closely connected with this inequality are introduced and their main properties are given. By using some of these properties new refinements of Hermite–Hadamard inequality are obtained.

Joint work with Charles E. M. Pearce (Applied Mathematics Department, University of Adelaide, Australia) and Josip Pečarić.

On a Characterisation of Hilbert C^*–Modules

Franka Miriam Brückler

Abstract. A Hilbert C^*-module is a generalised Hilbert space: the inner product is allowed to take values in an arbitrary C^*-algebra. For Hilbert spaces there is a metric characterisation, the parallelogram identity, which allows the inner product to be reconstructed from the norm. For Hilbert C^*-modules the situation is more complicated, but there is also a metric characterisation: D. P. Blecher has shown (1995) that a Hilbert C^*-module V over a C^*-algebra $\mathcal{A} \subseteq \mathbf{B}(\mathcal{H})$ is characterised by a Hilbert space $\mathcal{H}_V$, obtained as a tensor product, and a map ϕ allowing the Hilbert C^*-module to be embedded into bounded operators between the Hilbert spaces $\mathcal{H}$ and $\mathcal{H}_V$, such that $\phi(x)^*\phi(x) \in \mathcal{A}$ for all $x \in V$. In this case the inner product can be reconstructed via $\phi : \langle x \mid y \rangle = \phi(x)^*\phi(y)$ for $x, y \in V$.

Many of the properties of Hilbert spaces cannot be obtained for general Hilbert C^*-modules, e.g., the decomposition into orthogonal complements. A special class of Hilbert C^*-modules, for which many of the difficulties can be resolved, are Hilbert C^*-modules over C^*-algebras of compact operators on a Hilbert space. One of the nice properties of such modules, as D. Bakić and B. Guljaš have shown in 1999, is the following: if V is a Hilbert C^*-module over the C^*-algebra $\mathbf{K}(\mathcal{H})$ of all compact operators on a Hilbert space $\mathcal{H}$, then there is a Hilbert subspace V_e of V such that the C^*-algebra of all adjointable (with respect to the Hilbert C^*-module inner product) maps on V is isomorphic to the C^*-algebra of all bounded linear maps on V_e, via the restriction map. It turns out that this Hilbert space V_e is isometrically isomorphic to the Hilbert space from the Blecher theorem.

On Clifford Manifolds of Type $\mathcal{CL}_{0,3}$

Ilie Burdujan

Abstract. An almost hypercomplex structure of type $\mathcal{CL}_{0,3}$ on an $8n$-dimensional manifold M is a 6-tuple $H = (J_\alpha)$, $(\alpha = \overline{1,6})$ of anticommuting almost complex structures $J_\alpha : TM \to TM$ satisfying the identities of $\mathcal{CL}_{0,3}$, i.e.,

$$(1, J_1, \dots, J_6, J_7 = J_1 \circ J_6)$$

respects the multiplication table of $\mathcal{CL}_{0,3}$. An almost Clifford structure of type $\mathcal{CL}_{0,3}$ on M is a rank-7 subbundle $Q \subset \mathbf{End}(TM)$ which is locally or globally spanned by almost hypercomplex structures $H = (J_\alpha)$. Consequently there exists a reduction of the structural group of the principal frame bundle of M to $GL(n, \mathcal{CL}_{0,3}) \cdot (Sp_1 \times Sp_1)$ or $GL(n, \mathcal{CL}_{0,3})$, respectively. Using the prolongation of the Lie algebra g corresponding to each such a Lie group, one can define the Spencer cohomology

$H^{s,k}(g)$. Then the corresponding obstructions to integrability are either only in $H^{0,2}(g)$, $H^{1,2}(g)$, $H^{2,2}(g)$ or in $H^{0,2}(g)$, $H^{1,2}(g)$, respectively.

The set of all almost Clifford connections is determined.

The Kähler–Clifford manifolds are characterized by a non-zero harmonic $-form. Consequently if such a manifold is compact some information about its Betti numbers is obtained.

Morphisms and Quasinorms over Free Semigroups

Alexandru Cărăuşu

Abstract. The object of this paper is to investigate some properties of certain types of morphisms and operators defined on a free semigroup Σ^+ (or free monoid $\Sigma^\star = \Sigma^+ \cup \{\lambda\}$), in connection with some measures associated to parts thereof, that is to formal languages.

If Σ and Δ are two alphabets, a mapping $f : \Sigma^\star \to \Delta^\star$ is a homomorphism of free monoids if $(\forall u, v \in \Sigma^\star)\ f(uv) = f(u)f(v)$. Any homomorphism maps a language $L \subseteq \Sigma^\star$ on $f(L) \subseteq \Delta^\star$. An operator on the set of languages over the same alphabet Σ will be denoted by F. Some operators can be realized in terms of automata, transducers, etc.

A family $\mathcal{L}$ of formal languages may be endowed with a structure of vector space under the linear operations of "addition" (in fact, the disjoint union) $L_1 \dot{+} L_2 \triangleq (L_1 \cup L_2) \setminus (L_1 \cap L_2)$ and "multiplication by scalars" in the $\{0,1\}$ field, $0L \triangleq \emptyset$ & $1L \triangleq L$. If $u = a_{i_1} a_{i_2} \dots a_{i_m}$ is a word in $L \subset \Sigma^\star$, its length is $\ell(u) \triangleq m$. A way to define a norm of a language was proposed as $\|L\| \triangleq 2^{-\nu(L)}$, where $\nu(L) = \min\{\ell(u) : u \in L\}$. There are grounds for considering other types of norms or quasinorms, able to take into account the way L is generated/accepted by a formal device. We propose a couple of norms/quasinorms for languages generated by a formal grammar $G = (N, \Sigma, S, P)$ which take into account the structure of the words in L and their derivational complexity (using the notion of index). The definitions are too technical for being given in this abstract. The norms thus defined allow to classify operators on families of languages, e.g., to introduce bounded and contractive operators.

Construction of Symmetric Designs Admitting Automorphisms of Order $p \cdot q$

Dean Crnković

Abstract. Construction of symmetric (v, k, λ) designs admitting some automorphism group G consists of two steps. The first step is to find all orbit structures for parameters (v, k, λ) and the group G. The next step, called indexing, is to

determine which points are incident with the representatives of block orbits. We shall describe the construction of symmetric designs with automorphism groups $G \cong Z_p \times Z_q$, where p and q are primes. This method of construction is based on the use of principal series of these groups. The main idea is to decompose orbit structures for the group G to the corresponding orbit structures for the group Z_p, and then proceed with indexing of those decomposed structures, having in mind the action of the group Z_q on the designs. In each phase of the construction it is necessary to use elements of a normalizer to eliminate mutually isomorphic structures. Using this method, a number of symmetric designs are constructed, for example 2–$(71, 15, 3)$, 2–$(69, 17, 4)$ and 2–$(61, 16, 4)$ designs with an automorphism of order 6 and 2–$(66, 26, 10)$ designs with an automorphism of order 55.

Joint work with Sanja Rukavina.

On Properties of Rectangular Hyperbolas

Zvonko Čerin

Abstract. This work improves an old theorem about a rectangular hyperbola $\mathcal{H}$: its centre lies on the pedal circle of any point on $\mathcal{H}$ with respect to any triangle inscribed in $\mathcal{H}$. We also prove that an analogous result holds for Cevian circles. These results are used to obtain new characterisations of the Feuerbach, Jarabek, and Kiepert hyperbolas of a triangle.

Proper Shape Theory for C^*–Algebras

Zvonko Čerin

Abstract. We shall describe the proper shape category for arbitrary C^*-algebras and extend several results of Ball and Sher to the noncommutative case.

On Inequalities of Levin Type for Multidimensional Balls

Aleksandra Čižmešija

Abstract. In this talk a new approach, inspired by some results of V. Levin, to inequalities of Cochran–Lee type for multidimensional balls centered at the origin will be presented. We obtain a class of more general relations involving weighted geometric means with radial weight functions. The companion results will also be derived. Moreover, sharpness of all obtained inequalities will be discussed.

A Distributional Identity for Exchangeable Random Walks

Milica Čudina

Abstract. The aim of the talk is to give another perspective on proving a distributional identity for certain random variables related to chains with exchangeable increments.

The approach to this result, which is originally a corollary of the Arcsine law, is purely combinatorial. Therefore, certain chain-transformations are developed, and their characteristics are used in order to obtain the proof.

How to Make Paradoxes in Set Theory

Boris Čulina

Abstract. Frege's naive set theory is based on an assumption that for every property there is a set of all objects with this property. But, for some properties this assumption leads to a contradiction. Modern set theory resolved this problem in a pragmatic way, by postulating enough sets for mathematical purposes, but, it is believed, not too many, to avoid contradictions. So, there is no contradiction, but the paradox remains — why certain sets do not exist.

Sentence which claims that some property P doesn't have the associated set can be reformulated in a logically equivalent sentence which gives some logical understanding why it is so. It says that the property has a creative choice on it. It means that for every set of objects with property P there is an object with property P outside the set. Among other things it gives instructions how to make paradoxes — by investigating set operations we are looking for properties with a creative choice on them. Old paradoxes are investigated in this way, some new are made, and some general results are exhibited.

The Unified Treatment of Some Inequalities of Ostrowski and Simpson Types

Vera Čuljak

Abstract. Recently, a variety of inequalities of Ostrowski and Simpson types have been obtained with assumptions only on f and its first derivative. Here we shall give a unified treatment of these, deriving new generalizations and subsuming a variety of

known results as particular cases. We consider inequalities for lipschitzian functions, functions of bounded variation and functions with L_p derivative.

We denote $I^* : a \le t_1 < t_2 < t_3 \le b$,

$$R(f, I^*) = \int_a^b f(x)\,dx - [(t_1 - a)f(a) + (b - t_3)f(b) + (t_3 - t_1)f(t_2)].$$

Our basic result for functions with first derivative belonging to $L_p[a, b]$ is as follows.

Theorem. *Let $f : [a, b] \subseteq R \to R$ be differentiable. If $\nu > 1$, $\dfrac{1}{\nu} + \dfrac{1}{\mu} = 1$ and $f' \in L_\nu[a, b]$, then we have the inequality*

$$|R(f, I^*)| \le \left[\frac{(t_1 - a)^{\mu+1} + (t_2 - t_1)^{\mu+1} + (t_3 - t_2)^{\mu+1} + (b - t_3)^{\mu+1}}{\mu + 1}\right]^{1/\mu} \|f'\|_\nu.$$

Joint work with Charles E. M. Pearce (Applied Mathematics Department, University of Adelaide, Australia) and Josip Pečarić.

