Mali primjeri mozaika kombinatornih dizajna*

Vedran Krčadinac

PMF-MO

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Mozaici kombinatornih dizajna

O. W. Gnilke, M. Greferath, M. O. Pavčević, *Mosaics of combinatorial designs*, Des. Codes Cryptogr. **86** (2018), no. 1, 85–95.

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Definicija.

Neka su t_i-(v, k_i, λ_i), $i = 1, \ldots, c$ parametri kombinatornih dizajna koji svi imaju isti broj točaka v i blokova b te vrijedi $k_1 + \ldots + k_c = v$. Mozaik s parametrima t_1-$(v, k_1, \lambda_1) \oplus t_2$-$(v, k_2, \lambda_2) \oplus \cdots \oplus t_c$-$(v, k_c, \lambda_c)$ je $v \times b$ matrica s unosima iz $\{1, \ldots, c\}$ takva da unos i predstavlja incidencije t_i-(v, k_i, λ_i) dizajna.

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Teorem.

Ako postoji rastavljivi t-(v, k, λ) dizajn, onda postoji c-mozaik $(c = v/k)$

$$t\text{-}(v, k, \lambda) \oplus \cdots \oplus t\text{-}(v, k, \lambda).$$

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Teorem.

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$$t\text{-}(v, k, \lambda) \oplus \cdots \oplus t\text{-}(v, k, \lambda).$$

Definicija.

Mozaik u kojem svi dizajni imaju iste parametre zovemo homogenim, a inače ga zovemo nehomogenim mozaikom.

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Teorem.

Ako postoji rastavljivi t-(v, k, λ) dizajn, onda postoji c-mozaik ($c = v/k$)

$$t\text{-}(v, k, \lambda) \oplus \cdots \oplus t\text{-}(v, k, \lambda).$$

Definicija.

Mozaik u kojem svi dizajni imaju iste parametre zovemo homogenim, a inače ga zovemo nehomogenim mozaikom. Uniformni/neuniformni?

PAG

Prescribed Automorphism Groups

0.2.3

21 May 2024

2.8.7 AffineMosaic

▷ `AffineMosaic(k, n, q)` (function)

Returns mosaic of designs with blocks being k-dimensional subspaces of the affine space $AG(n,q)$. Uses the FinInG package. If the package is not available, the function is not loaded.

2.8.7 AffineMosaic

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Returns mosaic of designs with blocks being k-dimensional subspaces of the affine space $AG(n,q)$. Uses the FinInG package. If the package is not available, the function is not loaded.

2.8.1 MosaicParameters

▷ `MosaicParameters(M)` (function)

Returns a string with the parameters of the mosaic of combinatorial designs M. See [GGP18] for the definition. Entries 0 in the matrix M are considered empty, and other integers are considered as incidences of distinct designs.

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,2);
[ [ 1, 2, 1, 2, 1, 2 ],
  [ 2, 1, 1, 2, 2, 1 ],
  [ 1, 2, 2, 1, 2, 1 ],
  [ 2, 1, 2, 1, 1, 2 ] ]
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,2);
[ [ 1, 2, 1, 2, 1, 2 ],
  [ 2, 1, 1, 2, 2, 1 ],
  [ 1, 2, 2, 1, 2, 1 ],
  [ 2, 1, 2, 1, 1, 2 ] ]
gap> MosaicParameters(m);
"2-(4,2,1) + 2-(4,2,1)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,2);
[ [ 1, 2, 1, 2, 1, 2 ],
  [ 2, 1, 1, 2, 2, 1 ],
  [ 1, 2, 2, 1, 2, 1 ],
  [ 2, 1, 2, 1, 1, 2 ] ]
gap> MosaicParameters(m);
"2-(4,2,1) + 2-(4,2,1)"
```

Zadatak:

Ako k dijeli v, može li se uvijek napraviti (v/k)-mozaik potpunih dizajna?

$$k\text{-}(v,k,1) \oplus \cdots \oplus k\text{-}(v,k,1)$$

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,2);
[ [ 1, 2, 1, 2, 1, 2 ],
  [ 2, 1, 1, 2, 2, 1 ],
  [ 1, 2, 2, 1, 2, 1 ],
  [ 2, 1, 2, 1, 1, 2 ] ]
gap> MosaicParameters(m);
"2-(4,2,1) + 2-(4,2,1)"
```

Zadatak:

Ako k dijeli v, može li se uvijek napraviti (v/k)-mozaik potpunih dizajna?

$$k\text{-}(v, k, 1) \oplus \cdots \oplus k\text{-}(v, k, 1)$$

Jesu li potpuni dizajni rastavljivi?

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,3);
[ [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3 ],
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1 ],
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2 ],
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1 ],
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2 ],
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3 ],
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2 ],
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3 ],
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1 ] ]
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,3);
[ [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3 ],
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1 ],
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2 ],
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1 ],
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2 ],
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3 ],
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2 ],
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3 ],
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1 ] ]

gap> MosaicParameters(m);
"2-(9,3,1) + 2-(9,3,1) + 2-(9,3,1)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,4);
[ [ 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4 ],
  [ 3, 4, 1, 2, 3, 4, 1, 2, 1, 2, 3, 4, 3, 4, 1, 2, 3, 4, 1, 2 ],
  [ 2, 3, 4, 1, 2, 3, 4, 1, 1, 2, 3, 4, 2, 3, 4, 1, 2, 3, 4, 1 ],
  [ 4, 1, 2, 3, 4, 1, 2, 3, 1, 2, 3, 4, 4, 1, 2, 3, 4, 1, 2, 3 ],
  [ 1, 2, 3, 4, 2, 3, 4, 1, 3, 4, 1, 2, 3, 4, 1, 2, 4, 1, 2, 3 ],
  [ 3, 4, 1, 2, 4, 1, 2, 3, 3, 4, 1, 2, 1, 2, 3, 4, 2, 3, 4, 1 ],
  [ 2, 3, 4, 1, 1, 2, 3, 4, 3, 4, 1, 2, 4, 1, 2, 3, 3, 4, 1, 2 ],
  [ 4, 1, 2, 3, 3, 4, 1, 2, 3, 4, 1, 2, 2, 3, 4, 1, 1, 2, 3, 4 ],
  [ 1, 2, 3, 4, 4, 1, 2, 3, 2, 3, 4, 1, 2, 3, 4, 1, 3, 4, 1, 2 ],
  [ 3, 4, 1, 2, 2, 3, 4, 1, 2, 3, 4, 1, 4, 1, 2, 3, 1, 2, 3, 4 ],
  [ 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1, 1, 2, 3, 4, 4, 1, 2, 3 ],
  [ 4, 1, 2, 3, 1, 2, 3, 4, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1 ],
  [ 1, 2, 3, 4, 3, 4, 1, 2, 4, 1, 2, 3, 4, 1, 2, 3, 2, 3, 4, 1 ],
  [ 3, 4, 1, 2, 1, 2, 3, 4, 4, 1, 2, 3, 2, 3, 4, 1, 4, 1, 2, 3 ],
  [ 2, 3, 4, 1, 4, 1, 2, 3, 4, 1, 2, 3, 3, 4, 1, 2, 1, 2, 3, 4 ],
  [ 4, 1, 2, 3, 2, 3, 4, 1, 4, 1, 2, 3, 1, 2, 3, 4, 3, 4, 1, 2 ] ]
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,2,4);
[ [ 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4 ],
  [ 3, 4, 1, 2, 3, 4, 1, 2, 1, 2, 3, 4, 3, 4, 1, 2, 3, 4, 1, 2 ],
  [ 2, 3, 4, 1, 2, 3, 4, 1, 1, 2, 3, 4, 2, 3, 4, 1, 2, 3, 4, 1 ],
  [ 4, 1, 2, 3, 4, 1, 2, 3, 1, 2, 3, 4, 4, 1, 2, 3, 4, 1, 2, 3 ],
  [ 1, 2, 3, 4, 2, 3, 4, 1, 3, 4, 1, 2, 3, 4, 1, 2, 4, 1, 2, 3 ],
  [ 3, 4, 1, 2, 4, 1, 2, 3, 3, 4, 1, 2, 1, 2, 3, 4, 2, 3, 4, 1 ],
  [ 2, 3, 4, 1, 1, 2, 3, 4, 3, 4, 1, 2, 4, 1, 2, 3, 3, 4, 1, 2 ],
  [ 4, 1, 2, 3, 3, 4, 1, 2, 3, 4, 1, 2, 2, 3, 4, 1, 1, 2, 3, 4 ],
  [ 1, 2, 3, 4, 4, 1, 2, 3, 2, 3, 4, 1, 2, 3, 4, 1, 3, 4, 1, 2 ],
  [ 3, 4, 1, 2, 2, 3, 4, 1, 2, 3, 4, 1, 4, 1, 2, 3, 1, 2, 3, 4 ],
  [ 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1, 1, 2, 3, 4, 4, 1, 2, 3 ],
  [ 4, 1, 2, 3, 1, 2, 3, 4, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1 ],
  [ 1, 2, 3, 4, 3, 4, 1, 2, 4, 1, 2, 3, 4, 1, 2, 3, 2, 3, 4, 1 ],
  [ 3, 4, 1, 2, 1, 2, 3, 4, 4, 1, 2, 3, 2, 3, 4, 1, 4, 1, 2, 3 ],
  [ 2, 3, 4, 1, 4, 1, 2, 3, 4, 1, 2, 3, 3, 4, 1, 2, 1, 2, 3, 4 ],
  [ 4, 1, 2, 3, 2, 3, 4, 1, 4, 1, 2, 3, 1, 2, 3, 4, 3, 4, 1, 2 ] ]

