Hierarchical and Dynamic Threshold Paillier Cryptosystem without Trusted Dealer

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UnRAVeL Research Training Group – Uncertainty and Randomness in Algorithms, Verification, and Logic

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Introduction

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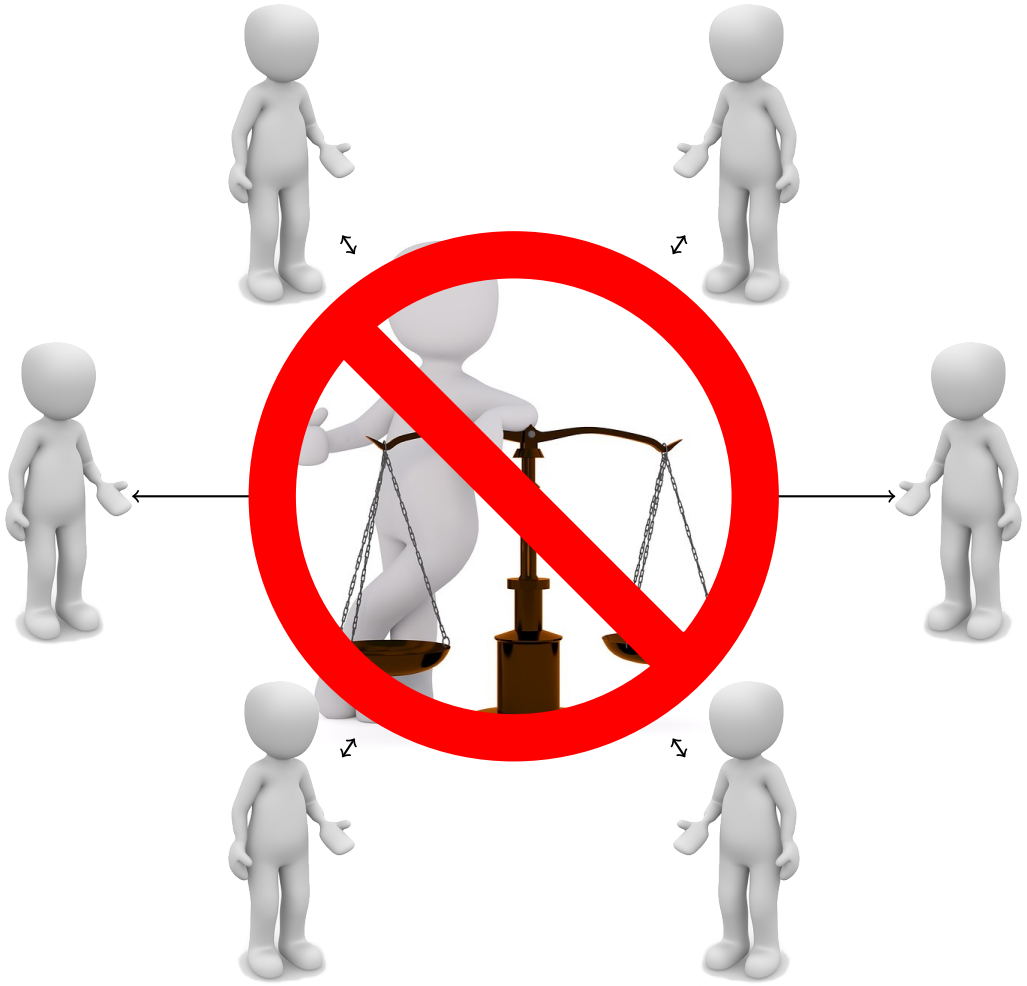
Secure Multi Party Computation



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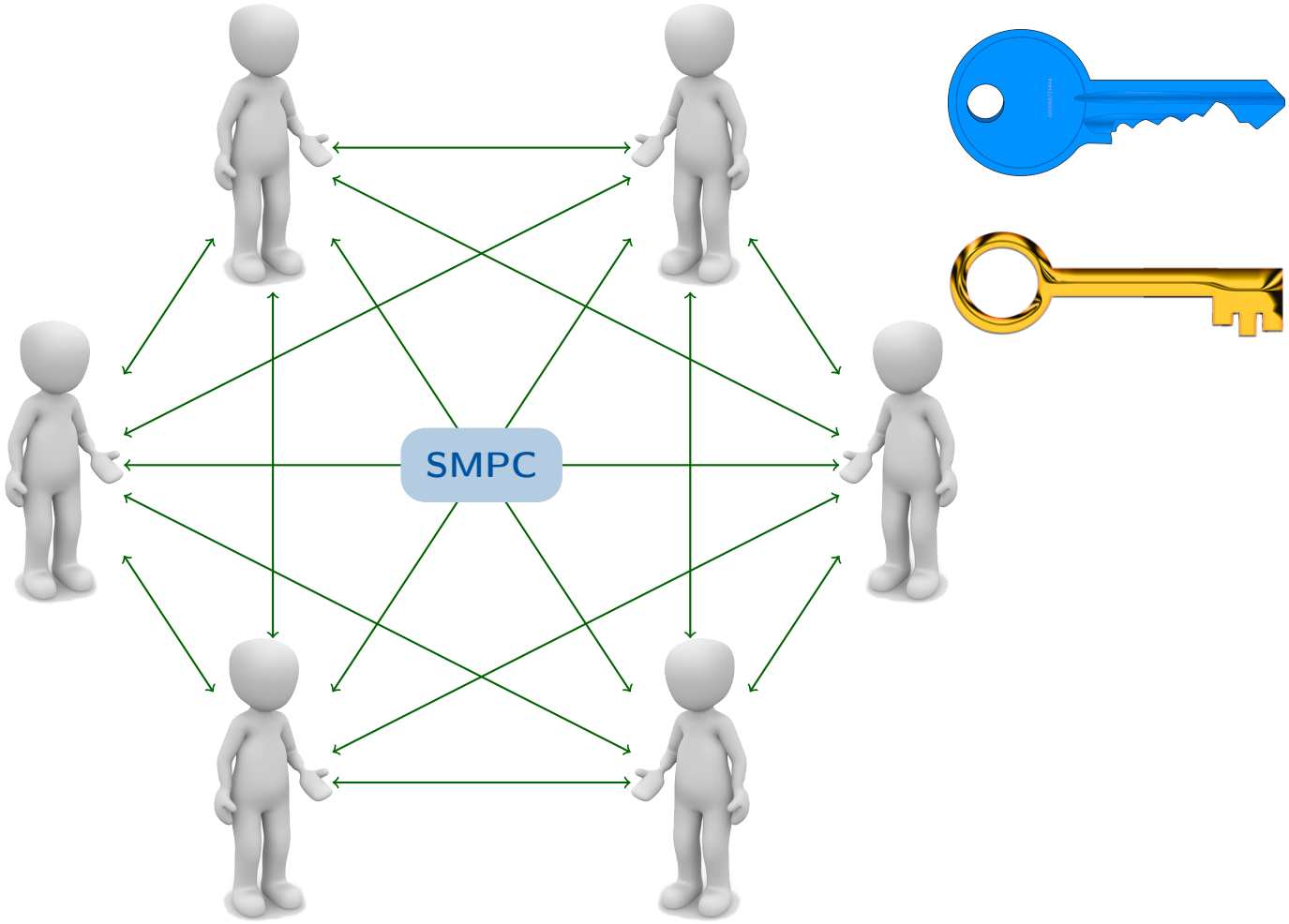
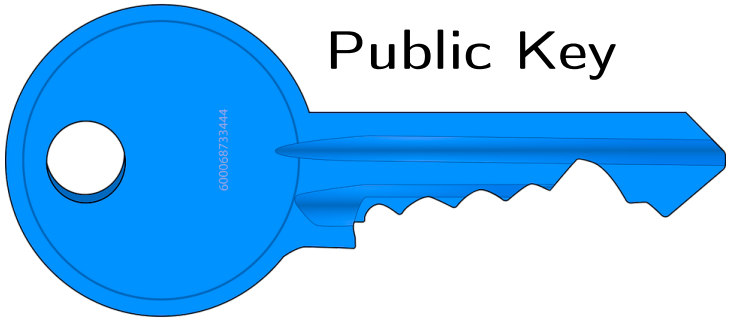


Public Key Cryptography



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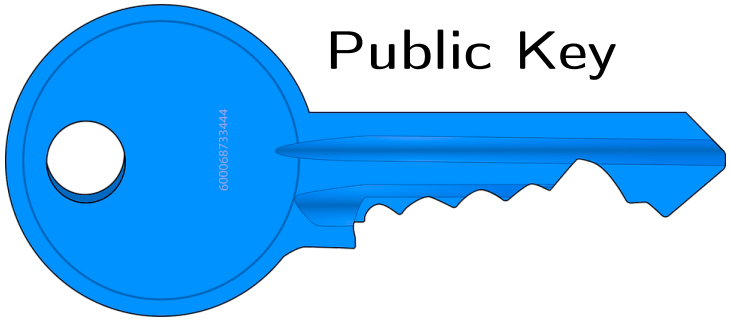