Some Results on Population Dynamics Modelling with Diffusion in an n–Patch Habitat

Joydip Dhar

Abstract. Biotic populations are usually distributed heterogeneously in their habitat, and the distribution itself is patchy. The patchiness also occurs due to geographical and topographical changes in the habitat. In modern times, due to various environmental factors such as industrialization, pollution, deforestation, etc., the animal populations are affected all around the world. In this paper, a single species dynamic model with continuous diffusion in an n-patch environment is proposed by assuming the carrying capacities are different in different patches. This model is analyzed both analytically and numerically. It is shown, both analytically and numerically, that there exists a positive, piece-wise monotonic, continuous steady state solution with continuous flux at the interface, in the case of reservoir boundary conditions, that is asymptotically stable in the both local and global cases under certain conditions. All the above results are also true for no-flux (closed habitat) boundary conditions. In the case of uniform equilibrium state it is globally asymptotically stable without any condition.

Minding's Isometries of Ruled Surfaces in Galilean and Pseudo–Galilean Space

Blaženka Divjak

Abstract. The Galilean and pseudo-Galilean geometry are the real Cayley–Klein geometries (of projective signature $(0,0,+,+)$ and $(0,0,+,-)$). The absolute of the Galilean (pseudo-Galilean geometry) is an ordered triple $\{\omega, f, I\}$, where ω is the ideal (absolute) plane, f line in ω and I is the fixed elliptic (hyperbolic) involution of the points of f.

There are three types of regular ruled surfaces in pseudo-Galilean space G_3^1, similarly to the Galilean case G_3^1. For any such surface a "Drall" and an associated trihedron are determined. In the same way as in the theory of curves in G_3 and G_3^1, the curvature and torsion for each type can be defined and associated derivative equations obtained.

Minding's isometry is such an isometric transformation between ruled surfaces Φ and Φ^* that transforms the direction field of Φ into the direction field of Φ^*. The Minding's isometries of all three types of ruled surfaces in G_3 and G_3^1 are described.

An Absolute Bound for the Size of Diophantine m–Tuples

Andrej Dujella

Abstract. A set of m positive integers $\{a_1, a_2, \ldots, a_m\}$ is called a Diophantine m-tuple if $a_i \cdot a_j + 1$ is a perfect square for all $1 \leq i < j \leq m$. The first Diophantine quadruple, $\{1, 3, 8, 120\}$, was found by Fermat. A famous conjecture is that there does not exist a Diophantine quintuple. There is an even stronger version of this conjecture, namely that if we fix a Diophantine triple $\{a, b, c\}$, then there is a unique positive integer d such that $d > \max\{a, b, c\}$ and $\{a, b, c, d\}$ is a Diophantine quadruple.

We are able to prove this stronger conjecture for a large class of Diophantine triples, namely for triples satisfying some gap conditions like $b > 4a$, $c > \max\{b^{13}, 10^{20}\}$ or $c > \max\{b^5, 10^{1029}\}$.

This result allows us to give an absolute bound for the size of Diophantine m-tuples. More precisely, we can prove that there does not exist a Diophantine 9-tuple and that there are only finitely many Diophantine 8-tuples.

Derivation of the Reynolds Type Equation with Memory Effects

Antonija Duvnjak

Abstract. We consider the lubrication process of slipper bearings. Starting from the nonstationary Stokes equations in a thin 3-dimensional domain and using the asymptotic expansion we obtain the Reynolds type lubrication equation with memory effects governing the 2-dimensional macroscopic flow. The law is nonlocal in time and gives a linear connection between velocity and pressure gradient. We present a convergence theorem for velocity and pressure in appropriate functional spaces, as the characteristic size of bearings tends to zero.

Linear Partial Differential Equations of Second Order in the Aleph Derivative with Variable Coefficients

Sergio Falcon

Abstract. From the generalized increment of a function, we find the Aleph derivative as a generalization of the classic derivative. Using this generalized derivative we obtain and solve any differential equation in a similar way to the classic differential equation. If we transform these equations in the Aleph derivative to equations in the classic derivative, we obtain expressions that are much more complex than the corresponding classic equations. In the same way, we can find the Aleph partial derivatives of a function of several variables, leading to the Aleph partial differential equations. In this paper we study the linear partial differential equations of second order in the Aleph derivative with variable coefficients and solve them in those cases in which it is possible.

Variation of an Operator Related to a Set Quantity

Sergio Falcon

Abstract. In this paper, using a set quantity defined in the family of Banach spaces, we associate with each operator $T \in \mathcal{L}(E, F)$ a number related with this set quantity in the same way as we define the Hausdorff or Kuratowsky noncompactness measure of an operator. Then, we analyse some properties of this variation with respect to the definition of an ideal of operators between Banach spaces. Finally,

we will prove a theorem which generalizes the result obtained by K. Astala for the ideal of compact operators and for the ideal of weakly compact operators.

Joint work with Kishin Sadarangani (University of Las Palmas de G.C., Spain).

On the Notion of Restriction in the Partial Lambda Calculus

Svitan Gaborovič

Abstract. The notion of restriction arises naturally when treating partial functions. It is formulated as an equational theory. Especially interesting is the notion of restriction in the scope of the partial lambda calculus where it is an important part of the syntax.

The partial lambda calculus was defined by Moggi in his thesis. Moggi proved that β_p reduction is confluent on the set of $p\lambda$-terms. The $\beta_p\eta_p$-reduction is confluent modulo an equivalence and congruence relation between terms which is induced by the theory of restrictions (denoted $\sim_\lceil$). Pérez proved that the theory of restriction is decidable and conjectured the confluence of β_p-reduction modulo this relation on the whole set of λ_p-terms.

Our intention is to study the notion of restriction in the scope of the partial lambda calculus. The idea is as follows. We impose directionality in the use of $\sim_\lceil$ by defining a relation of reduction (denoted $\underset{\lceil}{\rightarrow}$) such that $\sim_\lceil$ is a reflexive, symmetric, and transitive closure of it. Although $\underset{\lceil}{\rightarrow}$ is not a term rewriting, it has some nice properties making it a useful tool for investigating the syntax of the partial lambda calculus. The results we get cast some light to the proof of the Pérez's conjecture.

About 2–Partitions of Hartmanis

Radoslav Galić

Abstract. In this paper 2-partitions of Hartmanis are described by one class of hypergroupoids with division, whereby the connection between systems of Steiner's triplets and idempotent TS-quasigroups is generalized.

Joint work with Janez Ušan (Institute of Mathematics, University of Novi Sad, Yugoslavia).

Stabilization of Distribution of Markov's Process in Wide Sense

Sergey N. Gerasin

Abstract. The problems about focusing and stabilization of Markov's process states probabilities in wide sense are discussed in this report. Suppose that a set of elementary results is $\Omega \subseteq R^n$. The set of events consists of all Borel sets $B_i \in \Omega$ everywhere dense in Ω. On all sets B_i a probability measure is defined. Focusing of probability is such an evolution of the process in which probabilities reach the given values during the final period of time. If in the final period of time, the probabilities of states are localized in the small neighbourhood of components of the given distribution, then we can speak about stabilization of the process. Let's consider stochastic matrix of the process $P_{st} = \|P_{st}(B_i, B_j)\|$, whose elements are probabilities of transition between the state B_i of the process at time s and the state B_j at the moment t. Focusing and stabilization at the moment t_0 may be realized if some specially chosen elements of the matrix P_{st} are strongly perturbed (all matrix elements or a part of them).

This perturbation must provide an implementation of the following conditions:

1. There exist a sequence of indices j_k and a sequence $\{s_k\}_{k=1}^{\infty}$, $s_k \uparrow t_0$, $s < s_k$, such that

$$\sum_{k=1}^{\infty} \delta_{j_k}(s_k, s_{k+1}) = \infty, \quad \delta_{j_k}(s_k, s_{k+1}) = \inf_i P_{s_k s_{k+1}}(B_i, B_{j_k}).$$

2. A sequence of eigenvectors $\vec{P}(t)$ of the matrix P_{st}, $\vec{P}(t)P_{st} = \vec{P}(t)$, has a limit

$$\lim_{t \uparrow t_0} \vec{P}_{st} = \vec{P}_{t_0}.$$

Knowledge and Expectation Formulae for Parsing

Paola Glavan

Abstract. The mechanism of *knowledge* and *expectation formulae* can provide adequate syntactic analysis of German text, with smooth extension to at least elements of semantic analysis. It can be efficiently implemented, in the form of a running parser-generator, and integrated with the lexical analyzer being developed. It can be completely formally specified using distributed *Abstract State Machines* (ASM), and the correctness of the implementation with respect to formal specification can be proved.

As the dynamics of knowledge and expectation formulae is inherently distributed, it suggests connections to other formalisms, such as the *Chemical Abstract Machine* (CHAM) and versions of *Linear Logic*. Also, existence of the symmetries between expectation formulae suggests optimizations for parsing strategies.

By investigating the symmetries of expectations and the structure of the dynamics of verifying knowledge and expectation formulae for concrete nontrivial fragments of German grammar, we expect to achieve at least these two goals:

1. Derive complexity estimates and efficient strategies for both sequential and distributed parsers, verifying the optimizations introduced by purely logical means;
2. Relying on such structures, investigate the relations to formalisms such as CHAM, version of Linear Logic, Lambek calculus, leading to their eventual novel applications.

The Classification of the Pedal Surfaces of $(1,2)$ Congruences and their Construction with Mathematica 4.0

Sonja Gorjanc

Abstract. In the extended Euclidean space $\overline{E^3}$ the congruences of 1st order, 2nd class have been classified according to the section with the plane at infinity. In this way eight types of these congruences have been obtained.

The properties of the pedal surfaces of such congruences with the finite pole P have been studied. For the congruences of the type I–V the pedal surfaces are of 4th order, contain the absolute conic and double straight line. For the types VI–VIII pedal surfaces break up into the 3rd order surface and the plane at infinity. Basic classification is made according to the number of their real straight lines. Further classification is made according to the number and kind of singular points.