gap> MosaicParameters(m);
"2-(16,4,1) + 2-(16,4,1) + 2-(16,4,1) + 2-(16,4,1)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,3,2);
[ [ 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4 ],
  [ 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 1, 2, 3, 4, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2 ],
  [ 2, 3, 4, 1, 1, 2, 3, 4, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1 ],
  [ 4, 1, 2, 3, 3, 4, 1, 2, 4, 1, 2, 3, 3, 4, 1, 2, 4, 1, 2, 3, 1, 2, 3, 4, 4, 1, 2, 3 ],
  [ 1, 2, 3, 4, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1, 4, 1, 2, 3 ],
  [ 3, 4, 1, 2, 4, 1, 2, 3, 4, 1, 2, 3, 2, 3, 4, 1, 1, 2, 3, 4, 4, 1, 2, 3, 2, 3, 4, 1 ],
  [ 2, 3, 4, 1, 2, 3, 4, 1, 1, 2, 3, 4, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 3, 4, 1, 2 ],
  [ 4, 1, 2, 3, 4, 1, 2, 3, 3, 4, 1, 2, 4, 1, 2, 3, 2, 3, 4, 1, 2, 3, 4, 1, 1, 2, 3, 4 ] ]
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,3,2);
[ [ 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4 ],
  [ 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 1, 2, 3, 4, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2 ],
  [ 2, 3, 4, 1, 1, 2, 3, 4, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1 ],
  [ 4, 1, 2, 3, 3, 4, 1, 2, 4, 1, 2, 3, 3, 4, 1, 2, 4, 1, 2, 3, 1, 2, 3, 4, 4, 1, 2, 3 ],
  [ 1, 2, 3, 4, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 3, 4, 1, 2, 2, 3, 4, 1, 4, 1, 2, 3 ],
  [ 3, 4, 1, 2, 4, 1, 2, 3, 4, 1, 2, 3, 2, 3, 4, 1, 1, 2, 3, 4, 4, 1, 2, 3, 2, 3, 4, 1 ],
  [ 2, 3, 4, 1, 2, 3, 4, 1, 1, 2, 3, 4, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 3, 4, 1, 2 ],
  [ 4, 1, 2, 3, 4, 1, 2, 3, 3, 4, 1, 2, 4, 1, 2, 3, 2, 3, 4, 1, 2, 3, 4, 1, 1, 2, 3, 4 ] ]

gap> MosaicParameters(m);
"2-(8,2,1) + 2-(8,2,1) + 2-(8,2,1) + 2-(8,2,1)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(2,3,2);
[ [ 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 ],
  [ 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1 ],
  [ 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1 ],
  [ 2, 1, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2 ],
  [ 1, 2, 1, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 1 ],
  [ 2, 1, 1, 2, 2, 1, 2, 1, 2, 1, 1, 2, 1, 2 ],
  [ 1, 2, 2, 1, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2 ],
  [ 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1 ] ]
```

PAG 0.2.3

```
gap> m:=AffineMosaic(2,3,2);
[ [ 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 ],
  [ 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1 ],
  [ 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 2, 1 ],
  [ 2, 1, 2, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2 ],
  [ 1, 2, 1, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 1 ],
  [ 2, 1, 1, 2, 2, 1, 2, 1, 2, 1, 1, 2, 1, 2 ],
  [ 1, 2, 2, 1, 2, 1, 2, 1, 1, 2, 2, 1, 1, 2 ],
  [ 2, 1, 2, 1, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1 ] ]

gap> MosaicParameters(m);
"3-(8,4,1) + 3-(8,4,1)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(2,3,3);
[ [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3,
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1,
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2,
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2,
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3,
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1,
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1,
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2,
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3,
  [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 3, 1, 2, 2, 3, 1, 2, 3, 1,
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 3, 1, 2, 2, 3, 1, 1, 2, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 3, 1, 2,
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3,
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1, 1, 2, 3, 2, 3, 1, 1, 2, 3,
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1,
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 3, 1, 2,
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2,
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3, 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3,
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 1, 2, 3, 2, 3, 1,
  [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 2, 3, 1, 3, 1, 2, 3, 1, 2,
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3,
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 2, 3, 1, 3, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 1, 2, 3, 1,
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1,
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 1, 2, 3, 3, 1, 2,
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3, 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3,
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2, 1, 2, 3, 3, 1, 2, 1, 2, 3,
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 2, 3, 1,
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2,
```

PAG 0.2.3

```
gap> m:=AffineMosaic(2,3,3);
[ [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3,
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1,
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2,
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2,
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3,
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1,
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1,
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2,
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3,
  [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 3, 1, 2, 2, 3, 1, 2, 3, 1,
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 3, 1, 2, 2, 3, 1, 1, 2, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 3, 1, 2,
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3,
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1, 1, 2, 3, 2, 3, 1, 1, 2, 3,
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1,
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 3, 1, 2,
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2,
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 1, 2, 3, 1, 2, 3, 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3,
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 2, 3, 1, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 1, 2, 3, 2, 3, 1,
  [ 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 2, 3, 1, 2, 3, 1, 2, 3, 1, 3, 1, 2, 3, 1, 2,
  [ 2, 3, 1, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3,
  [ 3, 1, 2, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 2, 3, 1, 3, 1, 2, 1, 2, 3, 1, 2, 3, 1, 2, 3, 2, 3, 1, 2, 3, 1,
  [ 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 2, 3, 1,
  [ 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 3, 1, 2, 1, 2, 3, 3, 1, 2, 2, 3, 1, 1, 2, 3, 1, 2, 3, 3, 1, 2,
  [ 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3, 3, 1, 2, 3, 1, 2, 1, 2, 3, 1, 2, 3, 3, 1, 2, 2, 3, 1, 2, 3, 1, 1, 2, 3,
  [ 1, 2, 3, 3, 1, 2, 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 2, 3, 1, 3, 1, 2, 1, 2, 3, 3, 1, 2, 1, 2, 3,
  [ 2, 3, 1, 3, 1, 2, 3, 1, 2, 1, 2, 3, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 2, 3, 1,
  [ 3, 1, 2, 3, 1, 2, 1, 2, 3, 2, 3, 1, 3, 1, 2, 1, 2, 3, 2, 3, 1, 1, 2, 3, 2, 3, 1, 3, 1, 2, 2, 3, 1, 3, 1, 2,
gap> MosaicParameters(m);
"2-(27,9,4) + 2-(27,9,4) + 2-(27,9,4)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,4,2);;
gap> MosaicParameters(m);
"2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1)"
```

```
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gap> MosaicParameters(m);
"2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1)"

gap> m:=AffineMosaic(2,4,2);;
gap> MosaicParameters(m);
"3-(16,4,1) + 3-(16,4,1) + 3-(16,4,1) + 3-(16,4,1)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,4,2);;
gap> MosaicParameters(m);
"2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1) + 2-(16,2,1)"

gap> m:=AffineMosaic(2,4,2);;
gap> MosaicParameters(m);
"3-(16,4,1) + 3-(16,4,1) + 3-(16,4,1) + 3-(16,4,1)"

gap> m:=AffineMosaic(3,4,2);;
gap> MosaicParameters(m);
"3-(16,8,3) + 3-(16,8,3)"
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,4,3);;
gap> MosaicParameters(m);
"2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,4,3);;
gap> MosaicParameters(m);
"2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,

gap> m:=AffineMosaic(2,4,3);;
gap> MosaicParameters(m);
"2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13)
```

PAG 0.2.3

```
gap> m:=AffineMosaic(1,4,3);;
gap> MosaicParameters(m);
"2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,3,1) + 2-(81,

gap> m:=AffineMosaic(2,4,3);;
gap> MosaicParameters(m);
"2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13) + 2-(81,9,13)