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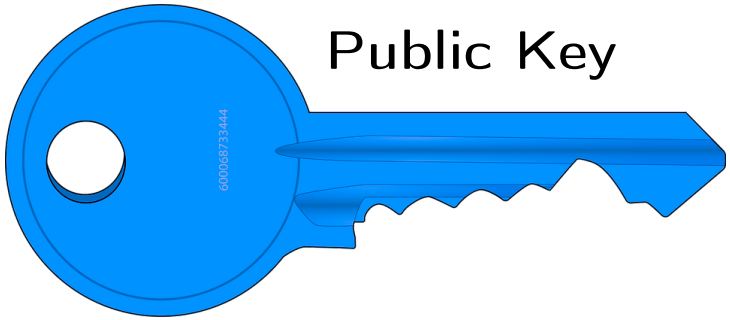

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Splitting the Key

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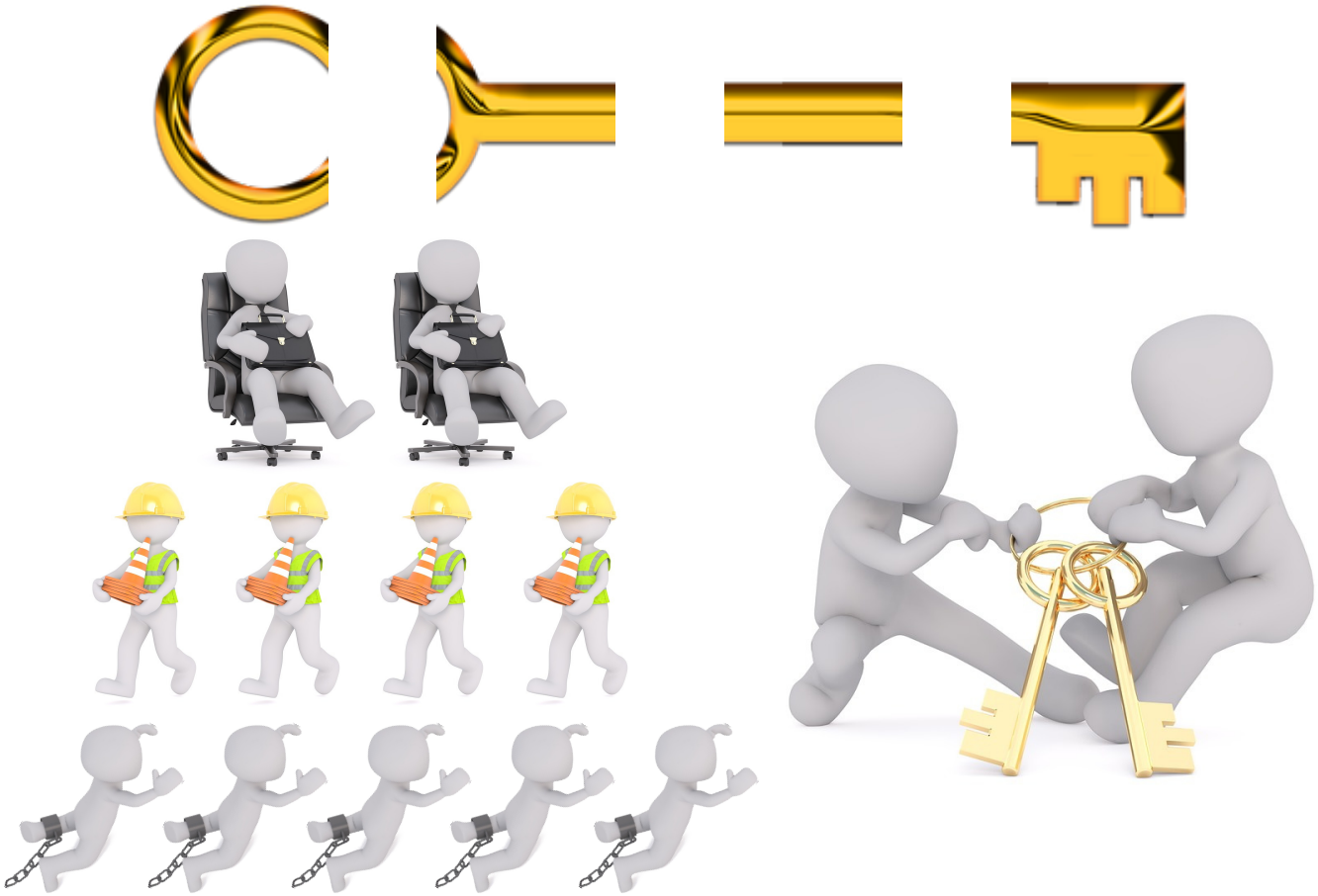
Splitting the Key



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Hierarchical and Dynamic Threshold Paillier Cryptosystem without Trusted Dealer

Main Properties

- Enables ***hierarchical*** access structures
- Allows to ***add*** and ***remove*** share holders
- No trusted-third party required
- Allows homomorphic ***addition***
- Secure in the presence of a ***malicious*** adversary

Usage Scenarios

- Enforce a certain hierarchy
  → Enforce the presence of certain parties
- Suitable for SMPC
  → Reuse of computations (encrypted results)
  → Audit computations

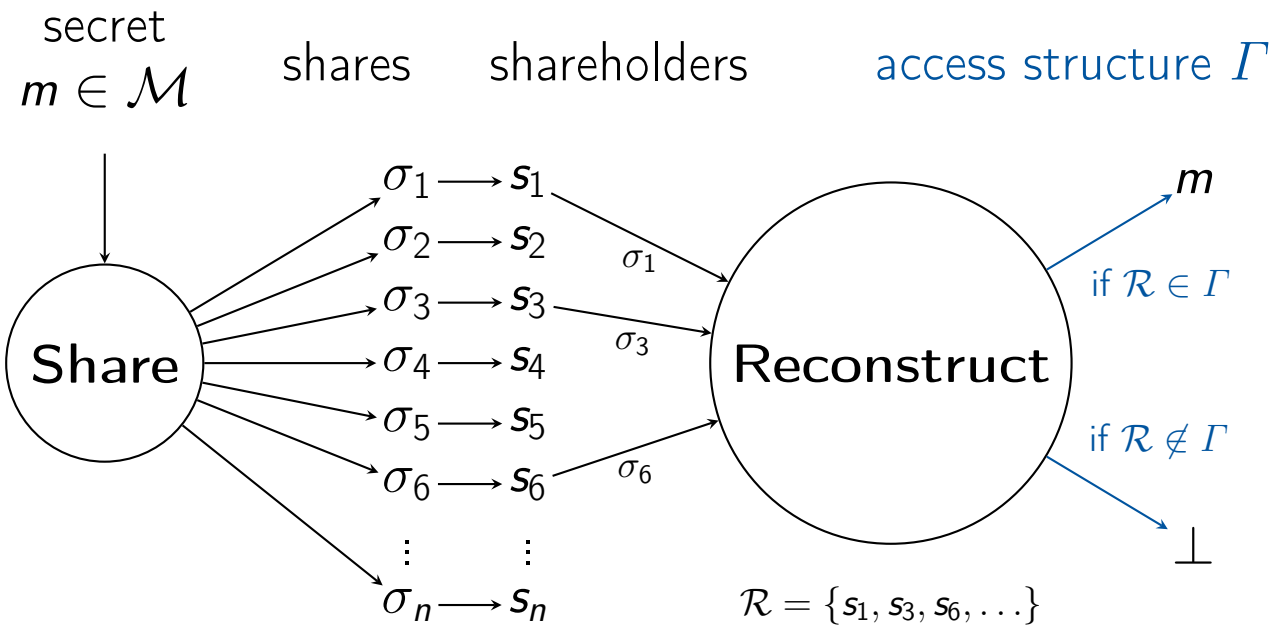
- Government, Electronic Votes
- Companies, Financial system

Secret Sharing Schemes

Secret Sharing Scheme

Definition (Secret Sharing Scheme)

A ***secret sharing scheme*** is a pair of PPT algorithms ***Share*** and *Reconstruct*.



