Multi-scale methods for reactive flow and transport in complex elastic media

Conference in memory of prof. Andro Mikelic

Dubrovnik, Croatia
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SCIENTIFIC PROGRAM

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INVITED LECTURES

Sharp interface limit of a Navier-Stokes/Allen-Cahn system with vanishing mobility

Helmut Abels (University of Regensburg)

Abstract: We consider the sharp interface limit of a Navier-Stokes/Allen-Cahn system, when a parameter $\varepsilon > 0$ that is proportional to the thickness of the diffuse interface tends to zero, in a two dimensional bounded domain. In dependence on the mobility coefficient in the Allen-Cahn equation in dependence on $\varepsilon > 0$ different limit systems or non-convergence can occur. In the case that the mobility vanishes as ε tends to zero slower than quadratic we prove convergence of solutions to a smooth solution of a classical sharp interface model for well-prepared and sufficiently smooth initial data. The proof is based on a relative entropy method and the construction of sufficiently smooth solutions of a suitable perturbed sharp interface limit system.

This is a joint work with Julian Fischer and Maximilian Moser (ISTA Klosterneuburg, Austria).

Coupled surface and subsurface flow simulation

Peter Bastian (University Heidelberg)

The simulation of coupled surface and subsurface water flow is important to assess flooding events, water scarcity and groundwater renewal. In the talk we will present a model coupling shallow surface flow described by the diffusive wave approximation with shallow groundwater flow. The model is naturally able to handle streams and lakes when a suitable bathymmetry is given. For the numerical solution we will compare cell- and vertex-centered finite volume and discontinuous Galerkin methods using different numerical fluxes. The numerical model is able to take reasonably large time steps and is parallelized by a domain decomposition approach. As an application a small, high-altitude catchment in Ladakh, India, is considered, where we have calibrated the model on a given hydrograph and then reconstructed the run-off over nearly two decades.

This is a joint work with Panasun Manorost.

TBA

Anna Marciniak-Czochra (Heidelberg University)

Weak solutions to fluid–structure interaction: problems with poroelastic media

Boris Muha (University of Zagreb)

We consider the interaction between an incompressible, viscous fluid and a poroelastic medium modelled by a Biot equation. Both linear and nonlinear cases are considered. In the linear case, the fluid flow and the poroelastic structure are coupled across a fixed interface, while in the nonlinear case, they are coupled via the moving interface. We discuss the existence of weak solutions for both cases and highlight the primary mathematical challenges associated with the analysis of fluid-poroelastic structure interactions (FPSI), particularly focusing on the difficulties arising from studying the moving boundary problem. Furthermore, we explore numerical methods for solving FPSI problems.

This is joint work with L. Bociu (North Carolina State University), M. Bukač (University of Notre Dame), S. Čanić (University of California, Berkeley), J. Kuan (University of Maryland) and J. Webster (University of Maryland).

On the problem of the compressible fluid on the domain with rough boundary and the case with slip boundary conditions of friction type

Šárka Nečasová (Czech Academy of Sciences)

We study a problem of the compressible fluid on the domains with rough boundary. We define on the rough boundary the slip boundary conditions and on the smooth part of the boundary non-slip boundary condition. we show that if the roughness is going to zero that in the limit we get the no-slip boundary condition. Further, we consider the compressible flow with slip boundary conditions of friction type and we show the existence of weak solutions to this model.

It is a joint work with K. Bhandari, M. Gahn, M. Neuss-Radu, M. Rodríguez-Bellido, J. Scherz, J. Ogorzaly.

TBA

Felix Otto (Max Planck Institute for Mathematics in the Sciences in Leipzig)

Homogenization of non-stationary convolution type operators in environments periodic in spatial variables and random in time

Andrey Piatnitski (The Arctic University of Norway)

The talk will focus on homogenization of a Cauchy problem for convolution type evolution equations with an integrable convolution kernel and rapidly oscillating coefficients which are periodic in spatial variables and random stationary in time.

We assume that the convolution kernel has finite second moments, the operator satisfies the uniform ellipticity (coerciveness) conditions, and the coefficients have sufficiently good mixing properties in time.

