Proceedings of the 1. Conference on
Applied Mathematics and Computation
Dubrovnik, Croatia, September 13–18, 1999
pp. 263–271

Review of Analytical Methods for Calculating Currents and Potentials along Buried Conductors in an Impressed Field

Damir Šljivac[*], Srete Nikolovski[†], Boris Carić[‡], and Selva Moorthy[§]

Abstract. This paper presents the analysis of the behaviour of a buried pipeline located in an impressed field due to DC traction systems, toroidal ground electrodes and cathodic protection systems, and is directed primarily towards developing accurate expressions for pipeline currents and potentials arising from the interference. Typical cases of stray current conduction have been treated using a field theory approach. Simulation models for voltage gradients imposed upon buried pipelines are developed using the CDEGS software package.

AMS subject classification: 78A55

Key words: buried conductors, impressed field, currents and potentials, field theory approach, CDEGS

1. Introduction

It is common practice to use ground in order to provide the return path for direct current between two substations. The voltage gradient, arising where direct current is injected in the ground, ultimately results in stray currents in exposed metallic structures, affecting the buried pipelines and cable sheaths most significantly. Stray current propagation along buried conductors causes differences in the contact potential to earth, with the long term effects including corrosion of the conductor in the region where current is discharged into the surrounding soil. It is not, however, the intention of this paper to discuss these problems in detail. In the given analysis, an attempt has been made to derive, as accurately as possible, the formulæ for currents and potentials in pipelines in the vicinity of some common sources of stray current, such as DC traction systems and DC ground electrodes. The general expressions for currents in pipelines exposed to electrified tracks have been published by Sunde [8],

[*]Faculty of Electrical Engineering, University of Osijek, Kneza Trpimira 2b, 31000 Osijek, Croatia, e–mail: sljivac@etfos.hr

[†]Faculty of Electrical Engineering, University of Osijek, Kneza Trpimira 2b, 31000 Osijek, Croatia, e–mail: srete@etfos.hr

[‡]Electrical Distribution and Transmission Pty Ltd, 2–4 Burleigh Court, Blackburn, Vic. 3130, Australia, e–mail: bcaric@edt.com.au

[§]Department of Electrical Engineering, Royal Melbourne Institute of Technology, GPO Box 2476V, Melbourne, Vic. 3001, Australia, e–mail: moorthy@rmit.edu.au

who treats two separate cases — the case of a buried pipeline crossing an electrified track at right angles, and the case of long parallel stretches between the pipeline and the track. Based on these equations, currents and potentials in pipelines near a DC railway are evaluated by considering the case of two conductors in close proximity, with one of them (track) energized to a remote point over an insulated perpendicular wire [9]. Some previous investigations have presented the expressions for the electric field intensity along the pipeline in the vicinity of toroidal DC electrodes in a two-layer soil, produced by two infinite series of electrode images satisfying the boundary conditions at both surfaces [4, 6]. In the simpler case considered here, the electrode is located in a semi-infinite conducting space with a single electrode image above the ground surface, and the pipeline is assumed to be infinitely long, which makes it permissible to neglect the verge effects at the extremities of an actual long pipeline. The pipeline current and potential are obtained by assuming that a transmission line can adequately represent the electrical properties of a buried pipeline [5]. Following this reasoning, the expressions for stray currents in pipelines in an impressed field due to cathodic protection systems may be derived in a similar manner.

2. Basic equations

It is assumed that a straight insulated conductor of infinite length, radius a, unit length internal impedance Z_i, and coating leakage conductance G_i extends along x-axis on the surface of homogeneous earth. The conductor is located in an impressed electric field given by the expression:

$$E^i(x) = E'(x) - \frac{d}{dx}V'(x),$$

where the first term on the right hand side denotes a component due to stray current in the earth, and the reminder is the electric field due to variation in contact potential along the conductor. The current and the potential along the conductor are obtained as a solution to the general differential equation of propagation [9]:

$$G^{-1}\frac{d^2 i(x)}{dx^2} - Z \cdot i(x) = -E^i(x),$$

in the form:

$$i(x) = \frac{e^{-\Gamma x}}{2K}\int_{-\infty}^{x} E^i(\zeta)e^{\Gamma\zeta}\,d\zeta + \frac{e^{\Gamma x}}{2K}\int_{x}^{\infty} E^i(\zeta)e^{-\Gamma\zeta}\,d\zeta, \tag{1}$$

$$v(x) = \frac{e^{-\Gamma x}}{2}\int_{-\infty}^{x} E^i(\zeta)e^{\Gamma\zeta}\,d\zeta - \frac{e^{\Gamma x}}{2}\int_{x}^{\infty} E^i(\zeta)e^{-\Gamma\zeta}\,d\zeta. \tag{2}$$