gap> m:=AffineMosaic(3,4,3);;
gap> MosaicParameters(m);
"2-(81,27,13) + 2-(81,27,13) + 2-(81,27,13)"
```

Mozaici od rastavljivih dizajna

$$
\left[\begin{array}{ccccc|ccccc|ccccc|ccccc|ccccc|ccccc|ccccc}
1&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&1&0\\
0&1&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&1&0&0&0&0&1&0&0\\
0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&1&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&1&0&0&0\\
1&0&0&0&0&0&0&0&0&1&1&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&1\\
0&1&0&0&0&0&1&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&1\\
0&0&1&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&1&0&0\\
0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&1&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&0&0&1\\
1&0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0\\
0&1&0&0&0&0&0&1&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0\\
0&0&1&0&0&0&0&0&0&1&0&0&1&0&0&0&0&1&0&0&0&0&0&0&1&1&0&0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&0&0&1&0&0&0&0&0&1&0&0&0&1&0&0&0&0&0&1&0&1&0&0&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&1&0&0
\end{array}\right]
$$

$2\text{-}(15,3,1)$

Mozaici od rastavljivih dizajna

$$\left[\begin{array}{ccccc|ccccc|ccccc|ccccc|ccccc|ccccc|ccccc}
1&5&4&3&2&1&5&4&3&2&5&4&3&2&1&3&2&1&5&4&4&3&2&1&5&5&4&3&2&1&4&3&2&1&5\\
2&1&5&4&3&1&5&4&3&2&1&5&4&3&2&5&4&3&2&1&5&4&3&2&1&3&2&1&5&4&3&2&1&5&4\\
3&2&1&5&4&2&1&5&4&3&1&5&4&3&2&4&3&2&1&5&1&5&4&3&2&4&3&2&1&5&4&3&2&1&5\\
4&3&2&1&5&2&1&5&4&3&2&1&5&4&3&5&4&3&2&1&2&1&5&4&3&5&4&3&2&1&1&5&4&3&2\\
5&4&3&2&1&1&5&4&3&2&2&1&5&4&3&1&5&4&3&2&1&5&4&3&2&1&5&4&3&2&2&1&5&4&3\\
1&5&4&3&2&5&4&3&2&1&1&5&4&3&2&1&5&4&3&2&2&1&5&4&3&2&1&5&4&3&5&4&3&2&1\\
2&1&5&4&3&2&1&5&4&3&5&4&3&2&1&2&1&5&4&3&3&2&1&5&4&1&5&4&3&2&5&4&3&2&1\\
3&2&1&5&4&3&2&1&5&4&2&1&5&4&3&2&1&5&4&3&4&3&2&1&5&2&1&5&4&3&3&2&1&5&4\\
4&3&2&1&5&3&2&1&5&4&3&2&1&5&4&1&5&4&3&2&3&2&1&5&4&3&2&1&5&4&4&3&2&1&5\\
5&4&3&2&1&4&3&2&1&5&3&2&1&5&4&5&4&3&2&1&4&3&2&1&5&4&3&2&1&5&5&4&3&2&1\\
1&5&4&3&2&4&3&2&1&5&4&3&2&1&5&2&1&5&4&3&1&5&4&3&2&3&2&1&5&4&1&5&4&3&2\\
2&1&5&4&3&3&2&1&5&4&4&3&2&1&5&3&2&1&5&4&2&1&5&4&3&4&3&2&1&5&2&1&5&4&3\\
3&2&1&5&4&5&4&3&2&1&3&2&1&5&4&3&2&1&5&4&5&4&3&2&1&1&5&4&3&2&1&5&4&3&2\\
4&3&2&1&5&4&3&2&1&5&5&4&3&2&1&4&3&2&1&5&5&4&3&2&1&2&1&5&4&3&2&1&5&4&3\\
5&4&3&2&1&5&4&3&2&1&4&3&2&1&5&4&3&2&1&5&3&2&1&5&4&5&4&3&2&1&3&2&1&5&4
\end{array}\right]$$

$$2\text{-}(15,3,1) \oplus 2\text{-}(15,3,1) \oplus 2\text{-}(15,3,1) \oplus 2\text{-}(15,3,1) \oplus 2\text{-}(15,3,1)$$

Popločavanje grupe diferencijskim skupovima

A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

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Definicija.

Popločavanje grupe G je familija u parovima disjunktnih (v, k, λ) diferencijskih skupova $\{D_1, \ldots, D_c\}$ takva da je $D_1 \cup \cdots \cup D_c = G \setminus \{1\}$.

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Teorem.

"Simultani razvoj" popločavanja grupe G diferencijskim skupovima $\{D_1, \ldots, D_c\}$ je mozaik s parametrima

$$2\text{-}(v, k, \lambda) \oplus \cdots 2\text{-}(v, k, \lambda) \oplus 2\text{-}(v, 1, 0).$$

Taj mozaik ima grupu automorfizama izomorfnu s G koja djeluje regularno na retke i stupce, odnosno točke i blokove dizajna.

Autotopije i automorfizmi

Definicija.

Neka su $A = [a_{ij}]$ i $B = [b_{ij}]$ c-mozaici dimenzija $v \times b$. Kažemo da su izotopni ako postoji trojka permutacija $(\alpha, \beta, \gamma) \in S_v \times S_b \times S_c$ takva da vrijedi $b_{ij} = \gamma(a_{\alpha(i)\,\beta(j)})$ za sve $i = 1, \ldots, v$, $j = 1, \ldots, b$. Ako je $A = B$, trojku (α, β, γ) zovemo autotopijom mozaika.

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Skup svih autotopija s operacijom kompozicije po komponentama tvori grupu, takozvanu punu grupu autotopija mozaika.

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A. D. Keedwell, J. Dénes, *Latin squares and their applications, second edition*, Elsevier/North-Holland, Amsterdam, 2015.

Homogeni mozaici simetričnih dizajna

Zadatak: Što predstavlja homogeni mozaik simetričnih dizajna s $k = 1$?

$$\underbrace{2\text{-}(v,1,0) \oplus \cdots \oplus 2\text{-}(v,1,0)}_{v \text{ puta}}$$

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Propozicija.

Za $k \geq 2$ ne postoje "pravi homogeni" mozaici simetričnih dizajna.

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Za $k \geq 2$ ovakve mozaike simetričnih dizajna smatramo homogenim:

$$2\text{-}(v, k, \lambda) \oplus \cdots 2\text{-}(v, k, \lambda) \oplus 2\text{-}(v, 1, 0)$$

Homogeni mozaici simetričnih dizajna

Zadatak: Što predstavlja homogeni mozaik simetričnih dizajna s $k = 1$?

$$\underbrace{2\text{-}(v,1,0) \oplus \cdots \oplus 2\text{-}(v,1,0)}_{v \text{ puta}}$$

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$(7,3,1)$, $(11,5,2)$, $(13,4,1)$, $(15,7,3)$, $(16,6,2)$, $(19,9,4)$, $(21,5,1)$, $(22,7,2)$, $(23,11,5)$, $(25,9,3)$, $(27,13,6)$, $(29,8,2)$, $(31,6,1)$, $(31,10,3)$, $(31,15,7)$, $(34,12,4)$, $(35,17,8)$, $(36,15,6)$, $(37,9,2)$, $(39,19,9)$, ...

2.8.8 DifferenceMosaic

▷ `DifferenceMosaic(G, dds)` (function)

Returns the mosaic of symmetric designs obtained from a list of disjoint difference sets *dds* in the group *G*.

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2.8.10 MatAut

▷ `MatAut(M)` (function)

Computes the full autotopy group of a matrix *M*. It is assumed that the entries of *M* are consecutive integers. Permutations of rows, columns and symbols are allowed. Represents the matrix by a colored graph and uses `nauty/Traces 2.8` by B.D.McKay and A.Piperno [MP14].

Mozaici od popločavanja grupa

$$D_1 = \{1, 5, 11, 24, 25, 27\},$$
$$D_2 = \{2, 10, 17, 19, 22, 23\},$$
$$D_3 = \{3, 4, 7, 13, 15, 20\},$$
$$D_4 = \{6, 8, 9, 14, 26, 30\},$$
$$D_5 = \{12, 16, 18, 21, 28, 29\}.$$

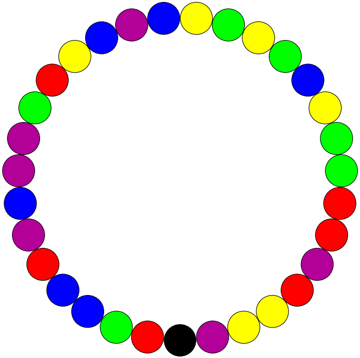


Figure 1: A $(31, 6, 1)$ tiling of $\mathbb{Z}_{31}$.

Mozaici od popločavanja grupa

```
gap> t:=[ [ 1, 5, 11, 24, 25, 27 ],
> [ 2, 10, 17, 19, 22, 23 ],
> [ 3, 4, 7, 13, 15, 20 ],
> [ 6, 8, 9, 14, 26, 30 ],
> [ 12, 16, 18, 21, 28, 29 ] ];;
```

Mozaici od popločavanja grupa

```
gap> t:=[ [ 1, 5, 11, 24, 25, 27 ],
> [ 2, 10, 17, 19, 22, 23 ],
> [ 3, 4, 7, 13, 15, 20 ],
> [ 6, 8, 9, 14, 26, 30 ],
> [ 12, 16, 18, 21, 28, 29 ] ];;
gap> m:=DifferenceMosaic(CyclicGroup(31),t);;
```

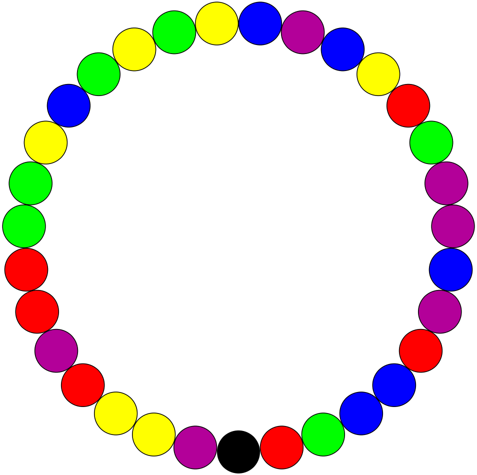
Mozaici od popločavanja grupa

```
gap> t:=[ [ 1, 5, 11, 24, 25, 27 ],
> [ 2, 10, 17, 19, 22, 23 ],
> [ 3, 4, 7, 13, 15, 20 ],
> [ 6, 8, 9, 14, 26, 30 ],
> [ 12, 16, 18, 21, 28, 29 ] ];;
gap> m:=DifferenceMosaic(CyclicGroup(31),t);;
gap> MosaicParameters(m);
"2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1)"
```

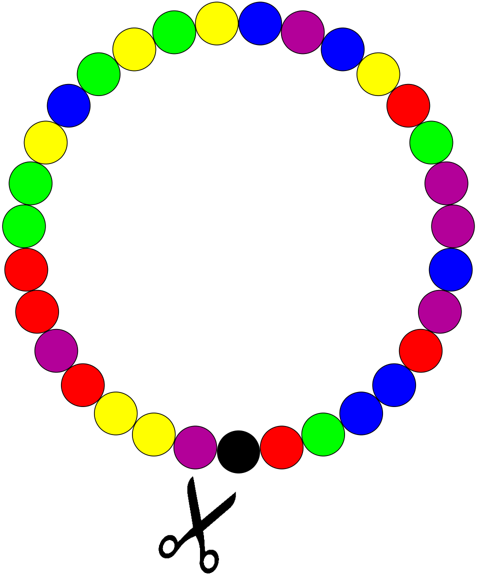
Mozaici od popločavanja grupa

```
gap> t:=[ [ 1, 5, 11, 24, 25, 27 ],
> [ 2, 10, 17, 19, 22, 23 ],
> [ 3, 4, 7, 13, 15, 20 ],
> [ 6, 8, 9, 14, 26, 30 ],
> [ 12, 16, 18, 21, 28, 29 ] ];;
gap> m:=DifferenceMosaic(CyclicGroup(31),t);;
gap> MosaicParameters(m);
"2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1)"
gap> aut:=MatAut(m);
<permutation group with 3 generators>
gap> Size(aut);
465
gap> StructureDescription(aut);
"C31 : C15"
```

Mozaici od popločavanja grupa

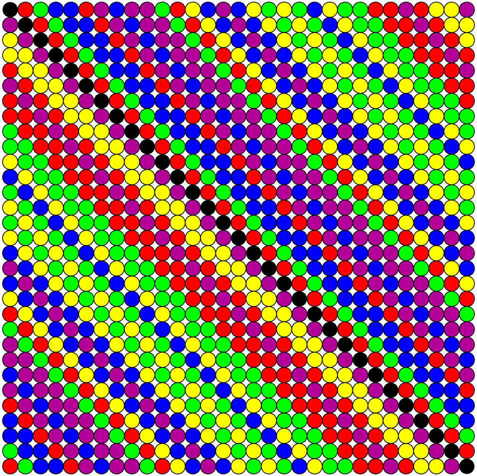


Mozaici od popločavanja grupa

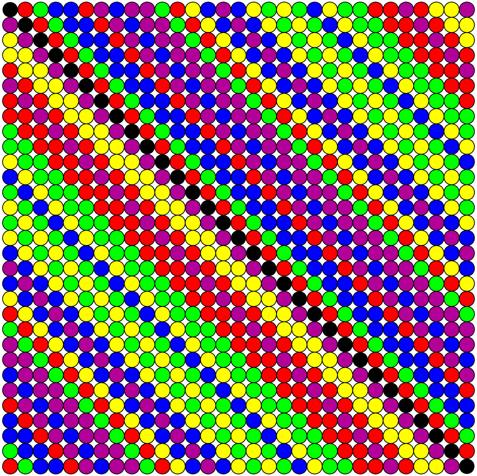


Mozaici od popločavanja grupa

Mozaici od popločavanja grupa



Mozaici od popločavanja grupa



$$(31,6,1) \oplus (31,6,1) \oplus (31,6,1) \oplus (31,6,1) \oplus (31,6,1) \oplus (31,1,0)$$

Mozaici od popločavanja grupa