Our goal is to show that, under the above conditions, a solution of the original Cauchy problem converges in law to a solution of the limit SPDE and to describe the properties of the effective problem.

This is a joint work with Elena Zhihina.

Stability and convergence of in time approximations of hyperbolic elastodynamics via stepwise minimization

Sebastian Schwarzacher (Uppsala University)

Step-wise time approximations of non-linear hyperbolic initial value problems are considered. The main applications are from elastodynamics namely so-called generalized solids, undergoing large deformations. The technique used here is a generalization of the minimizing movements method, using two time-scales: one for velocity, the other (potentially much larger) for acceleration. The evolution follows an underlying variational structure exploited by step-wise minimisation. We showed for a family of (elastic) energies that the introduced scheme is stable. If the highest order term can assumed to be linear, we show that the limit solutions are regular and that the minimizing movements scheme converges with optimal linear rate.

The talk is based on a joint work with A. Cesik.

Phase-field modeling of propagating fluid-filled fractures coupled to a surrounding porous medium

Thomas Wick (Leibniz University Hannover)

In this talk, we discuss the heritage of Andro Mikelić's co-authored work starting in the year 2013 on phase-field descriptions of fluid-filled fractures coupled to a surrounding solids and porous media. In phase-field fracture two principle variables couple: vector-valued displacements (mechanics) and a scalar-valued smoothed indicator function describing a single fracture, multiple fractures, or even fracture networks. This results into coupled variational inequality systems (CVIS). Based on the initial work, extensions towards multiphysics phase-field fracture have been performed, namely proppant-filled fractures, two-phase fracture flow, non-isothermal flow, up to thermal-hydraulic-mechanical-chemical phase-field fracture. From the numerical side adaptive discretizations, efficient and robust solvers such as error-controlled adaptivity, predictor-corrector mesh refinement, Newton-type techniques, multigrid methods, and non-intrusive global-local multiscale techniques have been developed. Most recently, phase-field is used as a predictor step, followed by a remeshing procedure to describe sharp interface conditions and Stokes fracture flow, which has been in Andro's interest since the beginning, but never realized together. Our focus in this presentation is on modeling aspects, numerical advancements, ongoing work and open questions.

CONTRIBUTED TALKS

Quantitative stochastic homogenization of variational models arising in fracture mechanics

Nicolas Clozeau (ISTA Austria)

I will present a recent quantitative result concerning the homogenization of the so-called Griffith type model arising in fracture mechanics. Since the work of Cagnetti, Dal Maso, Scardia and Zeppieri, the homogenized model has been identified qualitatively and in particular the two main constitutive properties of the system have been derived : the homogenized elastic energy and the homogenized fracture toughness, both given explicitly by means of cell-formulas. I will explain in this talk how we can derive quantitative estimates for the convergence of the cell-formula for the effective toughness.

This is based on a joint work with Julian Fischer and Antonio Agresti.

Asymptotic analysis of heat transfer in flow through dilated pipe

Andrijana Ćurković (University of Split)

We study the asymptotic behavior of the fluid temperature in a heat conduction problem in a dilated pipe with a small circular cross-section. The fluid is governed by the pressure drop, and the heat exchange between the fluid and the surroundings is described by Newton's cooling law. The temperature is the solution of the convection-diffusion equation with a stationary Poiseuille velocity. Due to pipe dilation, the fluid domain is no longer fixed and changes as a function of the unknown temperature. By introducing a suitable change of variables, the domain becomes fixed, but the PDE becomes nonlinear.

Using asymptotic analysis with respect to the small parameters (coefficient of heat expansion and ratio between pipe thickness and length), an asymptotic solution is observed. Although the approximation is initially defined on a fixed domain (defined by the change of variables), the significance of considering the approximate solution on the original extended domain is presented. By proving the error estimate for the approximation on the extended domain, the justification of the effective model is given.

This is a joint work with Eduard Marušić-Paloka.

Swelling of porous media: analysis and Homogenization

Michael Eden (Karlstad University, Sweden)

Swelling of porous media is an example of a type of problems where the geometry is allowed to evolve and where microscopic effects (growing pore structures) influence the macroscopic transport properties of the system. As a consequence of the changes in geometry, the resulting mathematical problems are generally highly nonlinear and complex in and of itself; where the scale separation adds further difficulties.