The propagation constant and the characteristic resistance appearing in the previous expressions are defined in a usual way as:

$$\Gamma = \sqrt{ZG}, \qquad K = \sqrt{\frac{Z}{G}}, \tag{3}$$

with the conductor impedance and total leakage conductance given by:

$$Z(\Gamma) = Z_i + 2i\,\omega \cdot 10^{-7} \ln \frac{1.85}{\alpha a_e}, \qquad G(\Gamma) = \left(G_i^{-1} + \frac{1}{\pi\kappa} \ln \frac{1.12}{a_e \Gamma} \right)^{-1},$$

where α is the attenuation constant, κ – conductivity of the earth, and $a_e = \sqrt{a^2 + 4h^2}$ is an equivalent radius of the conductor buried at a depth h.

3. Currents in pipelines exposed to electrified tracks

The first approximations for currents and potentials along the pipeline which parallels an electrified track energized to a remote point over an insulated perpendicular wire, as shown in Figure 1, are obtained by [8]:

$$i_2(x) = I_1(0) \frac{G_2 \Gamma_1^2 (1-\nu)}{\pi\kappa(\Gamma_1^2 - \Gamma_2^2)} \Big(\Psi'(\Gamma_2 x) - \Psi'(\Gamma_1 x) \Big), \tag{4}$$

$$v_2(x) = -I_1(0) \frac{\Gamma_1^2 (1-\nu)}{\pi\kappa(\Gamma_1^2 - \Gamma_2^2)} \Big(\Omega'(\Gamma_2 x) - \Omega'(\Gamma_1 x) \Big), \tag{5}$$

where ν is taken as a ratio of the earth return mutual impedance between the trolley and the track to the earth return self impedance of the track, and may be neglected in the case of a DC railway, when the conductor impedance is substituted by its resistance. Propagation constants appearing in the previous expressions are obtained in a usual way from (3), and the track leakage conductance may be taken with sufficient accuracy as $G_1 = 0.34\kappa \div 0.66\kappa$, κ being the earth conductivity.

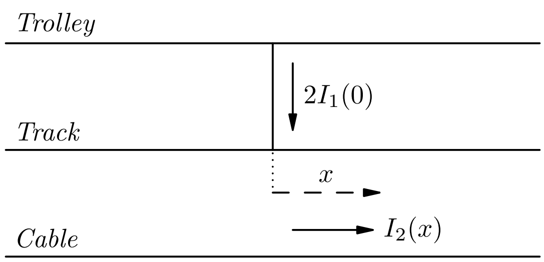

Figure 1. Current in a pipeline which parallels an electrified track.

The functions Ψ' and Ω' are defined in the Appendix, and are substituted by Ψ and Ω for DC railway.

In the case of the pipeline crossing the track at a right angle (Figure 2), the current and the potential are obtained in the form:

$$i_2(y) = I_1(0)\,\frac{G_2\Gamma_1}{2\pi\kappa\Gamma_2}\,(1-\nu)\cdot e^{-\Gamma_1|x|}\left\{e^{\Gamma_2 y}\,\mathrm{Ei}(\Gamma_2 r) - e^{-\Gamma_2 y}\big(\,\mathrm{Ei}(-\Gamma_2 r) + i\pi\big)\right\},$$
$$v_2(y) = -I_1(0)\,\frac{\Gamma_1}{2\pi\kappa}\,(1-\nu)\cdot e^{-\Gamma_1|x|}\left\{e^{\Gamma_2 y}\,\mathrm{Ei}(\Gamma_2 r) + e^{-\Gamma_2 y}\big(\,\mathrm{Ei}(-\Gamma_2 r) + i\pi\big)\right\},$$

where Ei is the exponential integral function given in the Appendix.

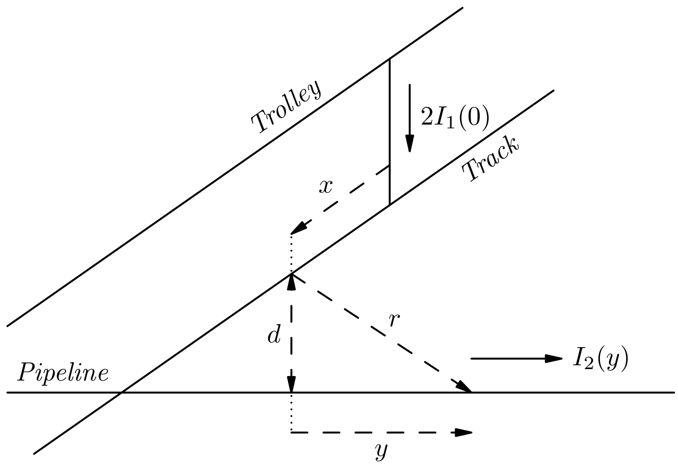

Figure 2. Current in a pipeline crossing an electrified track.