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Secret Sharing Scheme

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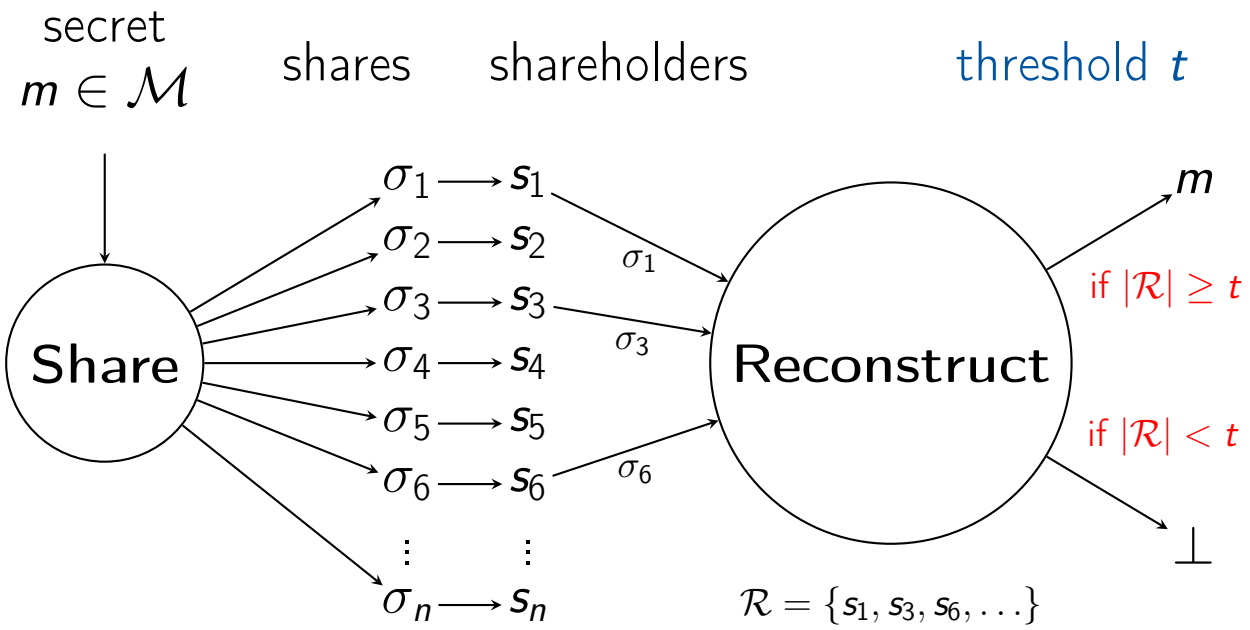
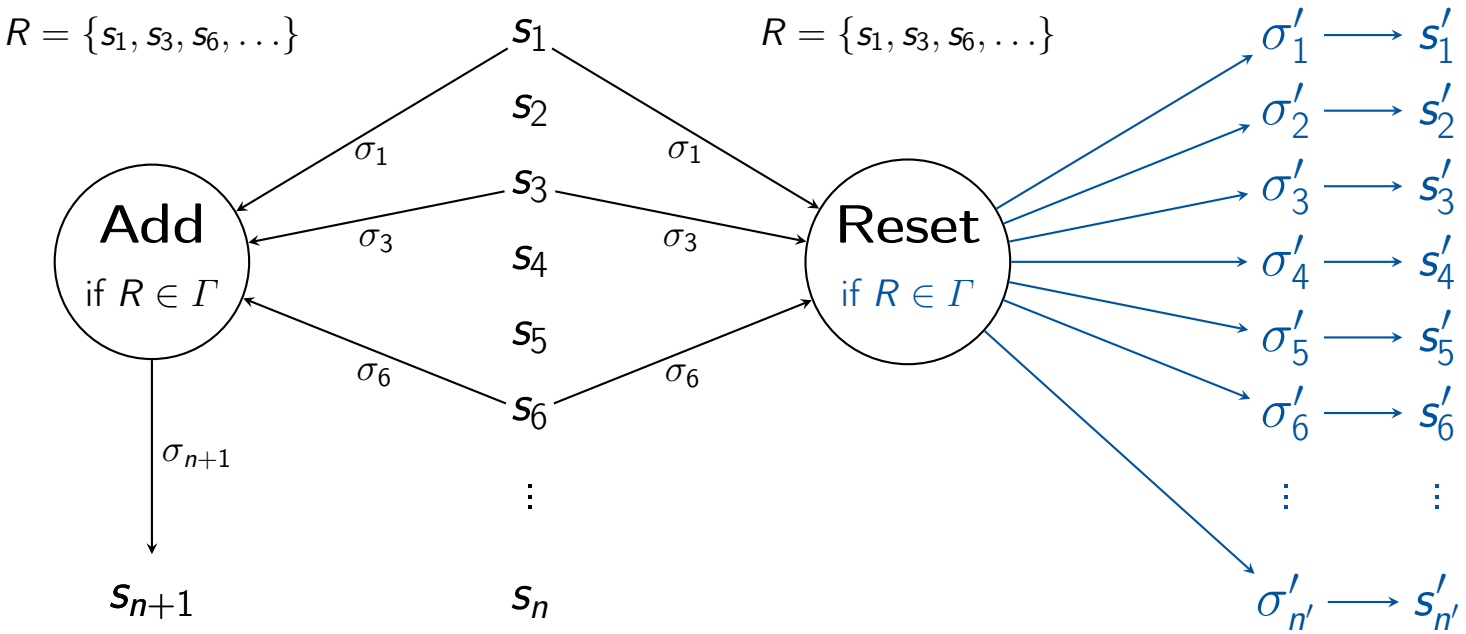


(t, n) Threshold Secret Sharing Scheme

Dynamic Secret Sharing

Definition (Dynamic Secret Sharing)

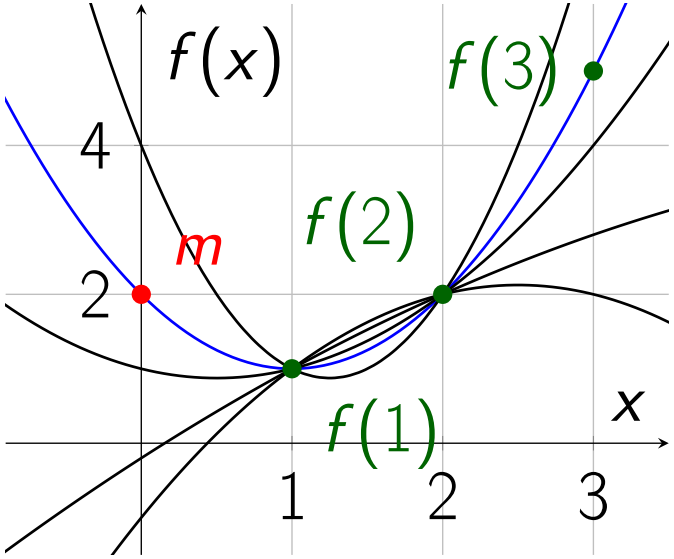
Same as secret sharing scheme (`Share` and `Reconstruct`), but with two additional PPT algorithms ***Add*** and ***Reset***.



(t, n) Threshold Secret Sharing Scheme

Example (Shamir's (t, n) Threshold Secret Sharing Scheme)

- Construct polynomial $f(x)$ of degree $t - 1$ where $f(0) = m$
- Distribute $f(i) = \sigma_i$ to shareholder s_i for $i \in \{1, \ldots, n\}$

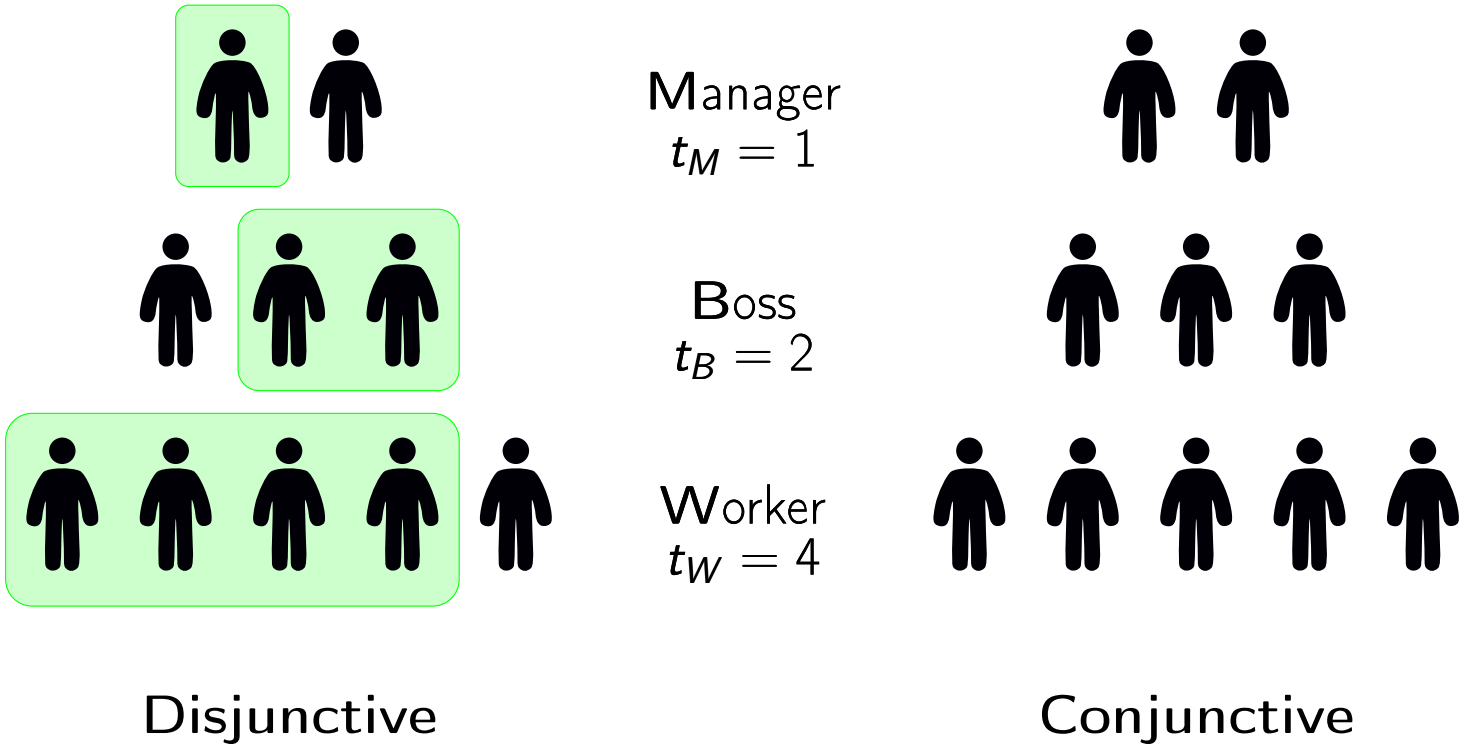


A $(3, 3)$ threshold scheme

$$f(x) = 2 - 2x + x^2$$
$$f(0) = 2 = m$$
$$f(1) = 1 \rightarrow s_1$$
$$f(2) = 2 \rightarrow s_2$$
$$f(3) = 5 \rightarrow s_3$$