In the simplified case of uniform cell evolutions, we show well-posedness of a poroelasticity system describing the effective fluid movement through swelling porous media and conduct a limit analysis with respect to the scale parameter. Finally, we mention some of the difficulties in extending these results to the case of more general cell evolutions.

Derivation of effective interface conditions for fluid flow in bulk domains separated by a thin porous elastic layer

Markus Gahn (University Heidelberg)

The aim of this talk is the rigorous derivation of a coupled Stokes-plate model for fluid flow across a thin porous elastic layer, which separates two fluid-filled bulk domains. The thin periodically perforated layer consists of a fluid and an elastic solid part. Thickness and periodicity of the layer are of order ϵ, where the parameter ϵ is small compared to the size of the bulk domains. We assume small deformations and therefore consider linear elasticity in the solid. The evolution of the fluid flow is described by an instationary Stokes system. At the fluid-solid interface we assume a linearized coupling condition for the velocities and the stresses. For $\epsilon \to 0$ the thin layer reduces to an interface Σ separating the two bulk domains. We proceed a simultaneous homogenization in the layer and dimension reduction to derive a macroscopic model with effective interface laws and homogenized coefficients on Σ.

To pass to the limit we use multi-scale techniques adapted to problems in continuum mechanics, including extension operators for perforated domains preserving ϵ-uniform bounds for the symmetric gradient, Korn-inequalities, and two-scale compactness of ϵ-dependent sets in Sobolev spaces on thin perforated domains. The macroscopic model consists of the instationary Stokes equations in the bulk domains coupled to a time dependent plate equation on the interface Σ including homogenized elasticity coefficients carrying information about the micro structure of the layer. The macroscopic displacement is given as a Kirchhoff-Love displacement. The macroscopic fluid velocity is continuous at the interface and equal to the vertical movement of the interface Σ.

This is a joint work with Willi Jäger and Maria Neuss-Radu.

Homogenization of the Navier-Stokes equations in perforated domains via relative energy estimates

Richard Höfer (Regensburg University)

We revisit homogenization problems of fluid flows in perforated domains which have applications in porous media and particulate flows. We consider the solution u_ε to the Navier-Stokes equations in $\mathbb{T}^3$ perforated by small particles centered at $(\varepsilon\mathbb{Z})^3$ with no-slip boundary conditions at the particles. We study the behavior of u_ε for small ε, depending on the diameter ε^α, $\alpha \geq 1$, of the particles and the viscosity ε^λ, of the fluid, both for compressible and incompressible flows. If the local Reynolds number on the length-scale of the particles is small and density fluctuations are negligible, one expects that the effective influence of the particles is governed by the asymptotic validity of a linear friction law (Stokes law). This reasoning can be made rigorous and a variety of effective macroscopic equations are obtained, depending on α and γ, including Darcy's law and Brinkman equations. The case $\alpha = 1$ for incompressible flows has been treated by Mikelic in 1991, and for compressible flows by Masmoudi in 2002, the latter giving rise to the porous medium equation. Based on relative energy estimates, we present new results regarding effective equations, their regimes of validity and convergence rates.

Multiscale problems in catalytic filters

Oleg Iliev (Fraunhofer Institute for Industrial Mathematics, ITWM, Kaiserslautern)

The catalytic membranes can degrade gaseous pollutants to clean gas via a catalytic reaction to achieve green emissions. A catalytic membrane is a three scale porous medium. The walls of catalytic filters are of size of centimeters or millimeters and consist of active (washcoat) particles, inert material and microscale, micron size pores. The washcoat particles are porous material with nanoscale pores. The catalytic reactions are heterogeneous (surface reactions) and they occur within the nanopores. Obviously, simulations at fully resolved pore scale are not feasible, and upscaling techniques have to be applied. It is known that the same microscale problem can be upscaled to different macroscale equations depending on the characteristic numbers. In this talk we present two studies: (A) Homogenization of reactive flow in the presence of strong absorption in the washcoat particles. Two flow regimes are studies, $P_{ef} = O(1)$ and $P_{ef} = O(\varepsilon - 1)$, and two different upscaled equations are obtained, respectively. The both derived upscaled equations are numerically validated comparing their solution to the solution of the microscale problem. (B) Three-way catalyst material was deposited inside the pores of a ceramic particulate filter and the pore geometry as well as the distribution of the catalyst in the pores was determined by xX-ray -microtomography (CT). On the resulting 3D geometry, the flow field through the pores was computed and the convection diffusion reaction equation in the open pores and the catalyst particles was solved assuming a first order reaction taking place in the catalyst. The conversion in the filter wall was compared to a homogeneous model with the same dimensions.