4. Impressed field calculations

This section briefly describes the effects of the voltage gradient in the vicinity of toroidal DC electrodes and cathodic protection systems on nearby pipelines. Assume a circular electrode of radius r_0 and uniform current leakage along the circumference, located at the origin of the coordinates with the z-axis through the center of the electrode and x- and y-axes in its plane. If the electrode is assumed to be at the surface of the earth, the scalar potential in the earth, expressed in cylindrical coordinates is obtained by [1, 9]:

$$V^i(r,z) = \frac{I_e}{2\pi^2\kappa\sqrt{r_0 r}}\,kK(k),$$

where I_e is the electrode return current and $K(k)$ is the complete elliptic integral of the first kind,

$$k = \sqrt{\frac{4r_0 r}{(r_0+r)^2+z^2}}$$

being its module.

The radial component of the electric field in the earth is given by [1]:

$$E_r(r,z) = \frac{V(0,0)}{2\pi r}\sqrt{\frac{r_0}{r}}\,\frac{k^3}{1-k^2}\left(k^{-2}\big(K(k)-E(k)\big)+\frac{r_0+r}{2r_0}\,E(k)-K(k)\right),$$

where $E(k)$ is the complete elliptic integral of the second kind, and

$$V(0,0) = \frac{I_e}{2\pi\kappa r_0}$$

is the potential in the center of the ring.

The impressed field along the pipeline extending parallel with the x-axis is obtained by differentiating the potential with respect to x [5, 6]:

$$E_x^i = -\frac{\partial V^i}{\partial r}\,\frac{\partial r}{\partial x} = \frac{x}{r}\,E_r,$$

where, for buried electrode, the contribution of an electrode image above the surface of the ground has to be included.

With the impressed field specified, the pipeline current and potential are obtained from (1) and (2), respectively. For a symmetrical voltage impressed along the pipeline, these equations may be expressed in the form:

$$i(x) = \frac{1}{2K}\int_0^{\infty} E_x^i(\zeta)\big(e^{-\Gamma|x-\zeta|}-e^{-\Gamma(x+\zeta)}\big)\,d\zeta,$$

$$v(x) = \frac{\Gamma}{2}\int_0^{\infty} V^i(\zeta)\big(e^{-\Gamma|x-\zeta|}+e^{-\Gamma(x+\zeta)}\big)\,d\zeta.$$

In the case of the pipeline exposed to impressed voltage gradient in the vicinity of an anode of the cathodic protection system, the same analysis applies. The potential due to an anode extending along z-axis between $-l/2$ and $l/2$ in a medium of infinite extent is determined by:

$$V^i(r,z) = \frac{I(0)}{4\pi\kappa l}\,\ln\frac{z+\frac{l}{2}+\sqrt{(z+\frac{l}{2})^2+r^2}}{z-\frac{l}{2}+\sqrt{(z-\frac{l}{2})^2+r^2}},$$

where l is the length of the electrode and $I(0)$ is the electrode current.

5. Computation results

Typical plots of the voltage gradients and impressed fields are obtained using the CDEGS software package [7].

The potential and the impressed field intensity along the pipeline buried at a depth $h = 1.5\,\mathrm{m}$, paralleled by an electrified track which is energized by the current

$2I_1(0) = 600\,\mathrm{A}$, with the region of parallelism extending over 4 km, are shown in Figure 3 and Figure 4, respectively.

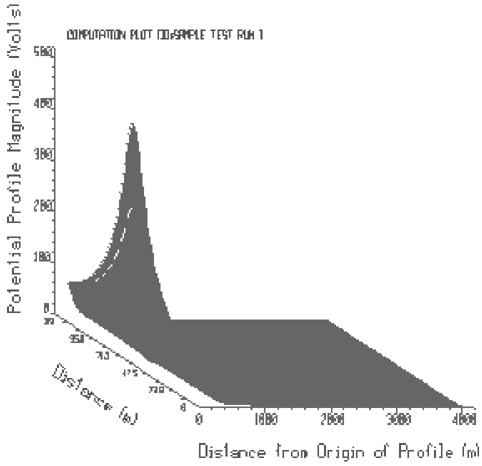

Figure 3. Impressed potential in earth along the pipeline profile.