```
[[0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4],
 [4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5],
 [5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5],
 [5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1],
 [1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4],
 [4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1],
 [1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1],
 [1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2],
 [2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2],
 [2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5],
 [5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3],
 [3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2],
 [2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5],
 [5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2],
 [2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5],
 [5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4,3],
 [3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3,4],
 [4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5,3],
 [3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1,5],
 [5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2,1],
 [1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4,2],
 [2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4,4],
 [4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3,4],
 [4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4,3],
 [3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1,4],
 [4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3,1],
 [1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3,3],
 [3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2,3],
 [3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1,2],
 [2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0,1],
 [1,2,3,3,1,4,3,4,4,2,1,5,3,4,3,5,2,5,2,3,5,2,2,1,1,4,1,5,5,4,0]]
```

Nehomogeni mozaici

O. W. Gnilke, M. Greferath, M. O. Pavčević, *Mosaics of combinatorial designs*, Des. Codes Cryptogr. **86** (2018), no. 1, 85–95.

Purely arithmetically, we may think of

$$2\text{-}(31,\ 15,\ 7)\ \oplus\ 2\text{-}(31,\ 10,\ 3)\ \oplus\ 2\text{-}(31,\ 6,\ 1),$$

however, so far, we have not been able to provide an example of a 3-valued incidence matrix giving rise to this decomposition.

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$$t\text{-}(v,k,\lambda)\oplus t\text{-}(v,v-k,\overline{\lambda})$$

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$$t\text{-}(v,k,\lambda)\oplus t\text{-}(v,v-k,\overline{\lambda})$$

Teorem.

Svaki parcijalni mozaik sim. dizajna $2\text{-}(v,k_1,\lambda_1)\oplus\cdots\oplus 2\text{-}(v,k_c,\lambda_c)$, $\sum\limits_{i=1}^{c} k_i < v$, može se dopuniti do potpunog dodavanjem $2\text{-}(v,1,0)$ dizajna.

Nehomogeni mozaici

$$2\text{-}(31, 6, 1) \oplus 2\text{-}(31, 15, 7)$$

Nehomogeni mozaici

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,15,7) \oplus 2\text{-}(31,1,0)^{10}$$

Nehomogeni mozaici

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Netrivijalnost znači: $c \geq 3, \quad k_i \geq 3$ za $i = 1, \ldots, c.$

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Pitanje. Postoje li ovi mozaici simetričnih dizajna?

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

$$2\text{-}(71,15,3) \oplus 2\text{-}(71,21,6) \oplus 2\text{-}(71,35,17)$$

$$2\text{-}(79,13,2) \oplus 2\text{-}(79,27,9) \oplus 2\text{-}(79,39,19)$$

Nehomogeni mozaici

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Pitanje. Postoje li ovi mozaici nesimetričnih dizajna?

$$2\text{-}(10,3,2) \oplus 2\text{-}(10,3,2) \oplus 2\text{-}(10,4,4) \quad (b = 30)$$

$$2\text{-}(11,3,6) \oplus 2\text{-}(11,4,12) \oplus 2\text{-}(11,4,12) \quad (b = 110)$$

$$2\text{-}(12,3,2) \oplus 2\text{-}(12,3,2) \oplus 2\text{-}(12,6,10) \quad (b = 44)$$

$$2\text{-}(13,3,1) \oplus 2\text{-}(13,4,2) \oplus 2\text{-}(13,6,5) \quad (b = 26)$$

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,15,7) \oplus 2\text{-}(31,1,0)^{10}$$

Netrivijalnost znači: $c \geq 3, \quad k_i \geq 3$ za $i = 1, \ldots, c$.

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Nehomogeni mozaici

V. Krčadinac, *Small examples of mosaics of combinatorial designs*, preprint, 2024. https://arxiv.org/abs/2405.12672

1	3	3	3	2	3	2	2	3	1	3	2	1	1	3	2	2	3	3	1	3	3	3	2	1	2
1	1	3	3	3	2	3	2	2	3	1	3	2	2	1	3	2	2	3	3	1	3	3	3	2	1
2	1	1	3	3	3	2	3	2	2	3	1	3	1	2	1	3	2	2	3	3	1	3	3	3	2
3	2	1	1	3	3	3	2	3	2	2	3	1	2	1	2	1	3	2	2	3	3	1	3	3	3
1	3	2	1	1	3	3	3	2	3	2	2	3	3	2	1	2	1	3	2	2	3	3	1	3	3
3	1	3	2	1	1	3	3	3	2	3	2	2	3	3	2	1	2	1	3	2	2	3	3	1	3
2	3	1	3	2	1	1	3	3	3	2	3	2	3	3	3	2	1	2	1	3	2	2	3	3	1
2	2	3	1	3	2	1	1	3	3	3	2	3	1	3	3	3	2	1	2	1	3	2	2	3	3
3	2	2	3	1	3	2	1	1	3	3	3	2	3	1	3	3	3	2	1	2	1	3	2	2	3
2	3	2	2	3	1	3	2	1	1	3	3	3	3	3	1	3	3	3	2	1	2	1	3	2	2
3	2	3	2	2	3	1	3	2	1	1	3	3	2	3	3	1	3	3	3	2	1	2	1	3	2
3	3	2	3	2	2	3	1	3	2	1	1	3	2	2	3	3	1	3	3	3	2	1	2	1	3
3	3	3	2	3	2	2	3	1	3	2	1	1	3	2	2	3	3	1	3	3	3	2	1	2	1

TABLE 1. A $2\text{-}(13,3,1) \oplus 2\text{-}(13,4,2) \oplus 2\text{-}(13,6,5)$ mosaic.

Mosaics of combinatorial designs

Mosaics of combinatorial designs were defined in [3]. Some interesting small examples are constructed in [5]. This web page contains files with the examples in a format suitable for GAP [2], where they can be analyzed using the PAG package [4].

2-(13,3,1) ⊕ 2-(13,4,2) ⊕ 2-(13,6,5)

These are the first nontrivial examples of inhomogenous mosaics, comprising designs with distinct parameters. The example from [5] is given in the first file, and the second file contains more mosaics with these parameters.

- 13-346ex.txt
- 13-346.txt

The mosaics were constructed from difference families in the cyclic group $\mathbf{Z}_{13}$. The files can be read into GAP by typing:

```
gap> LoadPackage("PAG");
gap> m:=ReadMat("13-346ex.txt");;
gap> MosaicParameters(m[1]);
"2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)"
```

Nehomogeni mozaici

```
13 26

1 3 3 3 2 3 2 2 3 1 3 2 1 1 3 2 2 3 3 1 3 3 3 2 1 2
1 1 3 3 3 2 3 2 2 3 1 3 2 2 1 3 2 2 3 3 1 3 3 3 2 1
2 1 1 3 3 3 2 3 2 2 3 1 3 1 2 1 3 2 2 3 3 1 3 3 3 2
3 2 1 1 3 3 3 2 3 2 2 3 1 2 1 2 1 3 2 2 3 3 1 3 3 3
1 3 2 1 1 3 3 3 2 3 2 2 3 3 2 1 2 1 3 2 2 3 3 1 3 3
3 1 3 2 1 1 3 3 3 2 3 2 2 3 3 2 1 2 1 3 2 2 3 3 1 3
2 3 1 3 2 1 1 3 3 3 2 3 2 3 3 3 2 1 2 1 3 2 2 3 3 1
2 2 3 1 3 2 1 1 3 3 3 2 3 1 3 3 3 2 1 2 1 3 2 2 3 3
3 2 2 3 1 3 2 1 1 3 3 3 2 3 1 3 3 3 2 1 2 1 3 2 2 3
2 3 2 2 3 1 3 2 1 1 3 3 3 3 3 1 3 3 3 2 1 2 1 3 2 2
3 2 3 2 2 3 1 3 2 1 1 3 3 2 3 3 1 3 3 3 2 1 2 1 3 2
3 3 2 3 2 2 3 1 3 2 1 1 3 2 2 3 3 1 3 3 3 2 1 2 1 3
3 3 3 2 3 2 2 3 1 3 2 1 1 3 2 2 3 3 1 3 3 3 2 1 2 1

1 3 3 3 2 3 2 2 3 1 3 2 1 1 2 3 3 3 3 1 2 3 2 2 1 3
1 1 3 3 3 2 3 2 2 3 1 3 2 3 1 2 3 3 3 3 1 2 3 2 2 1
2 1 1 3 3 3 2 3 2 2 3 1 3 1 3 1 2 3 3 3 3 1 2 3 2 2
3 2 1 1 3 3 3 2 3 2 2 3 1 2 1 3 1 2 3 3 3 3 1 2 3 2
1 3 2 1 1 3 3 3 2 3 2 2 3 2 2 1 3 1 2 3 3 3 3 1 2 3
3 1 3 2 1 1 3 3 3 2 3 2 2 3 2 2 1 3 1 2 3 3 3 3 1 2
2 3 1 3 2 1 1 3 3 3 2 3 2 2 3 2 2 1 3 1 2 3 3 3 3 1
2 2 3 1 3 2 1 1 3 3 3 2 3 1 2 3 2 2 1 3 1 2 3 3 3 3
3 2 2 3 1 3 2 1 1 3 3 3 2 3 1 2 3 2 2 1 3 1 2 3 3 3
2 3 2 2 3 1 3 2 1 1 3 3 3 3 3 1 2 3 2 2 1 3 1 2 3 3
3 2 3 2 2 3 1 3 2 1 1 3 3 3 3 3 1 2 3 2 2 1 3 1 2 3
3 3 2 3 2 2 3 1 3 2 1 1 3 3 3 3 3 1 2 3 2 2 1 3 1 2
3 3 3 2 3 2 2 3 1 3 2 1 1 2 3 3 3 3 1 2 3 2 2 1 3 1