This is a joint work with Torben Prill, Pavel Toktaliev, Arsha Sherly and Andro Mikelić.

The Dubrovnik-meetings on multiscale-systems - Looking back to the future: diffusion, transport and reactions in interacting fluids and poro-elastic media - Applications in life sciences

Willi Jäger (IWR, Universität Heidelberg)

This lecture is dedicated to the memory of Andro Mikelić. He was one of the main organizers of the previous conferences in Dubrovnik on the topic and influenced substantially their scientific focus and high quality as well as their stimulating atmosphere. He is known as an outstanding pioneer in modelling, analysis and numerics for multiscale systems. His concepts, ideas and results had and will have a strong influence on the future development in this field.

An overview of his contributions is followed by a discussion of selected open problems in the field of fluid-structure interaction, coupling the dynamics of fluids and poro-elastic structures, diffusion, transport and reaction of the chemical or biological species, and growth of the solid structure. In particular processes at interfaces and in layers gating and controlling the transmission between compartments are of interest. Applications to endothelial and epithelial layers in bio-medical systems are posing challenges to mathematical research, hot topics to be discussed in this meeting.

Poroelasticity and large strain fluid-structure interaction

Malte Kampschulte (Charles University Prague)

Porous media equations describe the flow of a fluid through a given fixed medium and how this flow is affected by that mediums properties. By now it is reasonably well understood how those models can not only be derived phenomenological, but actually arise as a rigorous homogenization limit of a fluid flow through a given micro-scale geometry. Poroelasticity in contrast deals with the case where the porous medium itself can deform elastically. While in that case there exists a similar wealth of models, these are much less well understood in terms of homogenization. The aim of this talk is to present a first result on such an homogenization, where the coupling of a fluid with a solid deforming at large strains leads to a macroscopic model of a poroelastic system.

This is joint work with Pei Su.

Efficient numerical simulation of effective micro-macro models for reactive transport in elastic perforated media

Jonas Knoch (Friedrich-Alexander-Universität Erlangen-Nürnberg)

In this talk we consider an effective elasticity-transport system of micro-macro type in a unified *Lagrangian* framework, derived by a formal asymptotic expansion from a microscopic model defined in an elastically deformable perforated medium and formulated in a mixed *Eulerian/Lagrangian* framework. The effective system, consisting of a macroscopic elasticity-transport problem and associated cell problems, is nonlinearly coupled through reaction terms as well as effective coefficients which take into account the periodic microstructure and, in the case of the transport problem, the deformation of the domain. As a result, the diffusion cell solutions have to be computed in each time step and for each quadrature point of the macroscopic grid, leading to high numerical effort. We develop and study an efficient numerical scheme for our problem, including the approximation of the effective coefficients using a feedforward neural network trained on precomputed coefficients. In the simulations, we reproduce key features of the energy metabolism in deforming tissue (e.g. lung or heart tissue) such as the metabolic reprogramming under hypoxic conditions. This is a well-known characteristic of various diseases including sepsis, cancer or Covid-19.

This is joint work with Markus Gahn (Heidelberg), Nicolas Neuß (Erlangen) and Maria Neuss-Radu (Erlangen).

Solvability of a multi-scale fluid-structure interaction problem for periodic filter media with semigroup theory

Maxime Krier (Fraunhofer ITWM, Kaiserslautern)

The two-way coupled interaction of the Stokes flow with a linear elastic thin plate made of long thin yarns in contact is considered. The thickness of the plate and the structure in-plane period are ε. The structure is described by geometric and mechanic parameters, like distances between yarns, their cross-section, elastic modulii of yarns and friction coefficients between yarns. The elastic properties are related to the fluid velocity as ε^3. Such problems typically arise e.g. in filtration applications with woven filter media.