The methods of synthetic and analytic geometry have been used.

The pictures of the surfaces have been constructed with Mathematica 4.0. They are obtained by using standard packages and make relatively sophisticated shapes easier to see. For the visualisation of synthetically obtained results the animation with Mathematica 4.0 is used.

Accuracy of Kogbetliantz Method

Luka Grubišić

Abstract. The known two-sided diagonalization method for computing the singular value decomposition of triangular matrices has been thoroughly analyzed in the

last two decades. Lately, the research concentrated on the quadratic convergence of scaled iterates and on the adaptation of the method for parallel computers. However, one problem has remained open: the relative accuracy of the method. The purpose of this communication is to present recent numerical investigation of the accuracy of the method.

Joint work with Vjeran Hari.

Pullback and Pushout Constructions for Hilbert C^*–Modules

Boris Guljaš

Abstract. We introduce pullback and pushout objects in the category of Hilbert C^*-modules. Both constructions are performed using the appropriate notion of morphism of Hilbert C^*-modules and the quotient structure over an ideal submodule.

Collocation Methods with Spline in Tension for Singularly Perturbed Volterra Integro–Differential Equations

Vilmoš Horvat

Abstract. The collocation methods with spline in tension are considered for singularly perturbed Volterra integro-differential equations (VIDE)

$$\varepsilon y'(t) = f(t, y(t)) + \int_0^t k(t, s, y(s))\, ds, \quad t \in I := [0, T],$$

with $y(0) = y_0$, where ε is a small parameter satisfying $0 < \varepsilon \ll 1$, $f : I \times \mathbf{R} \to \mathbf{R}$ and $k : S \times \mathbf{R} \to \mathbf{R}$ (with $S = \{(t, s) : 0 \le s \le t \le T\}$) are given functions. Under appropriate conditions on f and k, for every $\varepsilon > 0$ the above equation has a unique continuous solution on $[0, T]$. The well-known polynomial collocation methods are adequate only for $\varepsilon \approx 1$. A new collocation method is developed to approximate the solution of singularly perturbed VIDE for $0 < \varepsilon \ll 1$. The convergence results for the new method are given and its efficiency is illustrated by a few examples.

Stochastically Stable One–Step Approximations of Solutions of Stochastic Ordinary Differential Equations

Roža Horvat Bokor

Abstract. A series of numerical methods (schemes) for solving stochastic ordinary differential equations (SODEs) were developed. Under appropriate conditions the numerical solutions converge in mean square sense to the true solution. There is a general belief, supported by theorems for the Euler–Maruyama and Milstein methods, that the above statement implies the stochastic stability, i.e., the continuous dependence on initial values of the numerical solutions. In this talk this assertion is rigorously proved for methods widely used, but not investigated before from this aspect.

Problems in Embedding Theory of 2–Polyhedra

Krešimir Horvatić

Abstract. An embedding of a (compact) polyhedron is a PL injection of it into a given ambient space. Every n-polyhedron embeds into the Euclidean space E^{2n+1}, and, in general, the result is the best possible. Shapiro and Wu constructed an obstruction $Ob(P)$ for the embedability in spaces of lower dimensions and showed that, for $n \neq 2$, an n-polyhedron P embeds into E^{2n}, if and only if $Ob(P)$ vanishes. The condition is hard to verify in concrete situations.

Instead, a lot of sufficient conditions were found. Among them, there are two, which are independent. If $n \neq 2$, $H^n(P - \operatorname{Int} A) = 0$ (where A is an n-simplex in a triangulation of P; Horvatić) or $H^n(P) = \mathit{cyclic}$ (Rees; Wilson) imply embedability of n-polyhedron into E^{2n}. These theorems cover all other known results, as special cases.

The dimension $n = 2$ is an exception, because this case is $2n = n + 2$, i.e., the double dimension coincides with codimension 2, in which nonstandard phenomena in geometric topology (e.g., knots) appear. Freedman, Kruchal and Teichner (1994) found a polyhedron P (with 14 vertices!) showing, that $Ob(P) = 0$ is not a sufficient condition in the Shapiro–Wu theorem, when $n = 2$. On the other hand, Kranjc saved the Reed–Wilson theorem, in this dimension, too. So, natural questions arise:

Problem 1: Is $H^2(P) = \mathit{cyclic}$ group also necessary for the embedability of a 2-polyhedron P into E^4?

Problem 2: Is $H^2(P - \operatorname{Int} A) = 0$ a necessary and sufficient condition for a 2-polyhedron P to be embeddable into E^4?

Accelerating Bidiagonal Divide and Conquer by Fast Multipole Method

Ivana Hunjet

Abstract. Bidiagonal matrices arise when one computes the singular value decomposition (SVD) of a general matrix by first reducing it to a bidiagonal form. Currently, the fastest method for computing all singular values and singular vectors of a bidiagonal matrix of order $n \geq 25$ is the divide-and-conquer algorithm (BDC). BDC computes all singular values in $O(n^2)$ flops and all singular vectors in $O(n^3)$ flops. Using the fast multipole method (FMM), invented by Carrier, Greengard and Rokhlin for the problem of computing mutual forces on n electrically charged particles, BDC can be accelerated to compute all the singular values in $O(n \log_2 n \log_2^2 \epsilon)$ and all the singular values and vectors in $O(n^2 \log_2^2 \epsilon)$ flops.

Spine Based Construction of Column–Convex Polyominoes

Goran Igaly

Abstract. A unit square with all vertices at integer points in Cartesian plane is called a cell. A polyomino is a finite connected union of cells whose interior is also connected. A column-convex (or vertically convex) polyomino is a polyomino in which all intersections with vertical lines are connected.

We will construct a regular expression for coding the set of all column-convex polyominoes. This construction is based on the spine of a polyomino — the thin column-convex polyomino situated inside the original polyomino. As one step of this construction, we will show one useful bijection on the collection of $\{-1, 0, 1\}$-walks on y-axis that preserves the length of a walk and the number of its zero-steps.

Joint work with Dragutin Svrtan.

Extension Shape Theory

Ivan Ivanšić

Abstract. We develop a theory of P-shape for the class of compact Hausdorff spaces and for certain CW-complexes P. The complexes P which are allowed are those which, for each given weight α, admit a P-invertible map of a compact Hausdorff space of weight $\leq \alpha$ and of extension dimension $\leq P$ onto the Tychonoff cube I^α. In particular, it will be seen that the classical shape theory comes from the case $P = \{pt\}$.

Our concept is based on extension theory, and hence, for any extension equivalent CW-complex P', P-shape and P'-shape will be identical. Indeed, if the extension theories of P and P' are related so that $P \leq P'$, then we shall obtain a relation between their shape functors, one factoring through the other.

Joint work with Leonard R. Rubin (University of Oklahoma, USA) and Philip J. Schapiro (University of Oklahoma, USA).

Varieties of Groupoids with Axioms of the Form $x^{m+1}y = xy$ and/or $xy^{n+1} = xy$

Biljana Janeva

Abstract. The subject of this paper are varieties $\mathcal{U}(M,N)$ of groupoids defined by the following system of identities:

$$\{x^{m+1}y = xy : m \in M\} \cup \{xy^{n+1} = xy : n \in N\},$$

where M, N are sets of positive integers such that $M \cup N \neq \emptyset$. The equation $\mathcal{U}(M,N) = \mathcal{U}(M',N')$ for any given pair (M,N) is solved and, among all solutions, one called canonical, is singled out. Applying a result of Evans ("The Word Problem for Abstract Algebras", J. London Math. Soc., 26 (1951)), it is shown that, if M and N are finite nonempty and $\gcd(M) = \gcd(M \cup N)$, or exactly one of the sets M, N is nonempty and finite, then the word problem is solvable in $\mathcal{U}(M,N)$.

Joint work with Ǵorgi Čupona (Macedonian Academy of Sciences and Arts, Skopje, Macedonia) and Naum Celakoski (Faculty of Mechanical Engineering, Skopje, Macedonia).

Synchronization in Chaotic Systems

Krešimir Josić

Abstract. The problem of synchronizing chaotic systems has received much attention in the last decade. In this lecture I will give an overview of rigorous results that have recently been obtained in the area. The phenomenon of phase synchronization which already attracted the attention of Huygens will receive particular attention. A theoretical explanation of the phenomenon of chaotic phase synchronization based on Kuramoto's work will be proposed. The analytic predictions of this theory were tested in an electric circuit exhibiting a Rössler-like chaotic attractor and good agreement was found. This talk will be accessible to graduate students in the physical and mathematical sciences.

Joint work with Douglas J. Mar (Boston University, USA).

Partial Linearization of One Class of the Nonlinear Total Least Squares Problem by Using the Inverse Model Function

Dragan Jukić

Abstract. In this paper we consider a special nonlinear total least squares problem, where the model function is of the form $f(x; a, b) = \phi^{-1}(ax + b)$. Using the fact that, after an appropriate substitution, the model function becomes linear in parameters, and that the symmetry preserves the distances, this nonlinear total least squares problem can be greatly simplified. In this paper we give the existence theorem, propose an efficient parameters searching algorithm and give some numerical examples.

References

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Joint work with Rudolf Scitovski.

Homogenization of Convection–Diffusion Equation. Case of High Peclet Number

Mladen Jurak

Abstract. We consider convection-diffusion equation with spatially periodic coefficients. The period ε is a small quantity and the local Peclet number is taken to be of the order $O(1)$ when ε tends to zero. We homogenize the equation with respect to spatial oscillations and we make numerical comparison of the solutions of homogeneous and non-homogeneous equations.

k–Domination Numbers of Cardinal Products

Antoaneta Klobučar

Abstract. We represent k-domination numbers γ_k, $k \geq 1$, of the cardinal products of paths $P_m \times P_n$, $1 \leq m \leq 2k+2$, and cycles $C_m \times C_n$, $3 \leq m \leq k$. We will also give

$$\lim_{n\to\infty} \frac{\gamma_k(P_m \times P_n)}{mn}.$$

Heaps and Corresponding Algebraic and Geometric Structures

Zdenka Kolar–Begović

Abstract. This talk deals with a ternary algebraic structure called a heap and its properties. We investigate the connection between a heap (a laterally commutative heap) and corresponding algebraic and geometric structures such as groups, Desargues quasigroups, parallelogram quasigroups, Desargues systems, parallelogram spaces, medial Mal'cev functions, TST-spaces. Some properties of the ternary operation are illustrated on the specific example of a laterally commutative heap in an affine plane.