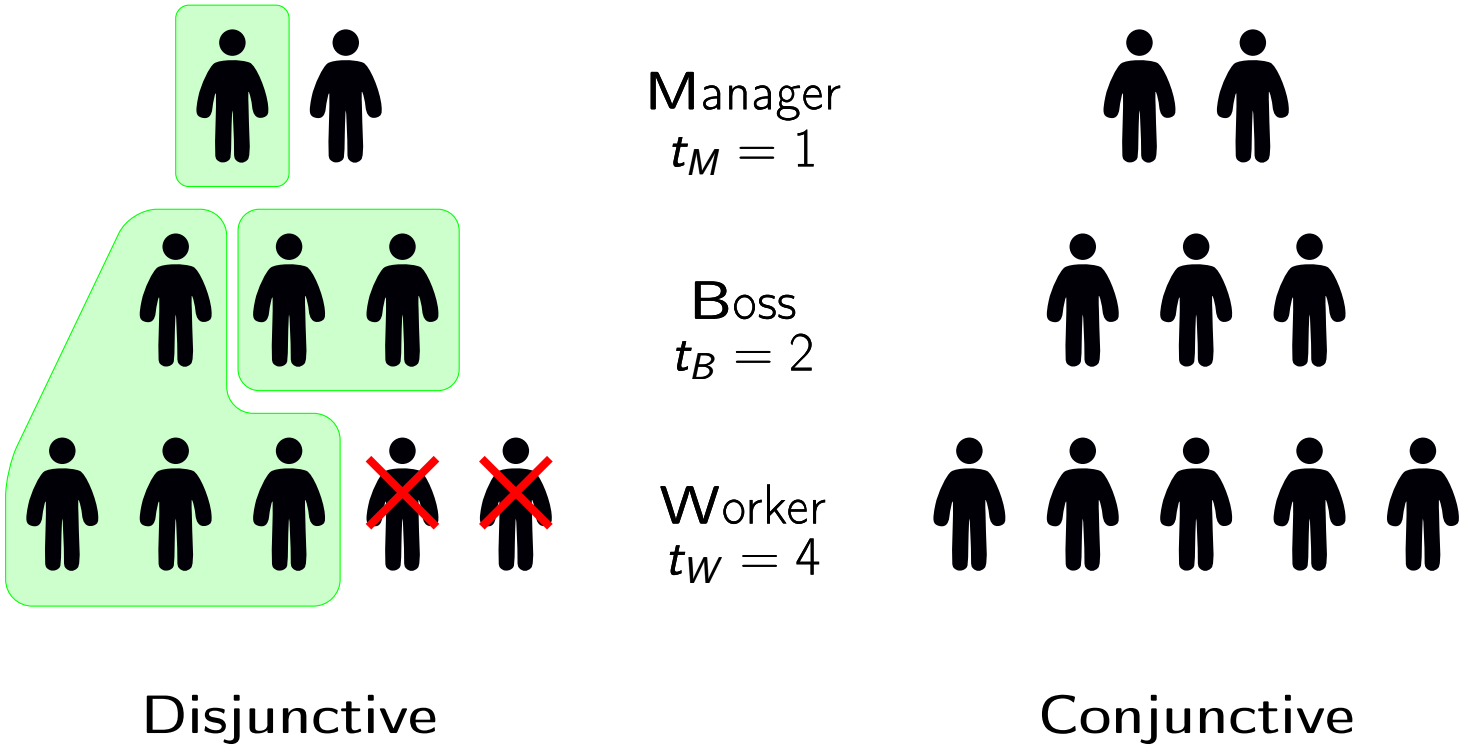
Adi Shamir. "How to Share a Secret". In: *Commun. ACM* 22.11 (1979)

Hierarchical Secret Sharing



Tamir Tassa. "Hierarchical Threshold Secret Sharing". In: *Journal of Cryptology* 20.2 (2007)

Hierarchical Secret Sharing



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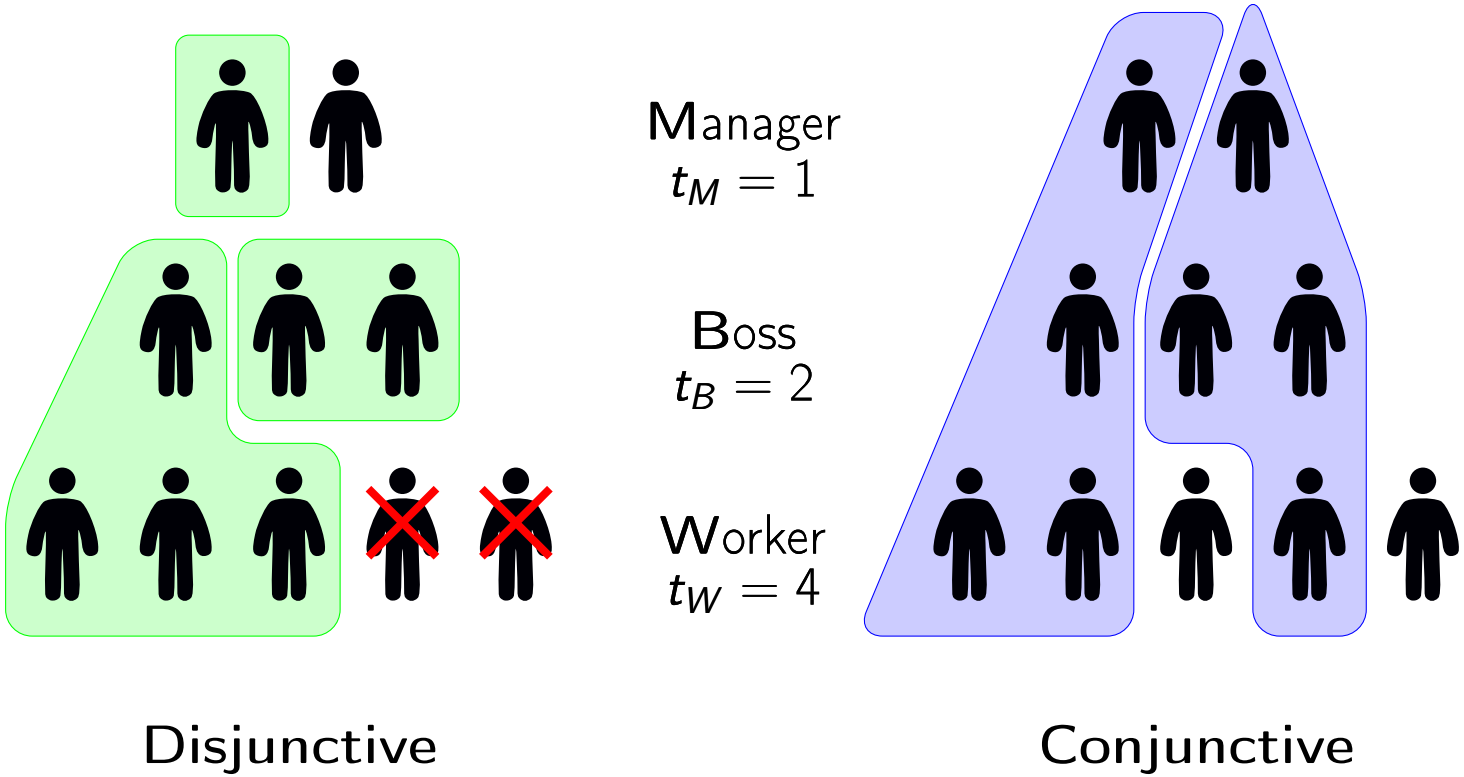
Hierarchical Secret Sharing