An asymptotic dimension reduction and homogenization as $\varepsilon \to 0$ was applied in [1, 2] to shrink the plate to an interface and obtain an effective condition for the pressure jump over this interface proportional to the 2D-plate vibration.

If the distance between yarns/fibers in the structure is much larger than their thickness, an interface flux term, similar to the Darcy's or Brinkmann's law appear, which incorporates the mass transport through the plate. The corresponding permeability tensor is attained by Stokes flow problems in the micro resolved periodic unit.

The existence proof for the presented FSI model is performed via a continuous semigroup approach, [3]. The application of the model is illustrated for real-life woven filter samples.

This is a joint work with Julia Orlik.

References

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[2] M. Gahn, W. Jäger, M. Neuss-Radu, *Derivation of Stokes-plate-equations modeling fluid flow interaction with thin porous elastic layers*, Applicable Analysis *101* (2022), 4319–4348.

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Properties of nonlinear Naghdi type shell model

Matko Ljulj (University of Zagreb)

In this talk a novel nonlinear shell model of Naghdi type will be presented. It is well defined for shells whose middle surface is parameterised by a Lipschitz function. Unknowns in the model are three-dimensional deformation of the middle surface and the rotation matrix function which describes the rotation of the shell cross–section. Its energy functional includes flexural, membrane, shear and drill energies.

Apart from the basic properties (restrictions to a particular subset of admissible functions gives as a model of flexural or Koiter type, it is frame indifferent, its linearization gives a linear model of Naghdi type, its differential formulation implies that it is a Cosserat model with one director...), we will analyse its asymptotic behaviour as the thickness of the shell tends to zero, in various regimes depending on the elastic properties of the material. The Gamma–limits are compared with rigorously derived nonlinear models obtained from 3d elasticity.

This is joint work with J. Tambača.

The Robin boundary condition for modeling heat transfer

Eduard Marušić-Paloka (University of Zagreb)

The heat exchange between a rigid body and a fluid is usually modeled by the Robin boundary condition saying that the heat flux through the interface is proportional to the difference between their temperatures. Such interface law describes only the unilateral heat exchange. The goal of this paper is to compare the Robin boundary condition starting with the transmission condition (the temperature and the flux continuity) using rigorously mathematical analysis. Our main results are the following. We first show, that a generalized version of the Robin boundary condition can be justified. Secondly, we prove that replacing the generalized by the standard Robin condition can be justified for high convection velocity if the conductivity of the surrounding liquid is much lower than that of the body. On the other hand, if the fluid conducts much better then the body, than the effective boundary condition is shown not to be the Robin one, but it involves second-order derivatives.

This is joint work with Igor Pažanin (University of Zagreb).

Compaction in a deformable cylindrical porous medium bounded by an elastic impermeable membrane

Richard Mcnair (The University of Manchester)

Hewitt et al. [1] showed experimentally, and with a one-dimensional theoretical model, that water driven vertically through a packed bed of hydrogel beads causes the medium to compact, reducing porosity and permeability such that the flow rate through the medium plateaus to a constant as the pressure head diverges to infinity. We investigate theoretically a two-dimensional extension of the problem in a tall, thin, axisymmetric porous cylinder where the radial boundaries are enclosed by a deformable, impermeable membrane with different material properties to the porous medium itself. Flow is driven through the medium by a fixed height of water maintained above the cylinder. We use the small ratio of radial to vertical length scales of the cylinder to find the leading-order flow rate and deformation as a function of the applied pressure head, initial porosity, and the ratio of linear elastic parameters of the porous bed to the bounding membrane, recovering Hewitt et al's [1] model as the membrane becomes much stiffer than the porous matrix. Our model predicts how a compliant bounding membrane augments compactions, but prevents unbounded increase in flow resistance, allowing flow rate to increase monotonically with applied pressure head for a relatively wide range of porous media. This research contributes to a better understanding of flow and deformation coupling in biological soft tissues, such as the brain and human placenta [2].

This is a joint work with Anne Juel and Igor Chernyavsky.