A New Steiner $S(2,5,41)$ Design

Vedran Krčadinac

Abstract. We report on a complete classification of Steiner 2-designs $S(2,5,41)$ with automorphisms of order 4. There are five such designs, all of which had been previously constructed. However, the search produced a new $S(2,5,41)$ with full automorphism group of order 2. Thus, the number of known $S(2,5,41)$ designs is now 15.

On Some Problems of Stochastic Analysis of Wiener Integrals Constructed by Fractional Brownian Motions

Yuriy Krvavych

Abstract. We solved three problems.

Problem 1: The differentiability of fractional integral $\Phi = \int\limits_0^t \phi(t,s)\,ds$, with kernel $\phi(t,s) = K_{H_0}(t,s)\int\limits_0^s \alpha(u)\,dB_u^H$, where B_s^H is a fractional Brownian motion (FBM) with the Hurst index $H \in \left(\frac{1}{2},1\right)$ and $K_{H_0}(t,s) = (t-s)^{\frac{1}{2}-H_0}\beta\left(\frac{s}{t}\right)$, $0<s<t$, $H_0 \in \left(\frac{1}{2},1\right)$. For this problem the mean-square differentiability and path differentiability conditions of the fractional integral Φ are obtained. These results are used in the proof of the Girsanov theorem for FBM and diffusion processes constructed by FBM.

Problem 2: Maximal inequalities for the moments of the Wiener integral $I_t = \int\limits_0^t f(s)\,dB_s^H$, $t>0$, where f is a deterministic, measurable, positive function that

satisfies the condition $\int_0^\infty \int_0^\infty f(s)f(t)|s-t|^{2H-2}\,ds\,dt < \infty$. The upper and lower maximal estimations for the moments of the Wiener integral I_t are established. They are applied to the solutions of stochastic differential equations involving FBM.

Problem 3: The presence and absence of arbitrage conditions on three types of (B, S)–market.

For the first type, which is defined as a "fractional model" of the (B, S)–market, the absence of equivalent martingale measure is proved. Further, the self-financing portfolio that represents the amount invested in the stock allows arbitrage opportunity on the (B, S)–market of the considered type.

In the second case, for modified "fractional model" of the (B, S)–market, we proved that our (B, S)–market is an arbitrage-free market.

For the third type of the (B, S)–market with stock price process

$$S_t = \exp\left(\int_0^t h(t-s)\,c(s)\,dW_s\right),$$

we proved that the (B, S)–market is an arbitrage-free market.

Calculating the Real Roots of a Rational Polynomial with Exact Real Arithmetic

Marko Krznarić

Abstract. The trisection algorithm, a generalization of the bisection technique adopted to exact real arithmetic, computes, with any given accuracy, any simple, real root of a function, implemented in exact arithmetic. By using a perturbation method and the well-known Sturm's sequence, we show that the trisection algorithm can, in fact, be used to obtain all real roots, including those with multiplicity, of any rational polynomial to any degree of accuracy.

Joint work with Abbas Edalat (Department of Computing, Imperial College, London, UK).

Some New Quartics

Kroacija Kučera

Abstract. On the authority of V. Niče's geometric quasi-zero function, some new curves of order 4 have been discovered. These curves are all unbounded and disconnected.

The first group of curves possesses 3 singular points in the finiteness, which are 2 nodes and 1 isolated node. The curves from this group seem to be the first known curves of fourth order (quartics) with such a property.

The second group contains curves with 3 singular points as well: one node and one isolated node in the finiteness and one node in the infiniteness. Each curve of this group has 2 asymptotes.

Any curve from these two described groups has 1 axis of symmetry.

Curves in the third group are all entirely irregular, but have 3 singular points in the finiteness, just like curves from the first group.

Optimized Versions of a Distributed Algorithm for Solving Path Problems

Robert Manger

Abstract. Path problems are a family of optimization and enumeration problems that reduce to generation or comparison of paths in graphs. Some well-known examples are: checking path existence, finding shortest or most reliable paths, listing all paths, etc. In our previous work we have already proposed a distributed algorithm for solving path problems. In this paper we present three optimized versions of the same algorithm. The new versions are faster than the original algorithm, but they are applicable only to certain instances of problems, i.e., to undirected, acyclic, and sparse graphs, respectively. We report on experiments, where the three versions have been implemented with the PVM package and evaluated on randomly generated problem instances.

Joint work with Goranka Nogo.

On Iterated Inverse Limits

Sibe Mardešić

Abstract. It is well known that every compact Hausdorff space is the limit of an inverse system of compact ANR's (polyhedra). If X is the limit of a (cofinite) inverse system $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of compact Hausdorff spaces X_λ and every X_λ is the limit of an inverse system $\boldsymbol{Y}_\lambda = (Y_\lambda^\mu, q_\lambda^{\mu\mu'}, M_\lambda)$ of compact ANR's (polyhedra), it is natural to ask if one can organize the collection $\{Y_\lambda^\mu, \lambda \in \Lambda, \mu \in M_\lambda\}$ of ANR's in such a way that one obtains an inverse system whose limit is X. This question was raised by Yu. T. Lisica during the Dubrovnik 1998 Topology conference.

Lisica's question has a negative answer. This can be seen by taking a compact Hausdorff space X with $\dim X = 1$ and $\operatorname{ind} X = 2$ and a system $\boldsymbol{X}$ of metric

compacta of dimension 1 with $\lim \boldsymbol{X} = X$ and by expressing each X_λ as the limit of a sequence of 1-dimensional compact polyhedra. Nevertheless, for every cofinite system $\boldsymbol{X}$ there exist systems $\boldsymbol{X}_\lambda$ of compact ANR's with limit X_λ, whose terms can be organized in a system with limit X. Using resolutions instead of limits, this positive result generalizes to arbitrary topological spaces X. The results of this talk are contained in a forthcoming joint paper with N. Uglešić.

Surface Fitting Using the Moving Total L_p Approximation

Tomislav Marošević

Abstract. We look at the problem of generating a surface on the basis of the given data (x_i, y_i, z_i), $i = 1, \ldots, m$. When the errors appear in all data variables, in general, it is suitable to use the so-called moving total L_p approximation ($1 \leq p \leq \infty$). Particularly, if $p = 2$ we have the moving total least squares method, and for $p = 1$ the moving total least absolute deviations.

The surface can be locally approximated by planes such that the weighted L_p distances from the points to the plane are minimized in the sense of the total least L_p-norms. Then a global approximant is obtained by means of a certain combination of local approximants, with a possible use of appropriate weight functions. We also consider some examples.

Joint work with Rudolf Scitovski.

Application of Tension Splines to Singularly–Perturbed Boundary Value Problems for ODE

Miljenko Marušić

Abstract. Besides applications in the shape preserving approximations, tension splines also found their role in the numerical solutions of the boundary value problems for the ODE. We consider a problem of the form $\varepsilon y'' + py' + qy = f$. It is known that classical polynomial methods fail when the perturbation parameter ε is small. Reason for this is exponential behaviour of the solution on the boundary layer. Using tension splines, i.e., functions that are piecewise combination of linear (polynomial) and hyperbolic functions, we obtain a method that approximates the solution well, even in the case when the perturbation parameter is small.

Here we present different collocation methods for numerical solution of singularly perturbed BVP for ODE. Theoretical and computational aspects of presented methods will be discussed. Especially, some improvements of existing methods will be pointed out, as well as possible applications to some other similar problems.

An Overview of Results on the Quadratic Convergence of Scaled Iterates by Diagonalization Methods

Josip Matejaš

Abstract. An overview of results on the quadratic convergence of scaled iterates by diagonalization methods will be presented. Importance of scaled iterates was first noticed by Demmel and Veselić in 1992 in their paper on accuracy of symmetric eigenvalue decompositions. In proving the quadratic rate of convergence of scaled iterates one uses the structure of scaled almost diagonal matrices. This structure was estimated by Hari and Drmač for Hermitian matrices and by Matejaš and Hari for general matrices in connection with the SVD.

The first quadratic convergence estimates of scaled iterates for the symmetric Jacobi method are proved in the author's Ph.D. thesis in 1996. In the meantime, new results have been proved for Hermitian and J–Hermitian Jacobi methods in the general case of multiple eigenvalues. Also, general estimates for Kogbetliantz method applying to complex triangular matrices have been obtained. All these results have nice applications, since in the convergence bounds there appear relative gaps instead of absolute gaps. The estimates hold under natural assumptions, which are somewhat more stringent than the standard ones, which is due to nonunitary transformations linking the consecutive scaled iterates.

Joint work with Vjeran Hari.

A Constructive Approach to the Conformal Field Theory in Genus 1

Antun Milas

Abstract. Nowadays, genus 0 conformal field theory is a well-founded theory which has both algebraic and geometrical flavour. The seminal work of Y. Zhu on the genus 1 meromorphic conformal field theory is a step forward in understanding higher genus theories. We present an approach to the construction of the general (non-meromorphic) genus 1 conformal field theory based on certain examples.

Minding's Bendings of Ruled Surfaces in Simply Isotropic Space

Željka Milin Šipuš

Abstract. Simply isotropic space I_3^1 is a Cayley–Klein space whose absolute figure consists of a fixed plane ω of a three dimensional real projective space and of a pair of conjugate lines f_1, f_2 in ω. In this work we study Minding's bendings of ruled surfaces in simply isotropic space I_3^1. Minding's bending is a local isometry of ruled surfaces which preserves the rulings of the surfaces. There are 3 types of ruled surfaces in I_3^1 and each type is treated separately.

Speed of Convergence of Classes of Sequences

Harry I. Miller

Abstract. It is elementary that, given any convergent sequence, there is another convergent sequence that converges more slowly than the given sequence. M. Bajraktarević and H. Miller have written a series of articles on "rates of convergence". The current work, joint with J. Fridy and C. Orhan, involves statistical convergence.

A typical result: Necessary and sufficient conditions are given for there to exist a null sequence z that "converges statistically slower" than each null sequence x in a given class of null sequences.