1 3 3 2 3 3 3 2 2 1 2 3 1 1 3 3 3 2 2 1 3 2 3 2 1 3
1 1 3 3 2 3 3 3 2 2 1 2 3 3 1 3 3 3 2 2 1 3 2 3 2 1
3 1 1 3 3 2 3 3 3 2 2 1 2 1 3 1 3 3 3 2 2 1 3 2 3 2
```

2.8.5 ReadMat

▷ `ReadMat(filename)` (function)

Reads a list of $m \times n$ integer matrices from a file. The file starts with the number of rows m and columns n followed by the matrix entries. Integers in the file are separated by whitespaces.

Nehomogeni mozaici

2.8.5 ReadMat

▷ `ReadMat(filename)` (function)

Reads a list of $m \times n$ integer matrices from a file. The file starts with the number of rows m and columns n followed by the matrix entries. Integers in the file are separated by whitespaces.

2.8.11 MatFilter

▷ `MatFilter(ml[, opt])` (function)

Eliminates equivalent copies from a list of matrices `ml`. It is assumed that all of the matrices have the same set of consecutive integers as entries. Two matrices are equivalent (isotopic) if one can be transformed into the other by permutating rows, columns and symbols. Represents the matrices by colored graphs and uses `nauty/Traces 2.8` by B.D.McKay and A.Piperno [MP14]. The optional argument `opt` is a record for options. Possible components of `opt` are:

- *Positions*:=`true/false` Return positions of inequivalent matrices instead of the matrices themselves.

Nehomogeni mozaici

```
gap> mm:=ReadMat("13-346.txt");;
```

Nehomogeni mozaici

```
gap> mm:=ReadMat("13-346.txt");;

gap> List(mm,MosaicParameters);
[ "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)" ]
```

Nehomogeni mozaici

```
gap> mm:=ReadMat("13-346.txt");;

gap> List(mm,MosaicParameters);
[ "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)" ]

gap> Size(mm);
5
```

Nehomogeni mozaici

```
gap> mm:=ReadMat("13-346.txt");;

gap> List(mm,MosaicParameters);
[ "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)",
  "2-(13,3,1) + 2-(13,4,2) + 2-(13,6,5)" ]

gap> Size(mm);
5

gap> Size(MatFilter(mm));
5
```

Nehomogeni mozaici

O. Gnilke, P. Ó Catháin, O. Olmez, G. Nuñez Ponasso, *Invariants of quadratic forms and applications in design theory*, Linear Algebra Appl. **682** (2024), 1–27.

Nehomogeni mozaici

O. Gnilke, P. Ó Catháin, O. Olmez, G. Nuñez Ponasso, *Invariants of quadratic forms and applications in design theory*, Linear Algebra Appl. **682** (2024), 1–27.

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

Nehomogeni mozaici

O. Gnilke, P. Ó Catháin, O. Olmez, G. Nuñez Ponasso, *Invariants of quadratic forms and applications in design theory*, Linear Algebra Appl. **682** (2024), 1–27.

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

$$\overline{2\text{-}(31,6,1)} = 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

Nehomogeni mozaici

O. Gnilke, P. Ó Catháin, O. Olmez, G. Nuñez Ponasso, *Invariants of quadratic forms and applications in design theory*, Linear Algebra Appl. **682** (2024), 1–27.

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

$$2\text{-}(31,25,20) = 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

Nehomogeni mozaici

O. Gnilke, P. Ó Catháin, O. Olmez, G. Nuñez Ponasso, *Invariants of quadratic forms and applications in design theory*, Linear Algebra Appl. **682** (2024), 1–27.

$$2\text{-}(31,6,1) \oplus 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

$$2\text{-}(31,25,20) = 2\text{-}(31,10,3) \oplus 2\text{-}(31,15,7)$$

4.3. Decomposition of symmetric designs

We consider the following question: when can the incidence matrix of a symmetric design be written as the sum of two disjoint $\{0,1\}$ matrices, each of which is the incidence matrix of a symmetric design? The obvious necessary condition is that designs with suitable parameters should exist individually. In this section, we develop a further necessary condition in terms of invariants of quadratic forms.

Nehomogeni mozaici

Proposition 35. *Suppose that M is the incidence matrix of a symmetric (v, k, λ) design, and that $M = M_1 + M_2$ where M_i is the incidence matrix of a (v, k_i, λ_i) design.*

Then $k = k_1 + k_2$ and $\lambda = \lambda_1 + \lambda_2 + \alpha$ where $\alpha = \frac{2k_1k_2}{v-1}$ is an integer. Furthermore, $M_1M_2^\top + M_2M_1^\top = \alpha(J - I)$.

Proposition 35. *Suppose that M is the incidence matrix of a symmetric (v,k,λ) design, and that $M = M_1 + M_2$ where M_i is the incidence matrix of a (v,k_i,λ_i) design.*

Then $k = k_1 + k_2$ and $\lambda = \lambda_1 + \lambda_2 + \alpha$ where $\alpha = \frac{2k_1k_2}{v-1}$ is an integer. Furthermore, $M_1M_2^\top + M_2M_1^\top = \alpha(J - I)$.

Theorem 37. *Suppose that $M = M_1 + M_2$ is a decomposition of symmetric designs. If v is even then*

$$(k_1 - \lambda_1)(k_2 - \lambda_2) - \frac{2k_1k_2}{v-1} + 1$$

is the square of an integer. If v is odd, then

$$(\sigma,\sigma)_p^{\binom{v-1}{2}}(\sigma,v)_p = (\sigma,(-1)^{v-1/2}v)_p = 1$$

for all odd primes p.

Corollary 38. *If v is even, there is no decomposition of symmetric designs on less than 10,000 points.*

Nehomogeni mozaici

Corollary 38. *If v is even, there is no decomposition of symmetric designs on less than 10,000 points.*

In contrast, the conditions at odd orders are rather weaker. We observe that the incidence matrix of a $(91, 81, 72)$-design (the complementary design of a projective plane of order 9) cannot be written as the sum of designs with parameters $(91, 36, 14)$-design and a $(91, 45, 22)$-design. The relevant parameters for the computation are

$$k_1 = 36,\ \ \lambda_1 = 14,\ \ k_2 = 45,\ \ \lambda_2 = 22,\ \ \alpha = 36,\ \ \sigma = 471$$

The local invariants are $(471, 471)_p(471, 91)_p$ for all primes p. The prime 3 divides 471, so the invariant at $p = 3$ simplifies to $(3, 3)_p(1, 3)_p = -1$. So Theorem 37 shows that this decomposition does not exist.

Nehomogeni mozaici

Corollary 38. *If v is even, there is no decomposition of symmetric designs on less than 10,000 points.*

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$$k_1 = 36,\ \lambda_1 = 14,\ k_2 = 45,\ \lambda_2 = 22,\ \alpha = 36,\ \sigma = 471$$

The local invariants are $(471, 471)_p(471, 91)_p$ for all primes p. The prime 3 divides 471, so the invariant at $p = 3$ simplifies to $(3, 3)_p(1, 3)_p = -1$. So Theorem 37 shows that this decomposition does not exist.

~~2-(91, 10, 1) ⊕ 2-(91, 36, 14) ⊕ 2-(91, 45, 22)~~

Corollary 38. *If v is even, there is no decomposition of symmetric designs on less than 10,000 points.*

In contrast, the conditions at odd orders are rather weaker. We observe that the incidence matrix of a $(91, 81, 72)$-design (the complementary design of a projective plane of order 9) cannot be written as the sum of designs with parameters $(91, 36, 14)$-design and a $(91, 45, 22)$-design. The relevant parameters for the computation are

$$k_1 = 36,\ \ \lambda_1 = 14,\ \ k_2 = 45,\ \ \lambda_2 = 22,\ \ \alpha = 36,\ \ \sigma = 471$$

The local invariants are $(471, 471)_p(471, 91)_p$ for all primes p. The prime 3 divides 471, so the invariant at $p = 3$ simplifies to $(3, 3)_p(1, 3)_p = -1$. So Theorem 37 shows that this decomposition does not exist.

~~2-(91, 10, 1) ⊕ 2-(91, 36, 14) ⊕ 2-(91, 45, 22)~~

~~2-(91, 36, 14)~~

These methods **cannot** rule out the existence of a $(31, 25, 20)$-design (the complement of a projective plane of order 5) which decomposes into a $(31, 15, 7)$-design and a $(31, 10, 3)$-design. This is the smallest open case for a decomposition. Finally, we observe that solutions to this problem do exist, the sum of a skew-Hadamard design with parameters $(4t-1, 2t-1, t-1)$ with a trivial $(4t-1, 1, 0)$-design gives a $(4t-1, 2t, t)$-design, so the concept is not vacuous [10]. This topic will be considered more fully in forthcoming work of the authors.

Nehomogeni mozaici

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??? $2\text{-}(31, 6, 1) \oplus 2\text{-}(31, 10, 3) \oplus 2\text{-}(31, 15, 7)$???

Primjene popločavanja grupa i mozaika

X. Chen, Y. Zhou, *Asynchronous channel hopping systems from difference sets*, Des. Codes Cryptogr. **83** (2017), no. 1, 179–196.

Z. Gao, C. Li, Y. Zhou, *Upper bounds and constructions of complete asynchronous channel hopping systems*, Cryptogr. Commun. **11** (2019), no. 2, 299–312.

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Z. Gao, C. Li, Y. Zhou, *Upper bounds and constructions of complete asynchronous channel hopping systems*, Cryptogr. Commun. **11** (2019), no. 2, 299–312.

M. Wiese, H. Boche, *Mosaics of combinatorial designs for information-theoretic security*, Des. Codes Cryptogr. **90** (2022), no. 3, 593–632.

M. Wiese, H. Boche, *ε-Almost collision-flat universal hash functions and mosaics of designs*, Des. Codes Cryptogr. **92** (2024), no. 4, 975–998.

M. Wiese, H. Boche, *ε-Almost collision-flat universal hash functions and mosaics of designs*, Des. Codes Cryptogr. **92** (2024), no. 4, 975–998.

6 Open questions

After our extension results (Theorems 3 and 4), we discussed how the original function g and the generated $\hat{g}$ or $\check{g}$ relate with respect to equalities in the lower bounds on the seed sizes. What remained open was whether every seed-optimal OCFU hash function can be derived from a seed-optimal OU hash function. Formulated in terms of mosaics and designs, the question is: *Are the members of every mosaic of BIBDs resolvable?* In other words, is the method of Gnilke, Geferath and Pavčević (Corollary 3) essentially the only way of constructing a mosaic of BIBDs? By Corollary 2, the members of a mosaics of BIBD(v, k, λ) certainly need to satisfy the necessary condition $b \geq v + r - 1$ for resolvable designs.

Homogeni mozaici

V. Krčadinac, *Small examples of mosaics of combinatorial designs*, preprint, 2024. https://arxiv.org/abs/2405.12672

$$\begin{bmatrix}
1&2&1&1&2&1&1&3&3&1&2&3&1&3&2&1&3&3&2&2&3&2&3&2\\
1&1&2&1&1&2&3&1&3&3&1&2&2&1&3&3&1&3&3&2&2&2&2&3\\
2&1&1&2&1&1&3&3&1&2&3&1&3&2&1&3&3&1&2&3&2&3&2&2\\
1&3&2&2&3&3&1&2&1&3&3&1&2&1&2&2&2&3&1&3&1&1&3&2\\
2&1&3&3&2&3&1&1&2&1&3&3&2&2&1&3&2&2&1&1&3&2&1&3\\
3&2&1&3&3&2&2&1&1&3&1&3&1&2&2&2&3&2&3&1&1&3&2&1\\
2&3&3&1&3&2&3&2&2&2&2&1&1&3&3&2&1&1&1&2&3&3&1&1\\
3&2&3&2&1&3&2&3&2&1&2&2&3&1&3&1&2&1&3&1&2&1&3&1\\
3&3&2&3&2&1&2&2&3&2&1&2&3&3&1&1&1&2&2&3&1&1&1&3
\end{bmatrix}$$

$$\begin{bmatrix}
1&2&1&1&2&1&1&3&3&1&3&3&1&3&2&1&2&3&3&2&2&3&2&2\\
1&1&2&1&1&2&3&1&3&3&1&3&2&1&3&3&1&2&2&3&2&2&3&2\\
2&1&1&2&1&1&3&3&1&3&3&1&3&2&1&2&3&1&2&2&3&2&2&3\\
1&3&2&3&3&1&2&2&1&2&1&3&3&3&2&2&3&2&1&2&1&1&3&1\\
2&1&3&1&3&3&1&2&2&3&2&1&2&3&3&2&2&3&1&1&2&1&1&3\\
3&2&1&3&1&3&2&1&2&1&3&2&3&2&3&3&2&2&2&1&1&3&1&1\\
2&3&3&2&3&2&2&3&1&1&2&2&1&1&2&3&1&1&1&3&3&3&1&2\\
3&2&3&2&2&3&1&2&3&2&1&2&2&1&1&1&3&1&3&1&3&2&3&1\\
3&3&2&3&2&2&3&1&2&2&2&1&1&2&1&1&1&3&3&3&1&1&2&3
\end{bmatrix}$$

TABLE 2. Two $2\text{-}(9,3,2) \oplus 2\text{-}(9,3,2) \oplus 2\text{-}(9,3,2)$ mosaics.

Homogeni mozaici

C.J. Colbourn, J.H. Dinitz (Eds.), Handbook of combinatorial designs, 2nd edition, Chapman & Hall/CRC, Boca Raton, FL, 2007.

No	v	b	r	k	λ	Nd	Nr	Comments, Ref	Where?
1	7	7	3	3	1	1	-	PG(2,2)	II.6.4
2	9	12	4	3	1	1	1	R#3,AG(2,3)	1.22
3	13	13	4	4	1	1	-	PG(2,3)	1.26
4	6	10	5	3	2	1	0	R#7,×3	1.18
5	16	20	5	4	1	1	1	R#6,AG(2,4)	1.31
6	21	21	5	5	1	1	-	PG(2,4)	VI.18.73
7	11	11	5	5	2	1	-		1.26
8	13	26	6	3	1	2	-	[1544]	1.27
9	7	14	6	3	2	4	-	2#1,D#20 [1665]	1.19
10	10	15	6	4	2	3	-	R#13 [1665]	1.25
11	25	30	6	5	1	1	1	R#12,AG(2,5)	
12	31	31	6	6	1	1	-	PG(2,5)	VI.18.73
13	16	16	6	6	2	3	-	[898]	1.32
14	15	35	7	3	1	80	7	PG(3,2) [1241, 1544]	1.28
15	8	14	7	4	3	4	1	R#20,$AG_2(3,2)$ [1241, 898]	1.21
16	15	21	7	5	2	0	0	R#19*,×2	
17	36	42	7	6	1	0	0	R#18*,×2,AG(2,6)	
18	43	43	7	7	1	0	-	×1,PG(2,6)	
19	22	22	7	7	2	0	-	×1	
20	15	15	7	7	3	5	-	$PG_2(3,2)$ [898]	1.30
21	9	24	8	3	2	36	9	2#2,D#40 [1547]	1.23
22	25	50	8	4	1	18	-	[1339, 1945]	1.34

2.8.4 MosaicToBlockDesigns

▷ `MosaicToBlockDesigns(M)` (function)

Transforms a mosaic of combinatorial designs M with c colors to a list of c block designs in the Design package format.

Homogeni mozaici

2.8.4 MosaicToBlockDesigns

▷ `MosaicToBlockDesigns(M)` (function)

Transforms a mosaic of combinatorial designs *M* with c colors to a list of c block designs in the Design package format.

2.4.2 BlockDesignFilter

▷ `BlockDesignFilter(dl[, opt])` (function)

Eliminates isomorphic copies from a list of block designs *dl*. Uses `nauty/Traces 2.8` by B.D.McKay and A.Piperno [MP14]. This is an alternative for the `BlockDesignIsomorphismClassRepresentatives` function from the Design package (**DESIGN: Automorphism groups and isomorphism testing for block designs**). The optional argument *opt* is a record for options. Possible components of *opt* are:

- *Traces*:=`true/false` Use `Traces`. This is the default.
- *SparseNauty*:=`true/false` Use `nauty` for sparse graphs.
- *PointClasses*:=*s* Color the points into classes of size *s* that cannot be mapped onto each other. By default all points are in the same class.
- *Positions*:=`true/false` Return positions of nonisomorphic designs instead of the designs themselves.