References

[1] D. R. Hewitt, J. S. Nijjer, M. G. Worster, and J. A. Neufeld, *Flow-induced compaction of a deformable porous medium*, Physical Review E **93** (2016), p. 023116.

[2] O. E. Jensen and I. L. Chernyavsky, *Blood flow and transport in the human placenta*, Annual Review of Fluid Mechanics **51** (2019), 25–47.

Improving Osher-Rudin shock filter noise sensitivity with degenerate diffusion

Andrej Novak (University of Zagreb & University of Vienna)

We investigate highly nonlinear modifications to the traditional Osher and Rudin shock filter PDE in image processing, specifically focusing on digital images affected by noise and blur. Addressing the noise sensitivity of these equations, we explore the use of degenerate diffusion as a means to improve the properties of the classical shock filter. This shock-diffusion equation selectively applies diffusion to regions with potential edge formation, thereby enhancing image features while simultaneously reducing noise. From a mathematical perspective, we utilize Alvarez and Mazorra regularization and a specific approximation argument, establishing the existence of a family of approximate solutions using the Galerkin method, the Leray-Schauder fixed-point theorem, and the Aubin-Lions lemma. Furthermore, we present numerical results on binary and grayscale images in two and three dimensions to validate our theoretical results and underscore the practical applications of the filter. These simulations demonstrate the effectiveness of the modified shock filters, particularly in contexts where traditional methods falter due to noise and edge preservation challenges.

This is joint work with Igor Pažanin (University of Zagreb).

Γ-convergence for nearly incompressible fluids

Florian Oschmann (Czech Academy of Sciences, Praha)

We consider the time-dependent compressible Navier–Stokes equations in the low Mach number regime in a family of domains $\Omega_\varepsilon \subset \mathbb{R}^d$ converging in the sense of Mosco to a domain $\Omega \subset \mathbb{R}^d,\ d \in \{2,3\}$. We show the limit is the incompressible Navier–Stokes system in Ω.

This is joint work with Peter Bella (TU Dortmund) and Eduard Feireisl (CAS).

A variational analysis of thin elastic periodic sheets

David Padilla-Garza (IST Austria)

This talk will be about a mathematically general and rigorous analysis of thin elastic prestrained sheets with a microscopically periodic structure. Our starting point is a general elastic energy functional modelling such structures. We will be interested in the Gamma limit of such functionals as the thickness and period tend to 0. We will completely characterize the Gamma limit in the case of thickness and period having a finite positive ratio. We will also analyze the commutativity and non commutativity of different limits.

Homogenisation of local colloid evolution induced by reaction and diffusion

Malte A. Peter (University of Augsburg)

We consider the homogenisation of a coupled reaction-diffusion process in a porous medium with evolving microstructure. A concentration-dependent reaction rate at the interface of the pores and the solid matrix induces a concentration-dependent evolution of the domain, which makes the evolution fully coupled with the reaction-diffusion process. In order to pass to the homogenisation limit, we employ the two-scale-transformation method. Thus, we homogenise the highly nonlinear problem in a periodic and in-time cylindrical domain instead. The homogenisation result is a reaction-diffusion equation, which is coupled with an internal variable, representing the local evolution of the pore structure.

This is joint work with D. Wiedemann (Technical University of Dortmund).

Thermomicropolar fluid flow through a thin channel

Marko Radulović (University of Zagreb)

We will consider the steady-state flow of the thermomicropolar fluid through a thin straight channel as well a thin curvilinear channel. The flow is governed by the prescribed pressure drop between the channel's ends. The heat exchange between the fluid inside the channel and the exterior medium is allowed through the upper wall, while the lower wall is insulated. Using asymptotic analysis with respect to the thickness of the channel, we derive an asymptotic solution acknowledging the effects of the fluid's microstructure as well as the curvature of the channel. The proposed model is rigorously justified by proving the error estimates in suitable norms. These results were published in [1] and [2].

This is a joint work with Professor Grzegorz Łukaszewicz (University of Warsaw, Poland), Professor Igor Pažanin (University of Zagreb, Croatia) and B. Rukavina (University of Zagreb, Croatia).