Univalence Criteria for Classes of Equiangular Polygons

Lejla Miller–Van Wieren

Abstract. Let D be a simply connected plane domain and B the unit disk. The inner radius of D, $\sigma(D)$, is defined by

$$\sigma(D) = \sup\{a : a \geq 0,\ \|S_f\|_D \leq a \text{ implies } f \text{ is univalent in } D\}.$$

Here S_f is the Schwarzian derivative of f, ρ_D the hyperbolic density on D and $\|S_f\|_D = \sup_{z\in D} |S_f(z)|\, \rho_D^{-2}(z)$. Domains for which the value of $\sigma(D)$ is known include disks, angular sectors and regular polygons, as well as certain classes of rectangles and equiangular hexagons.

All of the mentioned domains except non-convex angular sectors have an interesting property in common, namely that $\sigma(D) = 2 - \|S_h\|_B$, where h maps B

conformally onto D. Because of the importance of this property for computing $\sigma(D)$, we say that D is a Nehari disk if $\sigma(D) = 2 - \|S_h\|_B$ holds.

We generalize the arguments used in an earlier paper to compute the inner radii of certain rectangles and hexagons, in order to obtain analogous results for some general classes of polygons. We show that these polygons are also Nehari disks and we compute the values of their inner radii.

Approximation of Maps into the Lipscomb Space by Embeddings

Uroš Milutinović

Abstract. Let $\mathcal{J}(\tau)$ be the Lipscomb one-dimensional space and $L_n(\tau) \subseteq \mathcal{J}(\tau)^{n+1}$ the Lipscomb n-dimensional universal space of weight $\tau \geq \aleph_0$. We prove the following theorem:

Let X be a metrizable space, $\dim X \leq n$, $wX \leq \tau$, $f : X \to \mathcal{J}(\tau)^{n+1}$ a continuous map, and ε a positive number. Then there is an embedding $\psi : X \to L_n(\tau)$ such that $\mathrm{d}(f, \psi) \leq \varepsilon$.

Also, in the separable case an analogous result is obtained, in which the classic triangular Sierpiński curve (homeomorphic to $\mathcal{J}(3)$) is used instead of $\mathcal{J}(\aleph_0)$ (strengthening the results of *Ivan Ivanšić and Uroš Milutinović: A Universal Separable Metric Space of Lipscomb Type*).

Equivariant Strong Homology (G-Ssh Homology)

Corina Mohorianu

Abstract. For a compact Lie group G we consider the categories *G-Top*, *proG-Top*, *CPH*(*G-Top*) and we construct the category *G-Ssh*.

Using the fact that every *G-space* X admits a *G-ANR* resolution (or a G-coherent expansion), first we prove that $\underline{f} : \underline{X} \to \underline{Y}$ is a G-coherent equivalence iff $\underline{f}^H : \underline{X}^H \to \underline{Y}^H$ is an ordinary coherent equivalence for every closed subgroup H in G, where $\underline{X}^H$ and $\underline{Y}^H$ are the corresponding inverse systems induced by $\underline{X}$ and $\underline{Y}$, respectively. So, we have that, for every F a *G-Ssh* morphism, F is *G-Ssh* equivalence iff F^H is an ordinary *Ssh*-equivalence for every closed subgroup H in G.

Then we define equivariant homology ${}^G H_n^s(\underline{X})$ for equivariant inverse systems $\underline{X} = (X_\lambda, p_\lambda^{\lambda'}, \Lambda)$ and equivariant strong homology for G-spaces ${}^G H_n^s(X) = {}^G H_n^s(\underline{X})$.

Finally, we prove homotopic invariance of equivariant strong homology and we deduce some long sequences for equivariant strong homology.

Howe Correspondence for Discrete Series

Goran Muić

Abstract. In this talk I will describe the first occurrence indices and structure of theta lifts of discrete series for the dual pair $Sp(n) \times O(V)$, over a non-archimedean local field of characteristic different from 2, in terms of the recent classification of discrete series due to Moeglin–Tadić. In the case of archimedean local fields similar results were obtained by Adams–Barbasch, Li, and Moeglin.

1D Flow of a Compressible Viscous Micropolar Fluid: Regularity of the Solution

Nermina Mujaković

Abstract. We consider nonstationary 1D-flow of micropolar viscous compressible fluid that is in a thermodynamic sense perfect and polytropic. Initial boundary value problem for such flow is well posed. Assuming that the initial mass density, velocity, microrotation velocity and temperature are Hölder functions on $[0,1]$, we prove that the mass density, velocity, microrotation velocity and temperature are also Hölder functions on $[0,1] \times [0,T]$, $T > 0$.

Relative Perturbation Theory for Matrix Pairs

Ivica Nakić

Abstract. We present a relative perturbation theory for strongly definitizable matrix pairs (H,K), i.e., for Hermitian matrix pairs which have separated positive and negative parts of the spectrum.

It is shown that, in some sense, this is the widest possible class of Hermitian matrix pairs for which such a theory can be obtained. This result generalizes the known result for the case when K is positive definite.

New eigenvalue perturbation estimates are also given for definite pairs.

We hope that the new estimates will be useful in the analysis of numerical algorithms for eigenvalue computation.

On Regions of Efficiency in Data Envelopment Analysis

Luka Neralić

Abstract. For each inefficient Decision Making Unit (DMU), according to the Additive model of Data Envelopment Analysis (DEA), there is a corresponding efficient DMU at the efficiency frontier. A region of efficiency of efficient DMU can be obtained, in which its efficiency is preserved under the non-negative perturbations of inputs and outputs of all DMUs. For that region there is a corresponding region around inefficient DMU and around all other DMUs. We study the relation between these regions. A generalization for the case of all efficient DMUs, with the region of joint efficiency, is also considered. An illustrative example is provided.

Parallel Algorithm for Generalised Flow Problem

Goranka Nogo

Abstract. The generalised flow problem is given by a flow network $(G(V,E),u,s,t)$ and a *gain function* $\gamma : E \to \mathbb{R}^+$. If $f(v,w)$ units are sent from v to w, $\gamma(v,w)f(v,w)$ units arrive to w. The generalised flow problem is to find a function f that satisfies the conservation constraints for all vertices except s and t and maximizes the net flow into the sink

$$e_f(t) := \sum_{(v,t)\in E} \gamma(v,t)f(v,t) = -\sum_{(t,v)\in E} f(t,v).$$

In this talk I will present one parallel algorithm for solving the generalised network flow problem.

A Proof of Vogan's Conjecture about Dirac Operators

Pavle Pandžić

Abstract. Let G be a connected real reductive group with maximal compact subgroup K. Let $\mathfrak{g}_0 = \mathfrak{k}_0 + \mathfrak{p}_0$ be the Cartan decomposition of the Lie algebra of G. Let $C(\mathfrak{p}_o)$ be the Clifford algebra of $\mathfrak{p}_0$ (with respect to the Killing form), and let S be a complex simple module for $C(\mathfrak{p}_0)$ (the space of spinors). The double cover $\tilde{K}$ corresponding to $Spin(\mathfrak{p}_0)$ acts on S.

The Dirac operator can be defined as $D = \sum_i Z_i \otimes Z_i \in \mathcal{U}(\mathfrak{g}) \otimes C(\mathfrak{p}_0)$, where $\{Z_i\}$ is an orthonormal basis of $\mathfrak{p}_0$. If X is a $(\mathfrak{g},K)$-module, and τ a (genuine) $\tilde{K}$-type, then D induces an operator D_τ on the space $\mathrm{Hom}_{\tilde{K}}(V_\tau, X \otimes S)$.

D. Vogan conjectured that if X is irreducible unitary, and if the kernel of D_τ is not zero, then the infinitesimal character of X is determined by τ; modulo certain identifications, it is given by the highest weight of τ plus a ρ-shift. This was proved for the discrete series by W. Schmid.

Vogan also made an algebraic conjecture from which the one just described can be deduced. Namely, consider the diagonal embedding of $\mathfrak{k}$ into the algebra $\mathcal{U}(\mathfrak{g}) \otimes C(\mathfrak{p})$; let $\mathfrak{k}_\Delta$ be its image. The conjecture states that for any element Z in the center of $\mathcal{U}(\mathfrak{g})$, $Z \otimes 1$ can be written as a sum of an element of the center of $\mathcal{U}(\mathfrak{k}_\Delta)$ with an element of the form $aD + Db$, for some (K-invariant) a and b.

I will present a proof of this last conjecture (this then also proves the first conjecture). The idea is to look at a differential on the K-invariants of the above algebra, defined by $d(a) = Da - \mathrm{sgn}(a)aD$, where $\mathrm{sgn}(a)$ is 1 (resp. -1) if the Clifford part of a is even (resp. odd). The result follows once we identify the cohomology of this differential.

Joint work with Jing–Song Huang.

Generalized Frame Multiresolution Analysis of Abstract Hilbert Spaces

Manos Papadakis

Abstract. The generalized frame multiresolution analysis of abstract Hilbert spaces is an MRA structure that encompasses orthogonal, biorthogonal, both one-dimensional and multidimensional MRA structures. Their core subspace, usually denoted by V_0, has a frame which is produced by the action of an abelian group of unitary operators (called a Translation group) on a countable set of vectors, called a set of frame multiscaling vectors. We prove that there exists a countable set of frame multiwavelet vectors associated with this structure, in the sense that the action of the translation group on this set produces a frame for the orthogonal complement of the core subspace with respect to its immediate successor subspace (V_1). We construct the set of the associated frame wavelet vectors by producing a pair of Generalized Quadratic Mirror Filters. Those filters are operator-valued functions contained in a Von Neumann algebra induced by the commutant of the translation group. Finally, we prove that fast algorithms are associated with this type of MRAs. Also, we show how classical MRA structures and the Frame MRAs of $L^2(\mathbf{R})$ are derived from these more general structures.

We conclude with some applications of this type of theory in the construction of non-separable multidimensional scaling functions and their associated low pass filters, which emerged from our recent interactions with signal processing groups.

Differentiation of Functions on Vilenkin Groups

Medo Pepić

Abstract. In this paper we will consider a concept of differentiation of complex functions defined on Vilenkin groups, looking from the aspect of summability of Vilenkin series. In that sense we give a matrix interpretation of C. N. Onneweer's concept of differentiation and then our generalization of that concept. We also give main properties of this generalized derivative. Moreover, we show that every method of summation of Vilenkin series, in a certain way, defines some concept of differentiation of integrable functions on Vilenkin groups.