```
gap> m:=ReadMat("9-3-2ex1.txt")[1];

[ [ 1, 2, 1, 1, 2, 1, 1, 3, 3, 1, 2, 3, 1, 3, 2, 1, 3, 3, 2, 2, 3, 2, 3, 2 ],
  [ 1, 1, 2, 1, 1, 2, 3, 1, 3, 3, 1, 2, 2, 1, 3, 3, 1, 3, 3, 2, 2, 2, 2, 3 ],
  [ 2, 1, 1, 2, 1, 1, 3, 3, 1, 2, 3, 1, 3, 2, 1, 3, 3, 1, 2, 3, 2, 3, 2, 2 ],
  [ 1, 3, 2, 2, 3, 3, 1, 2, 1, 3, 3, 1, 2, 1, 2, 2, 2, 3, 1, 3, 1, 1, 3, 2 ],
  [ 2, 1, 3, 3, 2, 3, 1, 1, 2, 1, 3, 3, 2, 2, 1, 3, 2, 2, 1, 1, 3, 2, 1, 3 ],
  [ 3, 2, 1, 3, 3, 2, 2, 1, 1, 3, 1, 3, 1, 2, 2, 2, 3, 2, 3, 1, 1, 3, 2, 1 ],
  [ 2, 3, 3, 1, 3, 2, 3, 2, 2, 2, 2, 1, 1, 3, 3, 2, 1, 1, 1, 2, 3, 3, 1, 1 ],
  [ 3, 2, 3, 2, 1, 3, 2, 3, 2, 1, 2, 2, 3, 1, 3, 1, 2, 1, 3, 1, 2, 1, 3, 1 ],
  [ 3, 3, 2, 3, 2, 1, 2, 2, 3, 2, 1, 2, 3, 3, 1, 1, 1, 2, 2, 3, 1, 1, 1, 3 ]
 ]
```

Homogeni mozaici

```
gap> m:=ReadMat("9-3-2ex1.txt")[1];

[ [ 1, 2, 1, 1, 2, 1, 1, 3, 3, 1, 2, 3, 1, 3, 2, 1, 3, 3, 2, 2, 3, 2, 3, 2 ],
  [ 1, 1, 2, 1, 1, 2, 3, 1, 3, 3, 1, 2, 2, 1, 3, 3, 1, 3, 3, 2, 2, 2, 2, 3 ],
  [ 2, 1, 1, 2, 1, 1, 3, 3, 1, 2, 3, 1, 3, 2, 1, 3, 3, 1, 2, 3, 2, 3, 2, 2 ],
  [ 1, 3, 2, 2, 3, 3, 1, 2, 1, 3, 3, 1, 2, 1, 2, 2, 2, 3, 1, 3, 1, 1, 3, 2 ],
  [ 2, 1, 3, 3, 2, 3, 1, 1, 2, 1, 3, 3, 2, 2, 1, 3, 2, 2, 1, 1, 3, 2, 1, 3 ],
  [ 3, 2, 1, 3, 3, 2, 2, 1, 1, 3, 1, 3, 1, 2, 2, 2, 3, 2, 3, 1, 1, 3, 2, 1 ],
  [ 2, 3, 3, 1, 3, 2, 3, 2, 2, 2, 2, 1, 1, 3, 3, 2, 1, 1, 1, 2, 3, 3, 1, 1 ],
  [ 3, 2, 3, 2, 1, 3, 2, 3, 2, 1, 2, 2, 3, 1, 3, 1, 2, 1, 3, 1, 2, 1, 3, 1 ],
  [ 3, 3, 2, 3, 2, 1, 2, 2, 3, 2, 1, 2, 3, 3, 1, 1, 1, 2, 2, 3, 1, 1, 1, 3 ]
 ]

gap> MosaicParameters(m);
"2-(9,3,2) + 2-(9,3,2) + 2-(9,3,2)"
```

Homogeni mozaici