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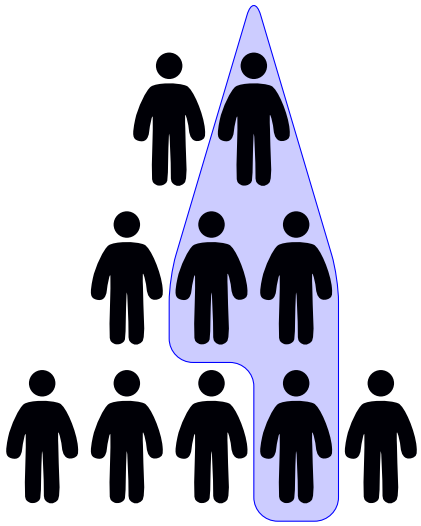
Hierarchical Secret Sharing

Example (Secret Sharing with Birkhoff Interpolation - Conjunctive)

- Setup: Secret $m = a_0$ and thresholds $0 < t_0 < t_1 < \ldots < t_\ell = t$
- Construct polynomial $f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_{t-1} x^{t-1}$
- The i-th shareholder on level j gets the **t_{j-1}-th derivative** $f^{t_{j-1}}(i)$



Conjunctive

$t_0 = 1$ $\quad f^0(1)$ $f^0(2)$

$t_1 = 2$ $\quad f^1(1)$ $f^1(2)$ $f^1(3)$

$t_2 = 4$ $\quad f^2(1)$ $f^2(2)$ $f^2(3)$ $f^2(4)$ $f^2(5)$

$$f^0(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$
$$f^1(x) = a_1 + 2a_2 x + 3a_3 x^2$$
$$f^2(x) = 2a_2 + 6a_3 x$$

Tamir Tassa. "Hierarchical Threshold Secret Sharing". In: *Journal of Cryptology* 20.2 (2007)

Hierarchical Secret Sharing

Reconstruction

- The secret is basically computed as $m = \sum_{i=1}^{4} \frac{\sigma_i \cdot \mathrm{A}_i}{\mathrm{A}}$
- A and A_i are determinants of publicly known matrices
- σ_i is the share of shareholder i

Add Shareholder

- Each shareholder computes a partial interpolation polynomial $f_i(x)$ s. t. $f(x) = \sum_i f_i(x)$
- The new share σ_y is then basically $\sum_i f_i(y)$

Reset Access structure

- Each shareholder computes a partial secret m_i s. t. $m = \sum_i m_i$
- Each shareholder shares m_i with a new polynomial $f_i'(x)$
- The new share σ_y' is then basically $\sum_i f_i'(y)$

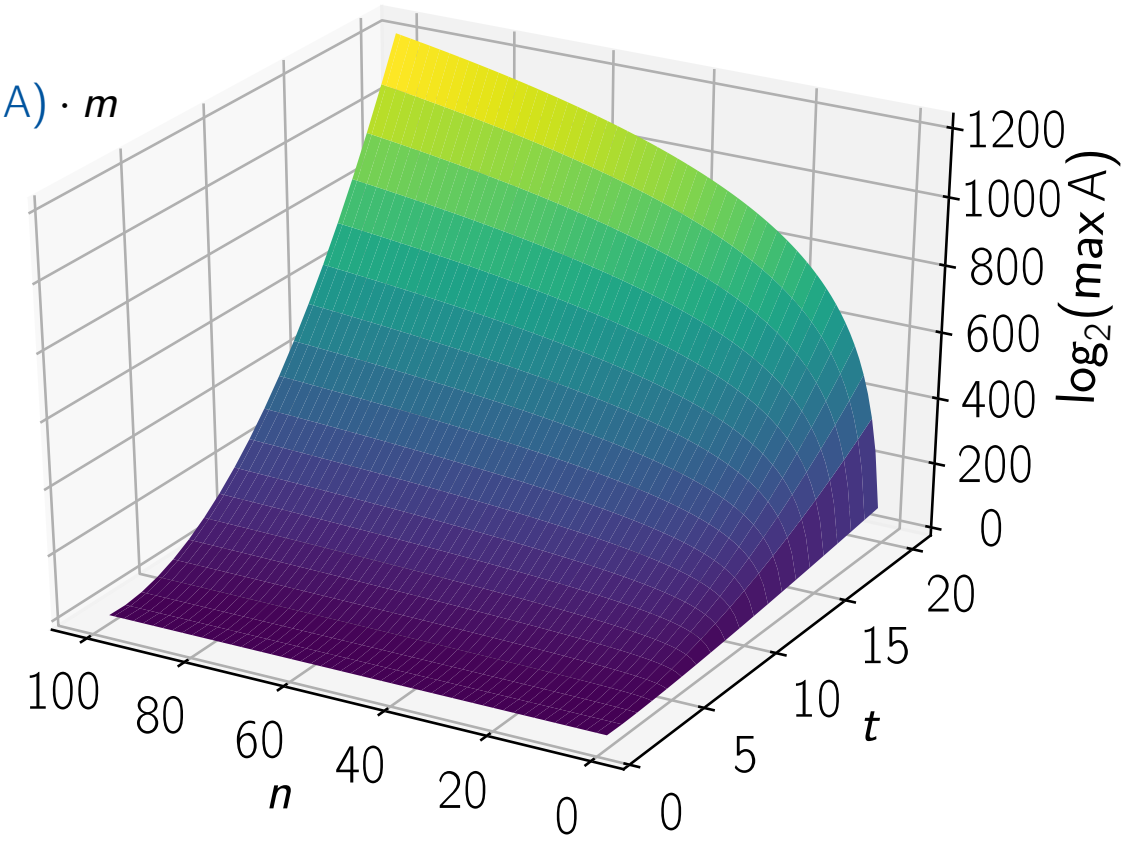
Note, we left out the derivatives/levels that are actually required.

Traverso et al. "Dynamic and Verifiable Hierarchical Secret Sharing". In: *ITS*. LNCS. Springer, 2016

“Secret Sharing” over the Integers

Basic Idea

- Choose coefficients from a **large** interval
- Flat: secret $= n! \cdot m$
- Hierarchical: secret $= \mathrm{lcm}(1, \ldots, A) \cdot m$



Verifiability

- Pedersen Commitments
- Zero-Knowledge Proofs

Paillier Cryptosystem

Paillier Cryptosystem - Properties

- **Asymmetric** and **probabilistic** cryptosystem
- **Semantically secure**
  $\rightarrow$ one message encrypts to many ciphertexts

- **Homomorphic addition**

$$E(m_1) +_h E(m_2) := E(m_1) \cdot E(m_2) = E(m_1 + m_2)$$

  - Homomorphic scalar multiplication

$$E(m) \times_h a := E(m)^a = E(m \cdot a)$$

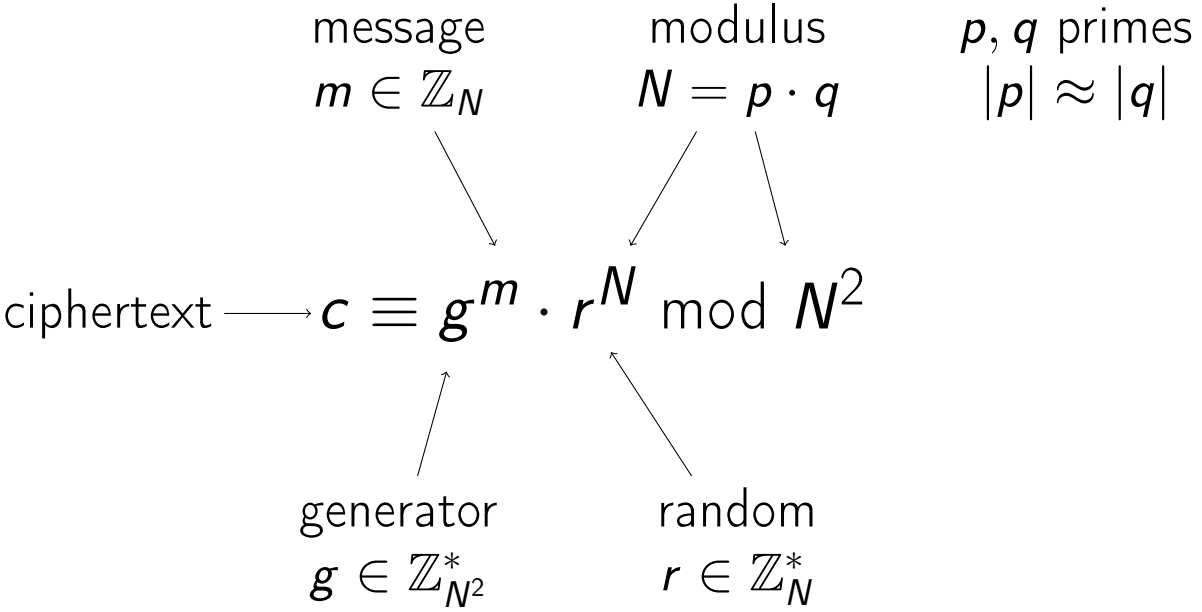
- **Self-Blinding** by adding a fresh encryption of "0"

$$E(m) +_h E(0)$$

Pascal Paillier. "Public-Key Cryptosystems Based on Composite Degree Residuosity Classes". In: *EUROCRYPT*. Lecture Notes in Computer Science. Springer, 1999