A Sharp Multidimensional Bergh–Type Inequality

Ivan Perić

Abstract. A sharp multidimensional integral inequality for functions satisfying two quasi-monotonicity conditions will be presented. Two proofs of this result will be given. This result generalizes Bergh's inequality valid for quasi-concave functions.

Testing the Roulette Wheel

Mihael Perman

Abstract. Testing random number generators is an ever recurring topic. The topics covered in the talk will deal with one particular random number generator: the roulette wheel. This seemingly simple problem involving using usual χ^2-statistics leads to questions about martingales, maximal inequalities, weak convergence and more.

Prevention — Model of Optimization

Dragutin Perše

Abstract. The paper presents a strict mathematical model of prevention as the model of optimization of reliability of the operation system. The basic precondition is to enable supervision (control) of the continuous functioning of the system by some other system (device). The aim is to substitute instantaneously the "ageing" or the damaged part (element) of the observed system in compliance with the warning from a device which is in charge of supervision. Therefore, the moment of prevention $S(X,t)$ must be planned. Further, this model is being generalized by the strategy (plan) of the form: $S_i(X,t)$, $i = 1, 2, \ldots$, where $S_i(X,t)$ is the strategy (plan) and prevention period in the rest of time t.

Realizations of Configurations

Tomaž Pisanski

Abstract. In his dissertation in 1894, E. Steinitz proved that any connected v_3 configuration can be drawn in the Euclidean plane with at most one curved line. Based on a construction by H. L. Dorwart and B. Grünbaum from 1992, we are able to produce a sequence of v_3 configurations $K(n)$ with the following property: For any integer m there exists an integer N such that for each $n > N$ one has to remove at least m lines from $K(n)$ in order to obtain an incidence structure that has a realization in the real projective plane (and hence in the Euclidean plane). We show how to reconcile this result with the theorem of Steinitz. However, in passing we produce straight-line drawings of various interesting configurations that were obtained as a joint work with numerous co-authors.

Derivative Sampling Reconstruction of Bandlimited Stochastic Processes in the Almost Sure Sense

Tibor K. Pogány

Abstract. The talk concerns the generalization of the Gaposhkin condition upon the a.s. sense uniformly spaced sampling reconstruction of the bandlimited weakly stationary stochastic processes. Namely, we generalize Gaposhkin's condition to the derivative sampling approach for stationary processes. So, let $\{\xi(t) : t \in \mathbf{R}\}$ be a band-limited (BL) weakly stationary stochastic process (WSSP). The sampling

reconstruction of a BL WSSP by the Kotel'nikov–Shannon formula which uses samples of its $r-1$ derivatives, $r \in \mathbf{N}$, as well as those of $\xi(t)$ itself, is called r^{th}-*order derivative sampling restoration*. Now, the derivative variant of the Gaposhkin's condition reads as follows.

Let $\{\xi(t) : t \in \mathbf{R}\}$ be a BL to $r\pi$ WSSP. Then it holds

$$\xi(t) = \sin^r(\pi t) \sum_{n \in \mathbf{Z}} (-1)^{nr} \sum_{j=0}^{\lceil \frac{r-1}{2} \rceil} \sum_{k=0}^{r-2j-1} \frac{\pi^{j-r}\, \Gamma_{r,2j}}{(2j)!\,(r-2j-k-1)!} \frac{\xi^{(r-2j-k-1)}(n)}{(t-n)^{k+1}}$$

in the a.s. sense, **iff**

$$\mathbf{P}\left\{ \lim_{m\to\infty} \left(\int\limits_{[-r\pi,-r\pi+2^{-m}]} \mathcal{P}^{+}_{r-1}(\lambda)\,\zeta(d\lambda) - \int\limits_{[r\pi-2^{-m},r\pi]} \mathcal{P}^{-}_{r-1}(\lambda)\,\zeta(d\lambda) \right) = 0 \right\} = 1,$$

where $\Gamma_{r,j} := \lim\limits_{z\to 0} \dfrac{d^j}{dz^j}\left(\dfrac{z}{\sin z}\right)^r$ and

$$\begin{aligned} \mathcal{P}^{\pm}_{r-1}(\lambda) \;&=\; \sum_{j=0}^{\lceil \frac{r-1}{2} \rceil} \sum_{k=0}^{r-2j-1} \frac{4^{\lceil k/2 \rceil} \pi^{2(j+\lceil k/2 \rceil)}\, \Gamma_{r,2j}\, (-1)^j\, i^{r-1}}{(2j)!\,(r-2j-k-1)!} \lambda^{r-2j-k-1} \\ &\quad \times \left[2\pi(1-(-1)^k)\beta_{2\lceil k/2 \rceil+2}(\lambda \pm r\pi) + i(1+(-1)^k)\beta_{2\lceil k/2 \rceil+1}(\lambda \pm r\pi) \right]. \end{aligned}$$

Here $\beta_s(\cdot)$ denotes the periodic continuation of the Bernoulli polynomial of the degree s from its basic interval to $[-r\pi, r\pi]$ and ζ is the orthogonally scattered random spectral measure of ξ.

Taking $r = 1$ in the previous formulae we get the Gaposhkin's result.

On the (k, f, l)–Chordal Polygons

Tibor K. Pogány

Abstract. The talk generalizes the concept of the k-chordal polygons and the so called (k, λ, l)-chordal polygons which are introduced by Radić, respectively Radić and Pogány. Namely, we describe the positive real function f such that $f(a_1), \ldots, f(a_n)$ are the side lengths of an l-chordal polygon and at the same time $a_1, \ldots, a_n$ are the side lengths of a k-chordal polygon. After that we give an exact upper bound for the constant $\gamma(k, n)$ defined by

$$a_1 + \cdots + a_n > \gamma(k, n) a^*,$$

so that this suffices for the existence of the k-chordal polygon with the side lengths $f(a_i)$, $i = 1, 2, \ldots, n$, where $a^* := \max_i a_i$.

Joint work with Mirko Radić (Vrbnik, Croatia).

On Medial–Like Quasigroups

Mirko Polonijo

Abstract. Starting from the medial law $ab \cdot cd = ac \cdot bd$, we are studying all identities of the form $ab \cdot cd = f(a)f(b) \cdot f(c)f(d)$ (where f is a permutation on $\{a, b, c, d\}$), on the set of all quasigroups. It can be shown that most of them characterise commutative medial quasigroups.

Asymptotic Behaviour of Ginzburg–Landau Functionals

Andrija Raguž

Abstract. We consider one-dimensional singularly perturbed problem which leads to multiple small scales depending on a small parameter ε. Our goal is to describe the limiting behaviour of the functional

$$I_\varepsilon(v) := \int_\Omega (\varepsilon^2 v''^2 + W(v') + a(v-g)^2).$$

Functional I_ε is a variation of a Cahn–Hilliard functional, namely the Ginzburg–Landau functional, which appears naturally in the Cahn–Hilliard model for phase transitions. We generalise a recent result by G. Alberti and S. Müller (which concerns the case $g = 0$) to the case of an arbitrary Lipschitz function g.

The approach relies on two important ingredients: rewriting the initial functional in terms of blow-ups of a function v and well-known Γ-convergence result of L. Modica and S. Mortola.

Genuine Representations of the Metaplectic Group

David Renard

Abstract. On one hand, many features in representation theory of real algebraic reductive groups are no longer available for non-linear groups. One example is the Langlands–Shelstad theory of stable conjugacy and endoscopic transfer, which has its root in automorphic form theory. Another example is Vogan's Kazhdan–Lusztig algorithm to compute characters of irreducible admissible representations of such groups, and Vogan's duality of characters. On the other hand, non-linear groups, specially the metaplectic group, appear in dual pair correspondences which play an important role in automorphic form theory. We will explain how these theories can

be extended to the non-linear group $\mathrm{Mp}(2n, \mathbb{R})$, and their connections with dual pair correspondences.

Joint work with P. Trapa.

An Operational Theory of Objects

Dean Rosenzweig

Abstract. Most object oriented programming languages, and practically all theoretical accounts, introduce a hierarchy of classes as (an extension of) a system of types. Notable counterexamples from the real world, however, demonstrate that object orientation and typing are entirely orthogonal notions.

In spite of apparently deep ontological differences between existing object oriented environments (even typed ones), in operational terms they function in very similar ways. The right abstraction from the idiosyncrasies of particular environments might thus be sought in operational terms. We suggest an operational semantics of objects intended to capture that similarity, based on Abstract State Machines of Gurevich. It is a design goal of the semantics to specialize naturally both to sequential and highly distributed object oriented environments, typed and type-free alike.

Joint work with Ante Đerek, (student, Department of Mathematics, University of Zagreb, Croatia).

Dynamics of Moving Contact Lines

Jan Rosenzweig

Abstract. This talk treats the notorious problem of the spreading of a thin film on a solid surface under the action of surface tension. The principal difficulty concerns the boundary conditions to be applied at the contact lines at the edges of the film.

I shall derive the formal asymptotic expansion for this problem away from the contact line and allow the intrinsic stress singularity to be relieved by any of a number of physical mechanisms, including evaporation/condensation, van der Waals forces, electrostatic potentials and some others. In all these cases, formal matching up to logarithmic orders results in a generic set of leading-order boundary conditions to be applied at the contact line, which depends only on two material parameters. The contact line condition involving the dynamic contact angle is similar to Tanner's law, but has a different limit for small capillary number. This difference may account for why Tanner's law is a poor approximation to some published experimental data. The results will be applied to derive the time dependence of surface-tension-driven spreading of droplets on solid surfaces.

Joint work with O. E. Jensen (Department of Applied Mathematics & Theoretical Physics, University of Cambridge, UK).

Total L_p Spline

Rudolf Scitovski

Abstract. It is known that the data smoothing problem can be solved by using the ordinary least squares spline. In this paper we consider the data smoothing problem where errors can appear in the measured values of both the dependent and the independent variable, and there is also a possibility that the so-called outliers appear. A method for the construction of the best total L_1, L_2 and L_∞ splines will be described. We will illustrate the above method on a few examples.

Joint work with Dragan Jukić, Kristian Sabo (Department of Mathematics, University of Osijek, Croatia) and Alfonzo Baumgartner (Faculty of Electrical Engineering, University of Osijek, Croatia).