References

[1] G. Łukaszewicz, I. Pažanin and M. Radulović, *Asymptotic analysis of the thermomicropolar fluid flow through a thin channel with cooling*. Applicable Analysis **101** (2022), 3141–3169.

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Uniform bounds for finite-energy sequences of one-dimensional Cahn-Hilliard functional endowed with pth power non-integrable multi-well potential

Andrija Raguž (Zagreb School of Economics and Management)

In this talk we present some uniform bounds as $\varepsilon \to 0$ for finite-energy sequences (u_ε) of one-dimensional Cahn-Hilliard functional $I_0^\varepsilon : \mathrm{H}^1(0,1) \to [0,+\infty)$ defined by

$$I_0^\varepsilon(u) := \int_0^1 \Big(\varepsilon^2 u'^2(s) + W(u(s))\Big) ds \ ,$$

where W is a multi-well potential which satisfies p-th power non-integrability condition

$$\int_0^{+\infty} V^p(\xi) d\xi = +\infty \ ,$$

with $p > \frac{1}{2}$, and where V is even symmetrization of W defined by

$$V(\xi) := \min\{W(\zeta) : |\zeta| = \xi\} \ .$$

Our results are generalizations of similar results in the paper G. Leoni, *A remark on compactness for the Cahn-Hilliard functional,* ESAIM COCV **20(2)** (2014), 517–523, and are complementary to results in the paper A. Raguž, *A priori estimates for finite-energy sequences of Cahn-Hilliard functional with non-standard multi-well potential,* Math. Commun. (2024) (to appear).

Strongly coupled parabolic-elliptic two-scale system: solvability and numerical studies

Vishnu Raveendran (Karlstad University)

We discuss the solvability and numerical simulations of a strongly coupled two-scale system with nonlinear dispersion. The two-scale system consists of a nonlinear reaction-dispersion problem coupled through a dispersion tensor to a system of elliptic cell problems. The problem is motivated by a rigorously derived effective model of the reaction-diffusion equation with large nonlinear drift, which indicates the concentration profile of the interacting particle crossing porous media.

This is a joint work with S. Nepal (Karlstad, SWE), R. Lyons (Karlstad, SWE), M. Eden (Karlstad, SWE) and A. Muntean (Karlstad, SWE).

Derivation and investigations of effective dispersion models for electroosmotic flow in an evovling thin strip

Nadja Ray (The Catholic University of Eichstätt – Ingolstadt)

Dispersion is one of the key parameters in transport in porous media. We derive novel effective dispersion models for reactive transport of electrically charged chemical species in a thin, potentially evolving strip by formal asymptotic expansion. These models include Taylor-Aris, electroosmotic induced dispersion and their cross coupling effects. We prove existence and uniqueness of strong solutions in the fixed geometry setting. Moreover, we numerically investigate scenarios for both the fixed and evolving geometry situation. Finally, we study the limits of vanishing channel width, precipitation layer thickness, and molecular diffusion. We show convergence of the solutions to the corresponding limit cases such as a hyperbolic model or the fixed geometry case.

This is joint work with Raphael Schulz and Stephan Gärttner.

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Remainder-based spatial refinement criteria of a moving mesh discontinuous Galerkin method for 2d unsteady convection-diffusion problems

Ezra Rozier (Universität Hamburg)

In convection-dominated flows, large-scale trends necessarily coexist with small-scale effects. While reducing the convection-dominance by moving the mesh, also called Arbitrary Lagrangian-Eulerian (ALE), already proved efficient, Adaptive Mesh Refinement (AMR) is able to catch the small scale effects. But ALE introduces uncertainties that cannot be neglected compared to the small-scale effects, so that it is unsatisfying to use AMR in an ALE situation in the same way as it is used on static meshes. Consecutively to the study of a remainder-based refinement criterion of a moving-mesh, interior-penalty discontinuous Galerkin (DG) semi-discretization of the 2D nonstationary convection-diffusion equation, we identify which component of the criterion catches the error propagation. This lightweight criterion is then tested and compared to the whole remainder-based criterion.