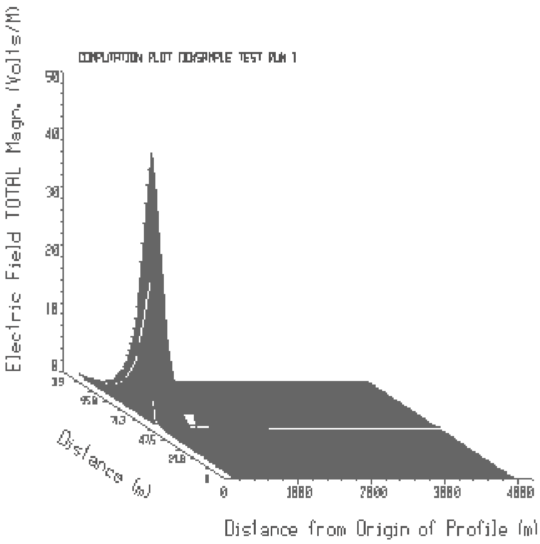

Figure 4. Impressed field in earth along the pipeline profile.

In order to obtain the potential and the electric field intensity in the vicinity of a DC ring of radius $r_0 = 300\,\mathrm{m}$, the circular electrode is represented by a twelve sided polygon. The ring is buried at a depth $h = 2.75\,\mathrm{m}$, and has the return current $I_e = 200\,\mathrm{A}$.

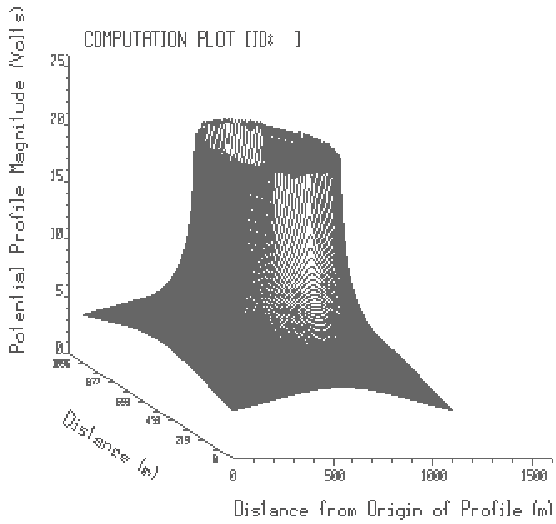

Figure 5. Potential distribution around a DC ring.

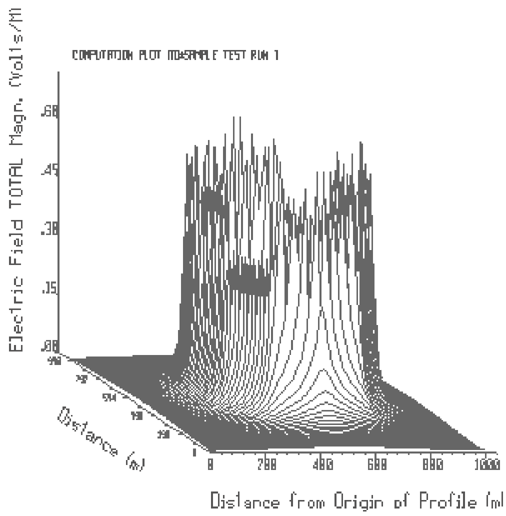

Figure 6. Electric field around a DC ring.

The voltage gradient around a vertical anode 1.2 m long, buried at a depth $h = 0.5$ m, with the current $I(0) = 2$ A through the anode, is shown in Figure 7.

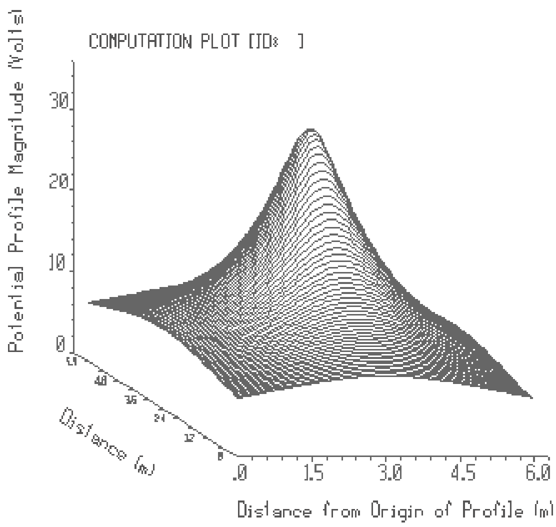

Figure 7. Voltage gradient around a vertical anode.