An Example of the Genetic Algorithm Usage for Human Vision System Sensitivity Function

Krešimir Seršić

Abstract. The results of the quantization noise visibility measurement for the wavelet subband coefficients are given. The quantization noise is simulated by a random number generator with the uniform distribution. We use biorthogonal wavelet for an example of a spline variant with less dissimilar length. With the wavelet filter coefficients the wavelet transfer functions are determined for the different levels of decomposition.

The function of the human visual system sensitivity to the frequency of the luminance variation on an image is given by

$$W(u,v) = \frac{A}{[1 + B(u^2 + v^2)]^C}.$$

To determine the unknown parameters A, B and C, we use the modified GENOCOP III program based on genetic algorithm. To do so, the program computes double integrals, defined via $W(u,v)$ and wavelet transfer functions, using Simpson's formula and Romberg's algorithm.

We compare the obtained values with the given measured values by calculating the evaluation function using norms L_1 ("Manhattan" norm), L_2 (Euclidean norm) and L_∞ (max norm).

Joint work with Snježana Rimac–Drlje (Faculty of Electrical Engineering, University of Osijek, Croatia) and Igor Urbiha (Department of Mathematics, University of Zagreb, Croatia).

A Basis of the Level 1 $A_2^{(2)}$–Module

Ivica Siladić

Abstract. In this work we present explicit parametrisation of the standard level 1 module for the twisted affine Lie algebra $A_2^{(2)}$. This description of the basis also gives some interesting Rogers–Ramanujan type combinatorial identities. Moreover, we show that this basis can be expressed in terms of the crystal base theory.

Joint work with Mirko Primc.

Some Perturbation Results for the Indefinite QR Factorization

Sanja Singer

Abstract. Indefinite QR factorization of a given matrix A is a generalization of the well-known QR factorization to the indefinite scalar product spaces.

We present componentwise and normwise perturbation bounds for the so called "triangular" case of the indefinite QR factorization. Similar perturbation analysis of the "ordinary" QR factorization has been done by Stewart, Sun and Zha.

These bounds fit well into the relative perturbation theory for Hermitian matrices given in factorized form.

Efficient Termination Tests for Simplex Based Direct Search Methods

Saša Singer

Abstract. Simplex based direct search methods are widely used algorithms to locate a point of minimum (or maximum) of a function $f : \mathbb{R}^n \to \mathbb{R}$ in applications where f is not smooth, or even not continuous.

Efficiency analysis of these methods shows that the domain convergence part of the termination test is a potential computational bottleneck, depending on the complexity of f.

We propose a simple domain convergence test based on the volume of the working simplex and show how to efficiently implement this test without sacrifice in accuracy.

Poincaré Sequences, Ergodic Theory and Fourier Analysis

Siniša Slijepčević

Abstract. Assume that $0 < s_1 < s_2 < \ldots$ is a strictly increasing sequence of integers, $S = \{s_1, s_2, \ldots\}$. Given an integer N, let A_N be the largest subset of $\{1, \ldots, N\}$ such that there are no two elements in A_N whose difference is in S, and let $a(N) = |A_N|/N$. If $\lim_{N\to\infty} a(N) = 0$, we say that S is a Poincaré sequence; the well-known examples are $(P(n))_{n\in\mathbb{N}}$, where $P(x)$ is a polynomial with integer coefficients and $P(0) = 0$; and the sequence $p_n - 1$, where p_n is the n-th prime (Sárközy, Furstenberg).

Combining the methods of ergodic theory and Fourier analysis on finite rings, we obtain a functional inequality for a given Poincaré sequence, whose every solution is an upper bound to $a(N)$. In particular, when S is the set of squares, by solving the inequality we improve the existing estimates and come close to the Erdős–Sárközy conjecture that $a(N) \leq N^{-1/2+\epsilon}$.

Joint work with Tonči Crmarić.

Moving Total L_p Method for Parameter Identification Problem

Renata Sotirov

Abstract. We consider the parameter identification problem for the mathematical model described by a system of ordinary differential equations. Parameters in the model will be estimated on the basis of the given measurement data. Thereby it is supposed that errors can appear in the measured values of the dependent and independent variable, and that outliers can also appear among the data. Therefore, for solving the parameter identification problem we will use the moving total L_1 method. The method will be illustrated on a few examples.

Shape Theories and the Kan Extension Property

Luciano Stramaccia

Abstract. A (generalized) shape theory is a pair $(\mathbf{C}, \mathbf{K})$, where $\mathbf{K}$ is a full proreflective subcategory of $\mathbf{C}$. The classical example is given by the pair $(\mathbf{HTop}, \mathbf{HCW})$. We give, as a main result, a characterization of shape theories $(\mathbf{C}, \mathbf{K})$ by means of their extension properties of functors $F : \mathbf{K} \to \mathbf{A}$ to functors $F' : \mathbf{C} \to \mathbf{A}$, $\mathbf{A}$ being any category having inverse limits.

On Involutory Automorphisms of a Hypothetical $(22, 8, 4)$ Design

Juraj Šiftar

Abstract. With regard to the number of points, the parameters $(22, 8, 4)$ correspond to the smallest possible block-design for which the problem of existence is still unresolved. Several serious attempts were made to that purpose, in particular the one by M. Hall jr., R. Roth, G. H. J. van Rees and S. A. Vanstone (J. Combin. Theory Ser. A, 47 (1988), pp. 157–175), but the final decision remained out of reach.

Another direction of investigation has shown that the full automorphism group of a hypothetical $(22, 8, 4)$ design had to be either trivial or a 2-group. In this report, some new restrictions on the action of involutory automorphisms are presented.

Kac Parameters and $\mathbb{Z}$–Gradations of Classical Affine Lie Algebras

Tomislav Šikić

Abstract. $\mathbb{Z}$-gradation of type s is well defined for any affine Lie algebra and any nonzero sequence $s = (s_0, s_1, \ldots, s_n)$ of integers. Using the Kac classification of finite order automorphims for a simple Lie algebra, we have developed a method to treat the problem of finding the sequence s_K of nonnegative, relatively prime Kac parameters for some $\mathbb{Z}$-gradations of type s. Our method applies in the case of classical affine algebras $(A_n^{(1)}, B_n^{(1)}, C_n^{(1)}, D_n^{(1)})$.

On Homeomorphisms of Knaster Continua

Sonja Štimac

Abstract. The Knaster continuum K_t is the inverse limit of the unit interval $I = [0, 1]$ using the standard tent map $g_t : I \to I$ with slope t as the bonding map. We study the corresponding composants and homeomorphisms between them. The main tool is symbolic dynamics. In particular, we code the Knaster continua using techniques of kneading theory.

One–Dimensional Eigenvalue Problem for Curved Rods

Josip Tambača

Abstract. In this work we derive a one-dimensional eigenvalue problem for curved rods. We start from the three-dimensional eigenvalue problem for the linearized elastic body. By the use of asymptotic analysis, as the thickness of the rod tends to zero, the one-dimensional eigenvalue problem is obtained. The appropriate convergence of eigenvalues and eigenfunctions is proved.

Joint work with Zvonimir Tutek.

Geometric Simplicity Theory

Ivan Tomašić

Abstract. Although no good analogues of "geometric stability theory" are known so far in a simple setting, we'll discuss the obstacles and give some ideas and strategies believed to lead to certain results, in an attempt to reach the first results in this, in our opinion, very interesting setting.

Things to be discussed: basic theorems of geometric stability theory, Zilber's theorem, Zilber's trichotomy, Hrushovski's amalgamation construction (and counterexamples it yields), group configuration, etc.

A standard reference for geometric stability is: Anand Pillay, *Geometric Stability Theory*, Oxford logic guides, Clarendon Press, Oxford, 1996.

New Generalizations of the Telyakovskii's Inequalities

Živorad Tomovski

Abstract. Generalizations of the theorems of Telyakovskii [1] have been obtained, by considering the conditions $S_{p\alpha r}$, $1 < p \leq 2$, $\alpha \geq 0$, $r \in \{0, 1, 2, \ldots [\alpha]\}$ and $S_p(\delta)$, $1 < p \leq 2$, instead of S.

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O iteriranim rezolventama

Nikica Uglešić

Abstract. (**A**) Neka je $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ inverzni sustav topoloških prostora i preslikavanja nad kofinitnim i antisimetričnim Λ. Tada postoji "sustav" kanonskih (poliedarskih) rezolvenata $(\boldsymbol{p}^{\lambda'}, \boldsymbol{p}^{\lambda\lambda'}, \boldsymbol{p}^{\lambda})$ veznih preslikavanja $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$, $\lambda \leq \lambda'$, $\boldsymbol{p}^{\lambda\lambda'} = [(p_\nu^{\lambda\lambda'}, p^{\lambda\lambda'})]$, s točno jednom kanonskom rezolventom $\boldsymbol{p}^\lambda = (p_\nu^\lambda) : X_\lambda \to \boldsymbol{X}^\lambda = (X_\nu^\lambda, p_{\nu\nu'}^\lambda, N^\lambda)$ za svaki prostor X_λ, $\lambda \in \Lambda$, i pritom je $\boldsymbol{p}^{\lambda\lambda''} = \boldsymbol{p}^{\lambda\lambda'}\boldsymbol{p}^{\lambda'\lambda''}$ čim je $\lambda \leq \lambda' \leq \lambda''$.

(**B**) "Sustav" $((\boldsymbol{p}^{\lambda'}, \boldsymbol{p}^{\lambda\lambda'}, \boldsymbol{p}^{\lambda}),\ \lambda \leq \lambda')$ iz (**A**) dopušta prirodno organiziranje (bez projekcija) u inverzni sustav $\boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$, pri čemu je $M = \{\mu = (\lambda, \nu) \mid \lambda \in \Lambda, \nu \in N^\lambda\}$, $Y_\mu = X_\nu^\lambda$ i $q_{\mu\mu'} : Y_{\mu'} \to Y_\mu$ kompozicija od $p_\nu^{\lambda\lambda'}$ i $p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'}$.

(**C**) Ako k tomu sustav $\boldsymbol{X}$ dopušta rezolventu $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X}$ nekog topološkog prostora X, onda je i $\boldsymbol{q} = (q_\mu = p_\nu^\lambda p_\lambda) : X \to \boldsymbol{Y}$ rezolventa od X.