Paillier Cryptosystem - Simplified

Encryption



Decryption (simplified)

with private key $\lambda(N) = \mathsf{lcm}(p-1, q-1)$

$$c^{\lambda(N)} \rightarrow m$$

Pascal Paillier. "Public-Key Cryptosystems Based on Composite Degree Residuosity Classes". In: *EUROCRYPT*. Lecture Notes in Computer Science. Springer, 1999

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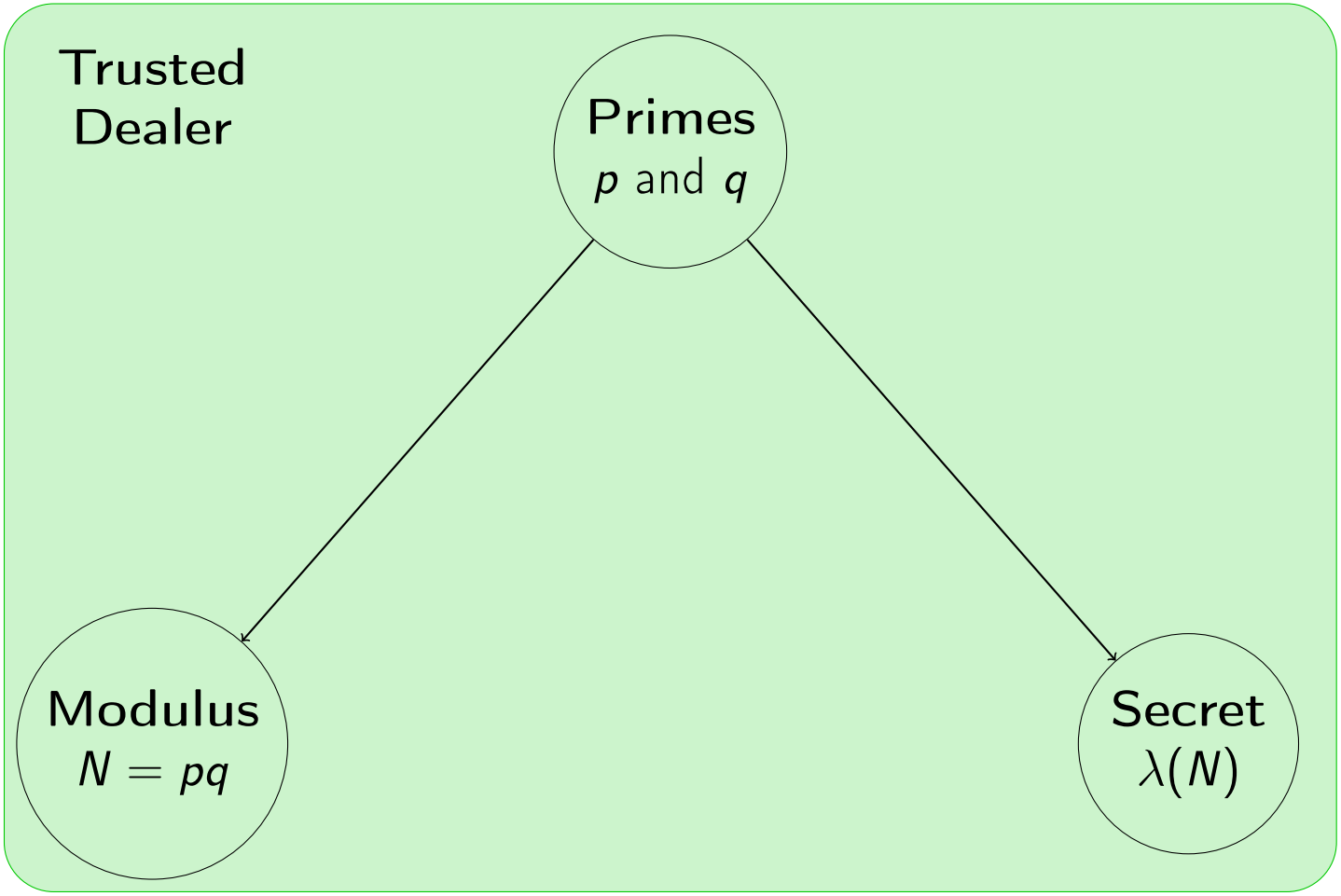
Threshold Paillier



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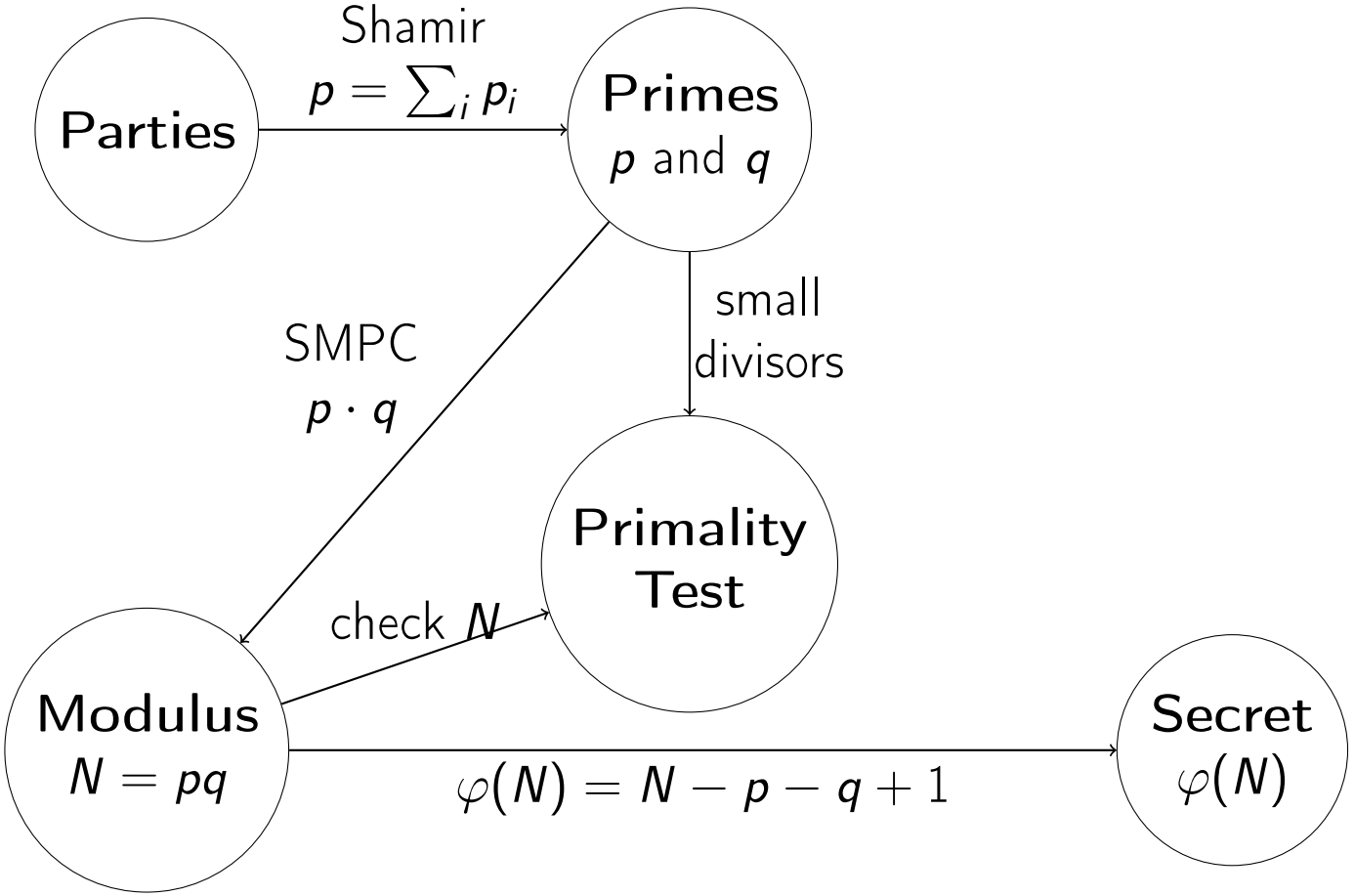
Threshold Paillier