```
gap> dd:=MosaicToBlockDesigns(m);
[ rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 7 ], [ 1, 3, 6 ], [ 1, 3, 9 ],
          [ 1, 4, 5 ], [ 1, 5, 8 ], [ 1, 6, 7 ], [ 1, 8, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 8 ], [ 2, 4, 8 ], [ 2, 5, 6 ], [ 2, 6, 9 ], [ 2, 7, 9 ],
          [ 3, 4, 6 ], [ 3, 4, 7 ], [ 3, 5, 9 ], [ 3, 7, 8 ], [ 4, 5, 7 ],
          [ 4, 6, 9 ], [ 4, 8, 9 ], [ 5, 6, 8 ], [ 5, 7, 9 ], [ 6, 7, 8 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 5 ], [ 1, 2, 7 ], [ 1, 3, 4 ], [ 1, 3, 9 ],
          [ 1, 4, 6 ], [ 1, 5, 9 ], [ 1, 6, 8 ], [ 1, 7, 8 ], [ 2, 3, 6 ],
          [ 2, 3, 8 ], [ 2, 4, 5 ], [ 2, 4, 9 ], [ 2, 6, 7 ], [ 2, 8, 9 ],
          [ 3, 4, 8 ], [ 3, 5, 6 ], [ 3, 5, 7 ], [ 3, 7, 9 ], [ 4, 5, 8 ],
          [ 4, 6, 7 ], [ 4, 7, 9 ], [ 5, 6, 9 ], [ 5, 7, 8 ], [ 6, 8, 9 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 9 ], [ 1, 3, 6 ], [ 1, 3, 8 ],
          [ 1, 4, 8 ], [ 1, 5, 6 ], [ 1, 5, 7 ], [ 1, 7, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 7 ], [ 2, 4, 6 ], [ 2, 5, 9 ], [ 2, 6, 8 ], [ 2, 7, 8 ],
          [ 3, 4, 5 ], [ 3, 4, 9 ], [ 3, 6, 7 ], [ 3, 8, 9 ], [ 4, 5, 8 ],
          [ 4, 6, 7 ], [ 4, 7, 9 ], [ 5, 6, 9 ], [ 5, 7, 8 ], [ 6, 8, 9 ] ],
      isBlockDesign := true, v := 9 ) ]
```

Homogeni mozaici

```
gap> dd:=MosaicToBlockDesigns(m);
[ rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 7 ], [ 1, 3, 6 ], [ 1, 3, 9 ],
          [ 1, 4, 5 ], [ 1, 5, 8 ], [ 1, 6, 7 ], [ 1, 8, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 8 ], [ 2, 4, 8 ], [ 2, 5, 6 ], [ 2, 6, 9 ], [ 2, 7, 9 ],
          [ 3, 4, 6 ], [ 3, 4, 7 ], [ 3, 5, 9 ], [ 3, 7, 8 ], [ 4, 5, 7 ],
          [ 4, 6, 9 ], [ 4, 8, 9 ], [ 5, 6, 8 ], [ 5, 7, 9 ], [ 6, 7, 8 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 5 ], [ 1, 2, 7 ], [ 1, 3, 4 ], [ 1, 3, 9 ],
          [ 1, 4, 6 ], [ 1, 5, 9 ], [ 1, 6, 8 ], [ 1, 7, 8 ], [ 2, 3, 6 ],
          [ 2, 3, 8 ], [ 2, 4, 5 ], [ 2, 4, 9 ], [ 2, 6, 7 ], [ 2, 8, 9 ],
          [ 3, 4, 8 ], [ 3, 5, 6 ], [ 3, 5, 7 ], [ 3, 7, 9 ], [ 4, 5, 8 ],
          [ 4, 6, 7 ], [ 4, 7, 9 ], [ 5, 6, 9 ], [ 5, 7, 8 ], [ 6, 8, 9 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 9 ], [ 1, 3, 6 ], [ 1, 3, 8 ],
          [ 1, 4, 8 ], [ 1, 5, 6 ], [ 1, 5, 7 ], [ 1, 7, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 7 ], [ 2, 4, 6 ], [ 2, 5, 9 ], [ 2, 6, 8 ], [ 2, 7, 8 ],
          [ 3, 4, 5 ], [ 3, 4, 9 ], [ 3, 6, 7 ], [ 3, 8, 9 ], [ 4, 5, 8 ],
          [ 4, 6, 7 ], [ 4, 7, 9 ], [ 5, 6, 9 ], [ 5, 7, 8 ], [ 6, 8, 9 ] ],
      isBlockDesign := true, v := 9 ) ]

gap> Size(BlockDesignFilter(dd));
1
```

Homogeni mozaici

```
gap> MakeResolutionsComponent(dd[1]);
gap> dd[1].resolutions.list;
[  ]
```

Homogeni mozaici

```
gap> MakeResolutionsComponent(dd[1]);
gap> dd[1].resolutions.list;
[  ]

gap> m:=ReadMat("9-3-2ex2.txt")[1];
```

```
[ [ 1, 2, 1, 1, 2, 1, 1, 3, 3, 1, 3, 3, 1, 3, 2, 1, 2, 3, 3, 2, 2, 3, 2, 2 ],
  [ 1, 1, 2, 1, 1, 2, 3, 1, 3, 3, 1, 3, 2, 1, 3, 3, 1, 2, 2, 3, 2, 2, 3, 2 ],
  [ 2, 1, 1, 2, 1, 1, 3, 3, 1, 3, 3, 1, 3, 2, 1, 2, 3, 1, 2, 2, 3, 2, 2, 3 ],
  [ 1, 3, 2, 3, 3, 1, 2, 2, 1, 2, 1, 3, 3, 3, 2, 2, 3, 2, 1, 2, 1, 1, 3, 1 ],
  [ 2, 1, 3, 1, 3, 3, 1, 2, 2, 3, 2, 1, 2, 3, 3, 2, 2, 3, 1, 1, 2, 1, 1, 3 ],
  [ 3, 2, 1, 3, 1, 3, 2, 1, 2, 1, 3, 2, 3, 2, 3, 3, 2, 2, 2, 1, 1, 3, 1, 1 ],
  [ 2, 3, 3, 2, 3, 2, 2, 3, 1, 1, 2, 2, 1, 1, 2, 3, 1, 1, 1, 3, 3, 3, 1, 2 ],
  [ 3, 2, 3, 2, 2, 3, 1, 2, 3, 2, 1, 2, 2, 1, 1, 1, 3, 1, 3, 1, 3, 2, 3, 1 ],
  [ 3, 3, 2, 3, 2, 2, 3, 1, 2, 2, 2, 1, 1, 2, 1, 1, 1, 3, 3, 3, 1, 1, 2, 3 ]
 ]
```

Homogeni mozaici

```
gap> MakeResolutionsComponent(dd[1]);
gap> dd[1].resolutions.list;
[  ]

gap> m:=ReadMat("9-3-2ex2.txt")[1];

[ [ 1, 2, 1, 1, 2, 1, 1, 3, 3, 1, 3, 3, 1, 3, 2, 1, 2, 3, 3, 2, 2, 3, 2, 2 ],
  [ 1, 1, 2, 1, 1, 2, 3, 1, 3, 3, 1, 3, 2, 1, 3, 3, 1, 2, 2, 3, 2, 2, 3, 2 ],
  [ 2, 1, 1, 2, 1, 1, 3, 3, 1, 3, 3, 1, 3, 2, 1, 2, 3, 1, 2, 2, 3, 2, 2, 3 ],
  [ 1, 3, 2, 3, 3, 1, 2, 2, 1, 2, 1, 3, 3, 3, 2, 2, 3, 2, 1, 2, 1, 1, 3, 1 ],
  [ 2, 1, 3, 1, 3, 3, 1, 2, 2, 3, 2, 1, 2, 3, 3, 2, 2, 3, 1, 1, 2, 1, 1, 3 ],
  [ 3, 2, 1, 3, 1, 3, 2, 1, 2, 1, 3, 2, 3, 2, 3, 3, 2, 2, 2, 1, 1, 3, 1, 1 ],
  [ 2, 3, 3, 2, 3, 2, 2, 3, 1, 1, 2, 2, 1, 1, 2, 3, 1, 1, 1, 3, 3, 3, 1, 2 ],
  [ 3, 2, 3, 2, 2, 3, 1, 2, 3, 2, 1, 2, 2, 1, 1, 1, 3, 1, 3, 1, 3, 2, 3, 1 ],
  [ 3, 3, 2, 3, 2, 2, 3, 1, 2, 2, 2, 1, 1, 2, 1, 1, 1, 3, 3, 3, 1, 1, 2, 3 ]
 ]

gap> MosaicParameters(m);
"2-(9,3,2) + 2-(9,3,2) + 2-(9,3,2)"
```

Homogeni mozaici

```
gap> dd:=MosaicToBlockDesigns(m);
[ rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 5 ], [ 1, 3, 4 ], [ 1, 3, 6 ],
          [ 1, 5, 8 ], [ 1, 6, 7 ], [ 1, 7, 9 ], [ 1, 8, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 6 ], [ 2, 4, 8 ], [ 2, 6, 9 ], [ 2, 7, 8 ], [ 2, 7, 9 ],
          [ 3, 4, 7 ], [ 3, 5, 9 ], [ 3, 7, 8 ], [ 3, 8, 9 ], [ 4, 5, 7 ],
          [ 4, 5, 9 ], [ 4, 6, 8 ], [ 4, 6, 9 ], [ 5, 6, 7 ], [ 5, 6, 8 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 5 ], [ 1, 2, 7 ], [ 1, 3, 4 ], [ 1, 3, 9 ],
          [ 1, 4, 7 ], [ 1, 5, 6 ], [ 1, 6, 8 ], [ 1, 8, 9 ], [ 2, 3, 6 ],
          [ 2, 3, 8 ], [ 2, 4, 6 ], [ 2, 4, 9 ], [ 2, 5, 8 ], [ 2, 7, 9 ],
          [ 3, 4, 5 ], [ 3, 5, 7 ], [ 3, 6, 9 ], [ 3, 7, 8 ], [ 4, 5, 8 ],
          [ 4, 6, 7 ], [ 4, 8, 9 ], [ 5, 6, 9 ], [ 5, 7, 9 ], [ 6, 7, 8 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 8 ], [ 1, 3, 6 ], [ 1, 3, 7 ],
          [ 1, 4, 5 ], [ 1, 5, 9 ], [ 1, 6, 7 ], [ 1, 8, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 9 ], [ 2, 4, 8 ], [ 2, 5, 6 ], [ 2, 6, 7 ], [ 2, 7, 9 ],
          [ 3, 4, 6 ], [ 3, 4, 8 ], [ 3, 5, 9 ], [ 3, 7, 8 ], [ 4, 5, 7 ],
          [ 4, 6, 9 ], [ 4, 7, 9 ], [ 5, 6, 8 ], [ 5, 7, 8 ], [ 6, 8, 9 ] ],
      isBlockDesign := true, v := 9 ) ]
```

Homogeni mozaici