Asymptotic analysis of the nonsteady micropolar fluid flow through a system of thin pipes

Borja Rukavina (University of Zagreb)

We analyze the time-dependent flow of an incompressible micropolar fluid in a multiple pipe system. Motivated by the applications, we assume that the pipes have circular cross-section and that the ratio between pipes' thickness and its length is small, denoted by the parameter ε. Far from the junction, the fluid exhibits different behavior depending on the magnitude of the viscosity coefficients with respect to the small parameter ε. Focusing on the critical case described by the strong coupling between velocity and microrotation, the complete asymptotic expansion of the solution (up to an arbitrary order) is built. To improve the accuracy of the asymptotic approximation, we introduce the boundary-layer correctors near the pipes' ends and take into account the interior-layer correction in the vicinity of the junction as well. The convergence is also proved via error estimates, providing the rigorous justification of the proposed effective model.

This is joint work with Igor Pažanin and Marko Radulović.

Homogenization of the planar one-dimensional periodic elastic rod structures

Josip Tambača (University of Zagreb)

In this work we consider a planar periodic network of elastic rods. For modeling of elastic rods we use a one-dimensional model of Naghdi type allowing for membrane, stretching and bending deformations. Using a mesh two-scale convergence we show that the equilibrium solutions of this elastic network with periodicity size δ converge when δ tensds to zero to the solution of the plate equation. The elasticity tensor in the effective plate equation is obtained as the solution of the network problem on the unit cell, as usual in homogenization.

This is a joint work with Matko Ljulj, Kersten Schmidt and Adrien Semin.

A simplified double porosity model of immiscible incompressible two-phase flow

Anja Vrbaški (University of Zagreb)

Immiscible incompressible two-phase flow in porous media with double porosity structure and with thin fractures is studied. We propose a new linear version of the local problem called the imbibition equation which appears in derivation of the homogenized model for such flow. This allows us to approximate the matrix-fracture source term by a convolution-type source term, decouple the imbibition equation from the global double porosity problem and finally to obtain a new simplified effective double porosity model for incompressible two-phase flow in double porosity media with thin fractures.

This is a joint work with Mladen Jurak, Leonid Pankratov.

Biot-Stokes interactions

Justin T. Webster (University of Maryland)

The problem of filtration of slow fluid flow near an adjacent poro-elastic region is classical. It is motivated by both geosciences and biological systems, where free flows may run adjacent to a saturated poro-elastic solid and the Beavers-Joseph-Saffman (slip) condition is in force. Mathematical results are largely numerical. We are concerned with the problem of a 3-D Stokes flow adjacent to a 3-D poro-elastic region, modeled by a nonlinear Biot dynamics and accounting for the slip coupling. From the mathematical point of view, such coupling has been rarely considered, as it presents technical challenges associated to the regularity of velocity traces at the interface. In this talk, we address two scenarios: (i) an intermediate, multilayered model, where a 2-D linear, poro-elastic plate mitigates the interaction between the two dynamics and (ii) when the poro-plate interface is not present.

For the multilayer case, we provide an existence result via a spatial semi-discretization that relies on the recent theory developed in the work of Bociu et al [2016]. After limit passage, we address a uniqueness result via a regularity criterion for weak solutions. Without the poro-plate interface, we demonstrate semigroup generation (providing strong and generalized solutions) for the coupled system. The definition of the generator is particularly delicate, which plays out in the proof of maximality. Following the elimination of pressure (using the approach of Avalos and Triggiani), we obtain a mixed variational formulation which is solved via Babuska-Brezzi, yielding semigroup generation. Finally, using the smooth solutions as approximants, we obtain weak solutions, including for the degenerate case of incompressible constituents.

Homogenisation of a system of Stokes flow and advection–reaction–diffusion transport in a porous medium with coupled evolving microstructure

David Wiedemann (Technical University of Dortmund)

We consider the homogenisation of the Stokes equations and an advection–reaction–diffusion equation in an porous medium with evolving microstructure. The microstructure's evolution is coupled with the unknown concentration of a substance, resulting in a free boundary value problem. We transform the problem into a fixed periodic domain, which results in a highly non-linear problem. By homogenising this substitute problem and transforming the limit problem back, we obtain as limit problem a Darcy law for evolving microstructure coupled with advective–reactive–transport.

This is a joint work with Markus Gahn, Malte A. Peter, Iulio Sorin Pop.

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