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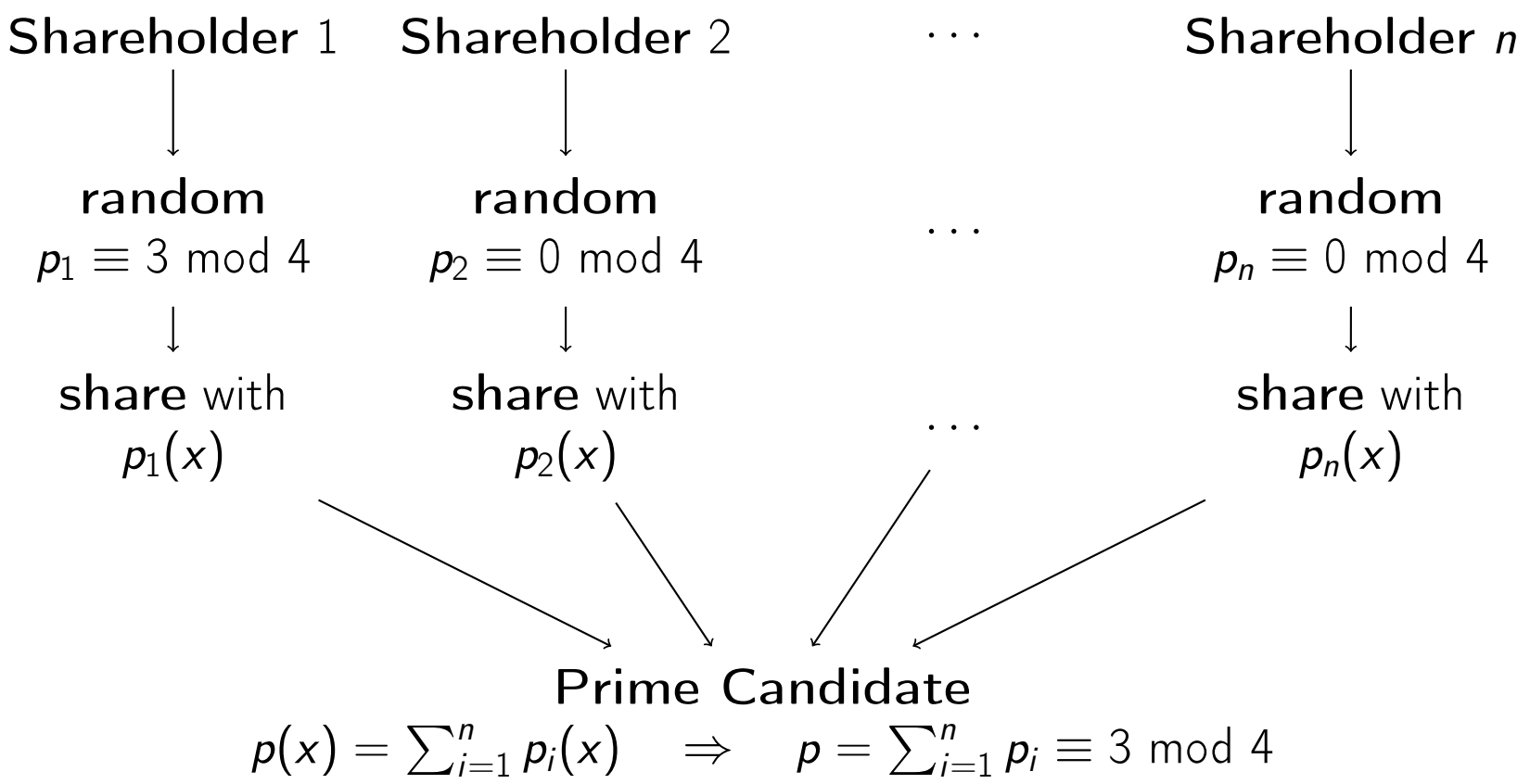
Prime Candidates



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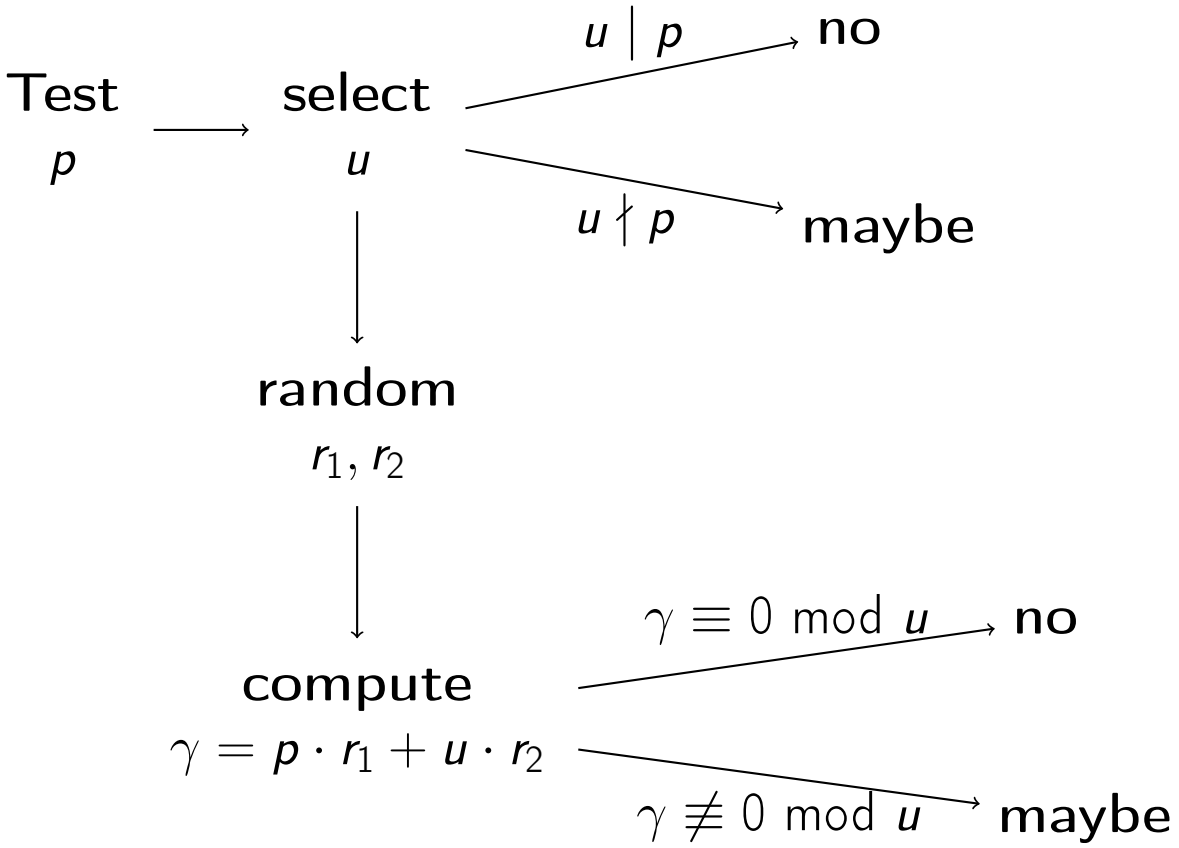
Trial Division



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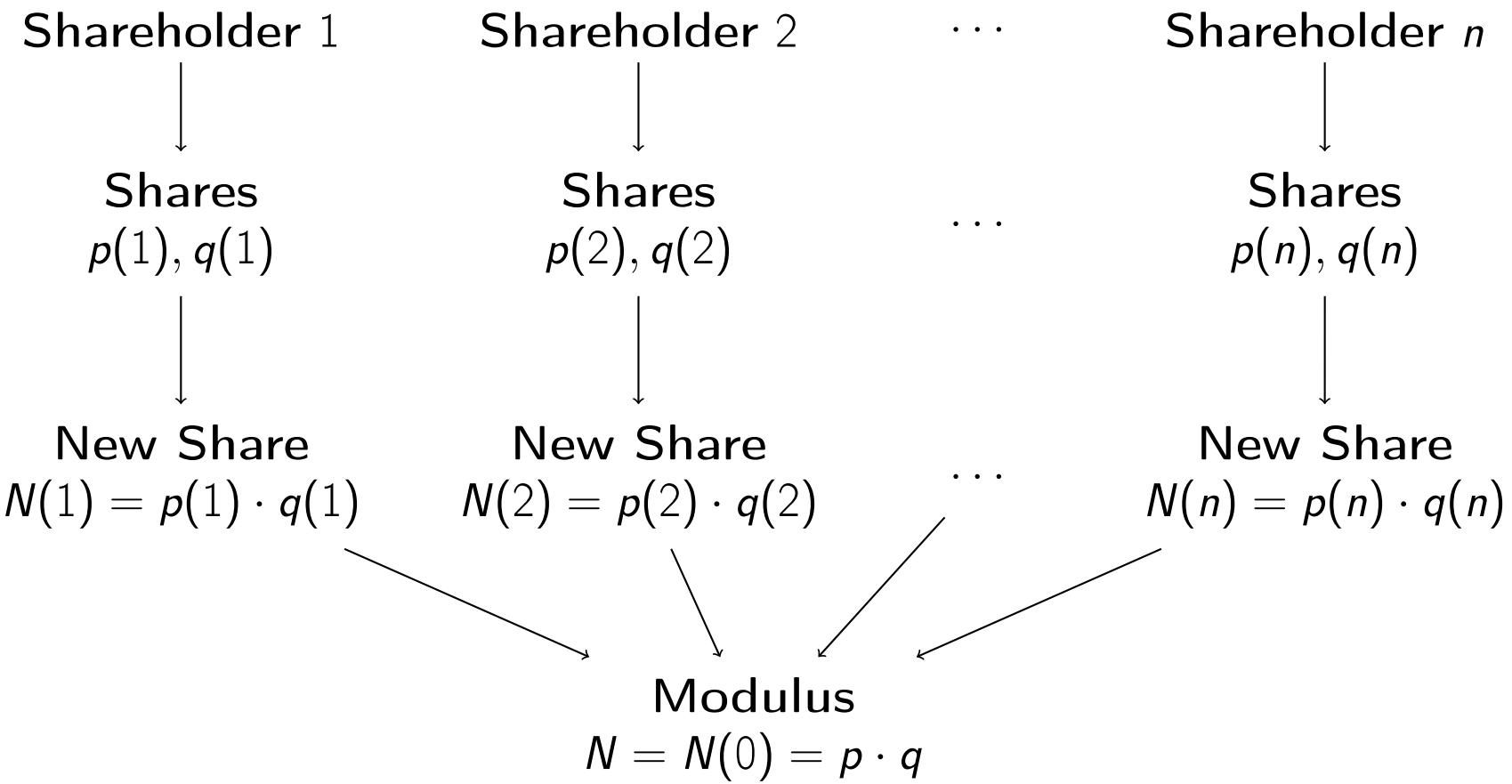
Multiplication - Modulus Generation