6. Conclusion

A mathematical model was developed to evaluate currents and potentials along ground return conductors in an impressed field due to electrified tracks and DC ground electrodes. The results obtained are found useful in the analysis of corrosion effects on buried steel pipelines, dependent upon the magnitude of the interference current.

Appendix

Exponential Integral Function

Exponential integral function is:

$$\mathrm{Ei}(\zeta) = \int_{\zeta}^{\infty} \frac{e^{-z}}{z}\, dz,$$

or

$$\mathrm{Ei}(\zeta) = -\gamma - \ln\zeta - \sum_{n=1}^{\infty} \frac{(-1)^n \zeta^n}{n \cdot n!},$$

where $|\arg\zeta| < \pi$ and $\gamma \approx 0.57722$ is Euler's constant.

Functions Φ, Ψ and Ω

$$\Phi(\xi,\zeta) = \int_{-\xi}^{\infty} \frac{e^{-\tau}\,d\tau}{\sqrt{\tau^2+\zeta^2}},$$

$$\Psi(\xi,\zeta) = \frac{1}{2}\left(e^{-\xi}\Phi(\xi,\zeta) - e^{\xi}\Phi(-\xi,\zeta)\right),$$

$$\Omega(\xi,\zeta) = \frac{1}{2}\left(e^{-\xi}\Phi(\xi,\zeta) + e^{\xi}\Phi(-\xi,\zeta)\right).$$

The function Φ is related to Bessel functions of the first and second kind, zero order, as follows:

$$\Phi(\xi,\zeta) = J_0(\zeta)\,\ln\frac{\zeta}{w-\xi} - \frac{\pi}{2}\,Y_0(\zeta) + \sum_{n=1}^{\infty}\frac{1}{n!}\,A_n,$$

where $w = \sqrt{\xi^2+\zeta^2}$, $A_0 = 0$, $A_1 = w$, and $A_n = \frac{1}{n}(w\xi^{n-1} - (n-1)\zeta^2 A_{n-2})$, $n \geq 2$.

Functions Ψ' and Ω'

These functions appear in the expressions (4) and (5), and may be expressed in terms of Ψ and Ω as:

$$\Psi' = \pm\pi\kappa Z_{12}\frac{1-e^{-\Gamma|x|}}{\Gamma^2} + \Psi(\Gamma x, \Gamma h),$$

$$\Omega' = \pi\kappa Z_{12}\frac{e^{-\Gamma|x|}}{\Gamma} - \Gamma\,\Omega(\Gamma x, \Gamma h).$$

References

[1] T. Bosanac, *Teoretska Elektrotehnika*, Tehnička knjiga, Zagreb, 1973.

[2] F. P. Dawalibi, *Electromagnetic fields generated by overhead and buried short conductors*, IEEE Transactions, PWRD–1, no. 4, October 1986, pp. 105–119.

[3] F. B. Hildebrand, *Methods of Applied Mathematics*, Prentice–Hall, Englewood Cliffs, NJ, 1965.

[4] P. J. Lagace, J. L. Houle, H. Greiss, and D. Mukhedkar, *Computer aided design of a toroidal ground electrodes in a two layer soil*, IEEE Transactions, PWRD–2, no. 3, July 1987, pp. 744–749.

[5] P. J. Lagace, J. L. Houle, H. Greiss, and D. Mukhedkar, *Computer aided evaluation of pipeline current near toroidal HVDC electrodes*, IEEE Transactions on Power Delivery, vol. 4, no. 1, January 1989, pp. 216–222.

[6] S. Moorthy, B. Carić, and T. Bergin, *Evaluation of the electric field intensity along a pipeline near a toroidal DC electrode in a two layer soil*, in Proceedings of the Third Biennial Engineering Mathematics and Applications Conference, Adelaide, July 1998, pp. 371–374.

[7] Safety Engineering Services and Technology, Ltd., *CDEGS Users Manual, HIFREQ Users Manual*, September 1997.

[8] E. D. Sunde, *Currents and potentials along leaky ground return conductors*, Electrical Engineering, December 1936, pp. 1338–1346.

[9] E. D. Sunde, *Earth Conduction Effects in Transmission Systems*, D. Van Nostrand, Toronto, New York, London, 1949.