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Intrinsic Geometry and Rigid Sets

Šime Ungar

Abstract. A metric space (X, d) is said to be *geometrically acceptable*, provided every two points can be joined by an arc of finite length, and the *intrinsic metric* d^* on X, defined as the infimum of lengths of all such arcs, is topologically equivalent to d.

An *intrinsic isometry* between two such spaces is an isometry with respect to their intrinsic metrics, which is, by a theorem of Borsuk, the same as being a homeomorphism preserving the lengths of arcs.

A geometrically acceptable subset $A \subseteq \mathbb{R}^n$, is said to be *rigid in* $\mathbb{R}^n$, provided every intrinsic isometry $h : A \to h(A) \subseteq \mathbb{R}^n$ is an isometry.

In 1981, Borsuk has proved that every open connected subset of $\mathbb{R}^n$ is rigid. In 1993, V. A. Aleksandrov gave an example of a one-dimensional rigid subset of

$\mathbb{R}^2$, and in 1994, I. Herburt showed how to construct some $(n-1)$-dimensional rigid subsets in $\mathbb{R}^n$, but the Hausdorff dimension of these sets remained n.

We give conditions under which an intrinsic isometry on a dense subset of an open set in $\mathbb{R}^n$ can be extended to an isometry of the whole open set. As a result, for each n we obtain examples of rigid sets in $\mathbb{R}^n$ with topological, and even Hausdorff dimension $n-1$.

For $k < n-1$ the problem still remains open.

Joint work with Irmina Herburt.

Pólya–Type Inequalities

Sanja Varošanec

Abstract. Generalizations of the Pólya inequality are given involving real functions which satisfy certain subordination and end-point conditions, but are otherwise arbitrary.

First, let us introduce two classes of functions as follows:

Let $\mathbf{w} = (w_1, \ldots, w_m)$ be a given constant real m-tuple. For I a given interval in $\mathbb{R}$, we say that a function $F : I^m \to \mathbb{R}$ belongs to the class $W(\mathbf{w})$, if and only if

$$F(x_1, \ldots, x_m) \le \sum_{i=1}^{m} w_i x_i \quad \text{for all } x_1, \ldots, x_m \in I.$$

If the inequality is reversed, we say that F belongs to the class $W^*(\mathbf{w})$.

The main result is the following theorem.

Theorem. *Let $F, G : I^m \to \mathbb{R}$ and $f : [a,b] \to \mathbb{R}$ be real functions and suppose $g_1, \ldots, g_m : [a,b] \to I$ are continuously differentiable functions such that $G(g_1, \ldots, g_m)$ is also continuously differentiable with*

$$G(g_1(a), \ldots, g_m(a)) = \sum_{i=1}^{m} w_i g_i(a), \qquad G(g_1(b), \ldots, g_m(b)) = \sum_{i=1}^{m} w_i g_i(b),$$

and $\int_a^b g_i'(t) f(t)\, dt \in I$, $i = 1, \ldots, m$.

(a) *If f is non-decreasing and $F, G \in W(\mathbf{w})$, then*

$$F\left(\int_a^b g_1'(t) f(t)\, dt, \ldots, \int_a^b g_m'(t) f(t)\, dt\right) \le \int_a^b (G(g_1(t), \ldots, g_m(t)))' f(t)\, dt.$$

If $F, G \in W^*(\mathbf{w})$, then the inequality is reversed.

(b) *Suppose f is non-increasing. If $F \in W(\mathbf{w})$ and $G \in W^*(\mathbf{w})$, then the inequality holds. If $F \in W^*(\mathbf{w})$ and $G \in W(\mathbf{w})$, the inequality is reversed.*

Using different types of the function F we obtain a number of known results, such as the standard Pólya inequality involving power functions, the Alzer result involving geometric means, inequalities involving weighted power, quasiarithmetic and logarithmic means, and derive some new results involving symmetric and complete symmetric means. Also, the previously mentioned theorem possesses a discrete and an operator analogue.

Symmetric and Skew–Symmetric Functions of Two Variables and a Bounded Mean Value Theorem

Radimir Viher

Abstract. For a real function of two real variables we obtain the following generalization of Lagrange's mean value theorem. Let $f \in C^3([a,b]^2)$ and $f(t,t) = 0$ for every $t \in [a,b]$. With every such function we can associate two symmetric functions s_1, s_2, and two functions of one variable $g(t) = s_1(t,t)$ and $h(t) = s_2(t,t)$. Let functions g, h fulfil the following conditions

$$g'(t) > 0\ (< 0); \quad h'(t) > 0\ (< 0), \quad t \in [a,b].$$

Then there exists $\delta > 0$ such that the following holds. If $x, y \in [a,b]$ and $0 < |y - x| < \delta$, then we can find two numbers $c_1, c_2 \in (x,y)$ $(\in (y,x))$ such that

$$f(x,y) = [g(c_1) + h(c_2)(y-x)]\,(y-x).$$

By means of this result we prove the following statement. For $a < b$ there exists $c \in (a,b)$ such that

$$\frac{e^a + e^b}{2} - e^{(a+b)/2} = \frac{e^c}{8}(b-a)^2.$$

Lot Sizing Problem with Equipment Replacement

Višnja Vojvodić Rosenzweig

Abstract. Each of N products is to be processed on a single machine in order to satisfy known demands in each of T periods. The object is to minimize the sum of the costs of production, storage and set-up. This problem is known as a Production Lot Sizing Problem.

On the other hand, an Equipment Replacement Problem concerns a machine which deteriorates with age and the decision to replace it. We assume that we must own such a machine during each of T periods and that the utility of operating a machine for one period is a known quantity and depends on the age of the machine.

The object is to decide when to replace the machine in order to maximize the utility of operating it.

Combining these two problems, we define a new multicriteria mixed 0–1 integer programming problem called a Lot Sizing Problem with Equipment Replacement. The object is to minimize the costs and maximize the utility of operating the machine. This paper studies the properties of efficient point set for this new problem.

Joint work with Kristina Šorić (Faculty of Economics, University of Zagreb, Croatia).

Optimal Microstructures for Relaxed Two–Well Energy

Marko Vrdoljak

Abstract. The relaxation of two-well energy associated to a system of two linearly elastic phases with different stress-free strains and different (well-ordered) elastic moduli is considered. We obtain a formula expressing this relaxation in terms of a finite-dimensional minimisation.

For the isotropic materials in two dimensions it is possible to obtain more information on the minimising microstructure. If the misfit strain is dilational, the optimal microstructure is a rank-one laminate, while the case of tetragonal misfit is more complex: the minimum is obtained either with rank-one or rank-two laminate.

The description of minimising microstructures in terms of H-measures will be given as well.

Joint work with Nenad Antonić.

On $(100, 45, 20)$ Symmetric Designs with Automorphism Groups of Order 100

Tanja Vučičić

Abstract. Nonabelian groups of order 100, which are extensions of elementary abelian group E_{25}, are considered as possible automorphism groups of symmetric designs with parameters $(100, 45, 20)$. By the action of these groups new $(100, 45, 20)$ symmetric designs are constructed. Their full automorphism groups and some other properties are given. In the cases where the determined full automorphism group is of order greater than 100, a possible construction of differential sets with the same parameters is more likely to be accomplished.

Joint work with Anka Golemac.

The Interpretability Logic and Generalized Veltman Models

Mladen Vuković

Abstract. The interpretability logic IL is the natural extension of the provability logic GL (Gödel–Löb). It is a (non-standard) propositional modal logic with two modal operators. We study generalized Veltman models as defined by D. de Jongh. We determine characteristic classes of the generalized Veltman frames of the principles M_0 and F. By using generalized Veltman semantics we prove many independencies between principles of interpretability. Each closed formula of interpretability logic is equivalent in the system ILF to a closed formula which contains only the modal operator $\Box$ (see [1]). We consider the problem of normal forms for closed fragment of the system IL (see [2]).

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A Filtered Version of the Bipolar Theorem of Brannath and Schachermayer

Gordan Žitković

Abstract. We obtain a version of the bipolar theorem for sets of supermartingales on a filtered probability space. We formulate the notion of fork-convexity as an analogue of convexity in this setting. As an intermediate step in the proof of our main result, we also establish a conditional version of the bipolar theorem. As an application to mathematical finance, we describe the structure of the enlarged dual domain of the utility maximization problem of Kramkov and Schachermayer (1997, to appear in Annals of Applied Probability) and give a *budget-constraint*-type of a characterization of admissible consumption processes in an incomplete semimartingale market.

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Existence, Nonexistence, and Qualitative Properties of Solutions for a Class of Quasilinear Elliptic Equations

Darko Žubrinić

Abstract. We obtain a result about control of essential supremum of supersolutions for quasilinear elliptic equations of Leray–Lions type with the natural growth in the gradient. From this we derive explicit local and global oscillation estimates, a lower bound on the constant appearing in Schauder's and Agmon–Douglis–Nirenberg a priori estimates (in terms of the inner radius of the domain), and a result about generating singularities. We provide an example showing that one obtains better oscillation estimates if deformation retracts of the domain are used, instead of balls.

We also study existence, nonexistence, and regularity of spherically symmetric solutions of quasilinear elliptic equations with the natural growth in the gradient, defined in a ball. A uniqueness result has been obtained, based on a comparison principle for integral operators of Volterra type, introduced by Mervan Pašić.

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Joint work with Luka Korkut (Faculty of Electrical Engineering and Computing, University of Zagreb, Croatia) and Mervan Pašić (Faculty of Electrical Engineering and Computing, University of Zagreb, Croatia).

Integrable Cubic Vector Fields

Vesna Županović

Abstract. We study planar cubic vector fields V_H with a center, having the Darboux first integral $H = P_1^2 Q_3$. The polynomials P_1 and Q_3, of degrees 1 and 3, respectively, are the invariant curves of V_H.

Let P_2 and Q_2 be polynomials of degree 2. We study the vector field

$$V_H = yP_2\frac{\partial}{\partial x} - xQ_2\frac{\partial}{\partial y}$$

with a center at the origin. Under these conditions we have two 3-parameter families of vector fields.

Our aim is to give the description of the global geometry of the vector fields V_H and the bifurcation diagram of the phase portraits in a 3-dimensional parameter space.

The main part of studying the dynamics of vector fields is the study of the nature of singular points of vector fields. This is done by blowing-up method, normal form method and using the Newton diagram.

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