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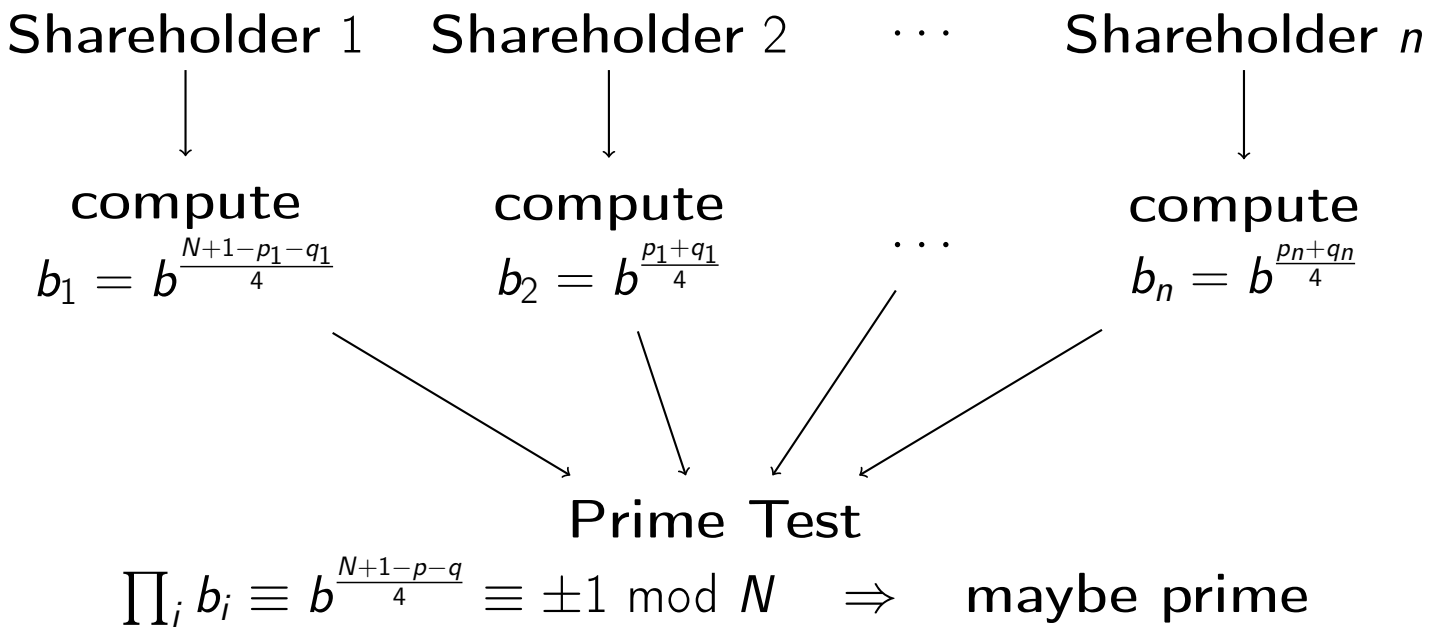
Biprimality Test

$$b^{\frac{N+1-p-q}{4}} \equiv \pm 1 \bmod N \quad \Rightarrow \quad \text{maybe prime}$$



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Final Steps

Private Key $\varphi(N)$

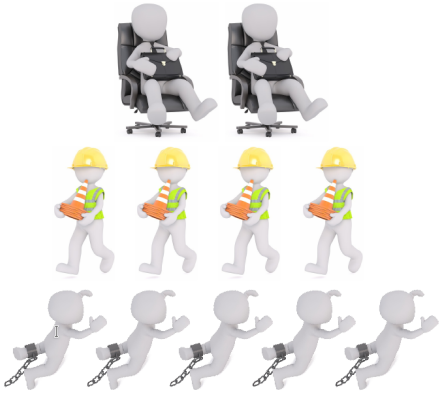
$$\begin{aligned}\varphi(N) &= (p-1)\cdot(q-1)\\ &= N-p-q+1\end{aligned}$$

Shares σ_i

$$\sigma_i = N - p(i) - q(i) + 1$$

Hierarchical Access Structure

Reset algorithm



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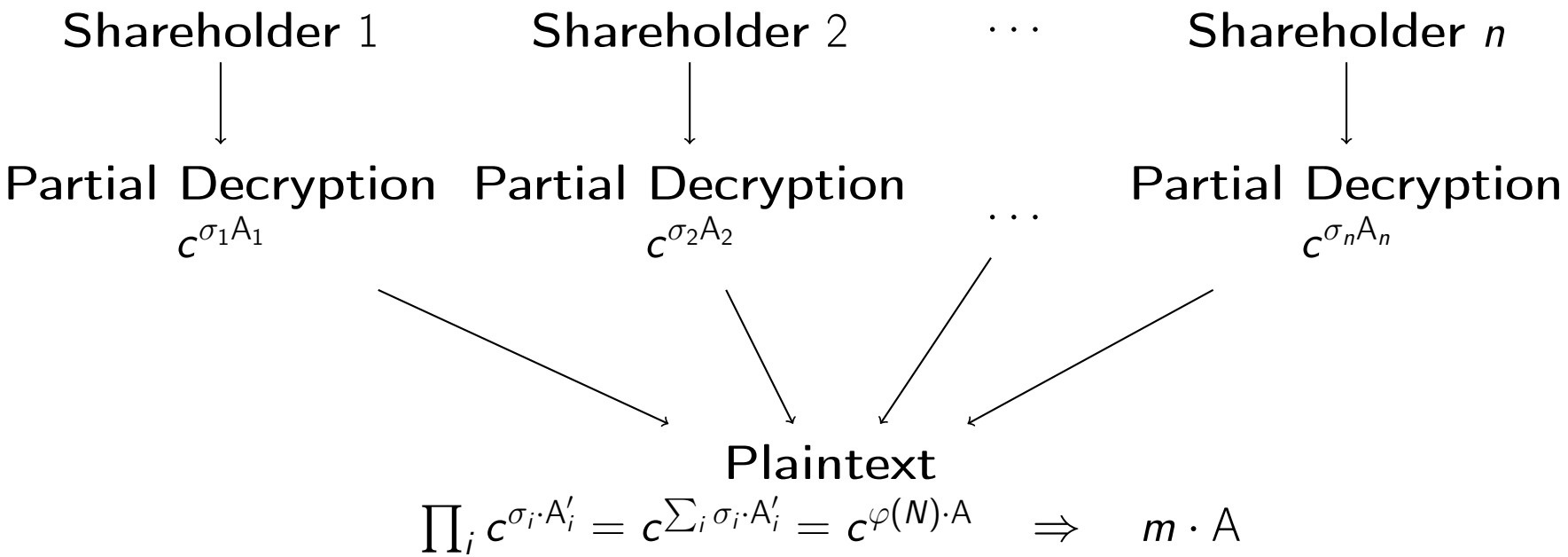
Decryption

$$c^{\varphi(N)} \quad \Rightarrow \quad m$$

$$\varphi(N) = \sum_i \frac{\sigma_i \cdot \mathrm{A}_i}{\mathrm{A}} \qquad \Rightarrow \qquad \varphi(N) \cdot \mathrm{A} = \sum_i \sigma_i \cdot \mathrm{A}_i$$



$$\prod_i c^{\sigma_i \cdot \mathrm{A}'_i} = c^{\sum_i \sigma_i \cdot \mathrm{A}'_i} = c^{\varphi(N) \cdot \mathrm{A}} \quad \Rightarrow \quad m \cdot \mathrm{A}$$

Conclusion

Contributions

- Verifiable hierarchical and dynamic secret sharing scheme over the integers
  - Add and remove shareholders
  - Hierarchical access structure
- Hierarchical and dynamic threshold Paillier cryptosystem without trusted dealer
  - Add and remove shareholders
  - Hierarchical access structure
  - No trusted dealer
  - Allows homomorphic addition
  - Secure against malicious adversaries

Future Work

- Implementation and real world performance evaluation
- Develop hierarchical threshold RSA

Discussion

Thank you for your attention!

References

- Takashi Nishide and Kouichi Sakurai. "Distributed Paillier Cryptosystem without Trusted Dealer". In: *ISA*. LNCS. Springer, 2010.
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