```
gap> dd:=MosaicToBlockDesigns(m);
[ rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 5 ], [ 1, 3, 4 ], [ 1, 3, 6 ],
          [ 1, 5, 8 ], [ 1, 6, 7 ], [ 1, 7, 9 ], [ 1, 8, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 6 ], [ 2, 4, 8 ], [ 2, 6, 9 ], [ 2, 7, 8 ], [ 2, 7, 9 ],
          [ 3, 4, 7 ], [ 3, 5, 9 ], [ 3, 7, 8 ], [ 3, 8, 9 ], [ 4, 5, 7 ],
          [ 4, 5, 9 ], [ 4, 6, 8 ], [ 4, 6, 9 ], [ 5, 6, 7 ], [ 5, 6, 8 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 5 ], [ 1, 2, 7 ], [ 1, 3, 4 ], [ 1, 3, 9 ],
          [ 1, 4, 7 ], [ 1, 5, 6 ], [ 1, 6, 8 ], [ 1, 8, 9 ], [ 2, 3, 6 ],
          [ 2, 3, 8 ], [ 2, 4, 6 ], [ 2, 4, 9 ], [ 2, 5, 8 ], [ 2, 7, 9 ],
          [ 3, 4, 5 ], [ 3, 5, 7 ], [ 3, 6, 9 ], [ 3, 7, 8 ], [ 4, 5, 8 ],
          [ 4, 6, 7 ], [ 4, 8, 9 ], [ 5, 6, 9 ], [ 5, 7, 9 ], [ 6, 7, 8 ] ],
      isBlockDesign := true, v := 9 ),
  rec( blocks := [ [ 1, 2, 4 ], [ 1, 2, 8 ], [ 1, 3, 6 ], [ 1, 3, 7 ],
          [ 1, 4, 5 ], [ 1, 5, 9 ], [ 1, 6, 7 ], [ 1, 8, 9 ], [ 2, 3, 5 ],
          [ 2, 3, 9 ], [ 2, 4, 8 ], [ 2, 5, 6 ], [ 2, 6, 7 ], [ 2, 7, 9 ],
          [ 3, 4, 6 ], [ 3, 4, 8 ], [ 3, 5, 9 ], [ 3, 7, 8 ], [ 4, 5, 7 ],
          [ 4, 6, 9 ], [ 4, 7, 9 ], [ 5, 6, 8 ], [ 5, 7, 8 ], [ 6, 8, 9 ] ],
      isBlockDesign := true, v := 9 ) ]
gap> Size(BlockDesignFilter(dd));
3
```

Homogeni mozaici

```
gap> MakeResolutionsComponent(dd[1]);
gap> MakeResolutionsComponent(dd[2]);
gap> MakeResolutionsComponent(dd[3]);
```

Homogeni mozaici

```
gap> MakeResolutionsComponent(dd[1]);
gap> MakeResolutionsComponent(dd[2]);
gap> MakeResolutionsComponent(dd[3]);

gap> dd[1].resolutions.list;
```

```
[ rec( autGroup := Group([ (1,5,8)(2,6,9)(3,4,7), (1,7,6)(2,8,4)(3,9,5), (1,2)(4,5)(7,9) ]),
      partition :=
        [ rec( blocks := [ [ 1, 2, 4 ], [ 3, 8, 9 ], [ 5, 6, 7 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 2, 5 ], [ 3, 7, 8 ], [ 4, 6, 9 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 3, 4 ], [ 2, 7, 9 ], [ 5, 6, 8 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 3, 6 ], [ 2, 7, 8 ], [ 4, 5, 9 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 5, 8 ], [ 2, 6, 9 ], [ 3, 4, 7 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 6, 7 ], [ 2, 4, 8 ], [ 3, 5, 9 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 7, 9 ], [ 2, 3, 5 ], [ 4, 6, 8 ] ], isBlockDesign := true, v := 9 ),
          rec( blocks := [ [ 1, 8, 9 ], [ 2, 3, 6 ], [ 4, 5, 7 ] ], isBlockDesign := true, v := 9 ) ]
     ) ]
```

```
gap> dd[2].resolutions.list;
[  ]
gap> dd[3].resolutions.list;
[  ]
```

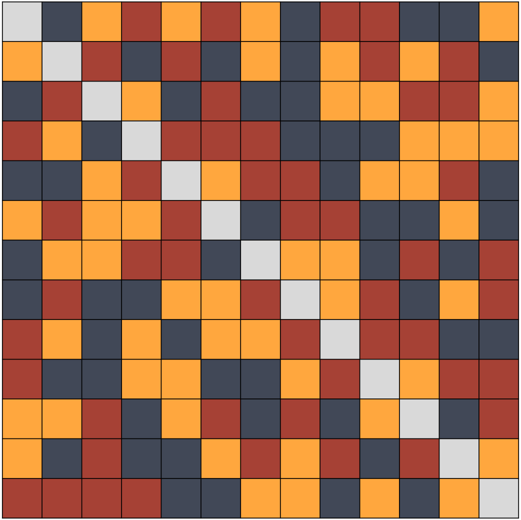
Mozaici projektivnih ravnina

V. Krčadinac, *Small examples of mosaics of combinatorial designs*, preprint, 2024. https://arxiv.org/abs/2405.12672

$$\begin{bmatrix} 0 & 1 & 2 & 1 & 3 & 2 & 3 & 1 & 1 & 3 & 3 & 2 & 2 \\ 3 & 0 & 2 & 3 & 2 & 1 & 2 & 1 & 2 & 3 & 1 & 1 & 3 \\ 3 & 1 & 0 & 2 & 1 & 3 & 3 & 3 & 2 & 2 & 1 & 2 & 1 \\ 3 & 3 & 1 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 \\ 2 & 1 & 1 & 2 & 0 & 2 & 2 & 3 & 3 & 1 & 3 & 1 & 3 \\ 2 & 3 & 2 & 3 & 3 & 0 & 1 & 3 & 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 2 & 3 & 3 & 0 & 2 & 1 & 1 & 1 & 3 & 3 \\ 3 & 2 & 3 & 1 & 3 & 1 & 2 & 0 & 3 & 1 & 2 & 2 & 1 \\ 1 & 1 & 3 & 2 & 2 & 1 & 1 & 3 & 0 & 3 & 2 & 3 & 2 \\ 1 & 3 & 3 & 1 & 1 & 2 & 3 & 2 & 2 & 0 & 2 & 1 & 3 \\ 1 & 2 & 1 & 3 & 2 & 2 & 3 & 1 & 3 & 2 & 0 & 3 & 1 \\ 2 & 2 & 3 & 3 & 1 & 3 & 1 & 1 & 2 & 1 & 3 & 0 & 2 \\ 2 & 3 & 1 & 1 & 2 & 3 & 1 & 2 & 3 & 3 & 1 & 2 & 0 \end{bmatrix}$$

Table 3. A $2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,4,1) \oplus 2\text{-}(13,1,0)$ mosaic.

Mozaici projektivnih ravnina



Mozaici projektivnih ravnina

Za koje redove q postoji q-mozaik projektivnih ravnina reda q?

$$(q^2+q+1, q+1, 1) \oplus \cdots \oplus (q^2+q+1, q+1, 1) \oplus (q^2+q+1, 1, 0)$$

q	2	3	4	5	7	8	9	…
Popločavanje	✓	✗	✗	✓	✓	✓	?	…
Mozaik	✓	✓	?	✓	✓	✓	?	…

Mozaici projektivnih ravnina

```
gap> m:=ReadMat("13-4-1.txt")[1];
[ [ 0, 1, 2, 1, 3, 2, 3, 1, 1, 3, 3, 2, 2 ],
  [ 3, 0, 2, 3, 2, 1, 2, 1, 2, 3, 1, 1, 3 ],
  [ 3, 1, 0, 2, 1, 3, 3, 3, 2, 2, 1, 2, 1 ],
  [ 3, 3, 1, 0, 1, 1, 2, 2, 1, 2, 3, 3, 2 ],
  [ 2, 1, 1, 2, 0, 2, 2, 3, 3, 1, 3, 1, 3 ],
  [ 2, 3, 2, 3, 3, 0, 1, 3, 1, 2, 2, 1, 1 ],
  [ 1, 2, 2, 2, 3, 3, 0, 2, 1, 1, 1, 3, 3 ],
  [ 3, 2, 3, 1, 3, 1, 2, 0, 3, 1, 2, 2, 1 ],
  [ 1, 1, 3, 2, 2, 1, 1, 3, 0, 3, 2, 3, 2 ],
  [ 1, 3, 3, 1, 1, 2, 3, 2, 2, 0, 2, 1, 3 ],
  [ 1, 2, 1, 3, 2, 2, 3, 1, 3, 2, 0, 3, 1 ],
  [ 2, 2, 3, 3, 1, 3, 1, 1, 2, 1, 3, 0, 2 ],
  [ 2, 3, 1, 1, 2, 3, 1, 2, 3, 3, 1, 2, 0 ] ]

gap> MosaicParameters(m);
"2-(13,4,1) + 2-(13,4,1) + 2-(13,4,1)"
```

```
gap> m:=ReadMat("13-4-1.txt")[1];
[ [ 0, 1, 2, 1, 3, 2, 3, 1, 1, 3, 3, 2, 2 ],
  [ 3, 0, 2, 3, 2, 1, 2, 1, 2, 3, 1, 1, 3 ],
  [ 3, 1, 0, 2, 1, 3, 3, 3, 2, 2, 1, 2, 1 ],
  [ 3, 3, 1, 0, 1, 1, 2, 2, 1, 2, 3, 3, 2 ],
  [ 2, 1, 1, 2, 0, 2, 2, 3, 3, 1, 3, 1, 3 ],
  [ 2, 3, 2, 3, 3, 0, 1, 3, 1, 2, 2, 1, 1 ],
  [ 1, 2, 2, 2, 3, 3, 0, 2, 1, 1, 1, 3, 3 ],
  [ 3, 2, 3, 1, 3, 1, 2, 0, 3, 1, 2, 2, 1 ],
  [ 1, 1, 3, 2, 2, 1, 1, 3, 0, 3, 2, 3, 2 ],
  [ 1, 3, 3, 1, 1, 2, 3, 2, 2, 0, 2, 1, 3 ],
  [ 1, 2, 1, 3, 2, 2, 3, 1, 3, 2, 0, 3, 1 ],
  [ 2, 2, 3, 3, 1, 3, 1, 1, 2, 1, 3, 0, 2 ],
  [ 2, 3, 1, 1, 2, 3, 1, 2, 3, 3, 1, 2, 0 ] ]

gap> MosaicParameters(m);
"2-(13,4,1) + 2-(13,4,1) + 2-(13,4,1)"

gap> aut:=MatAut(m);
Group([ (1,3,2)(4,6,5)(7,9,8)(10,12,11)
  (14,16,15)(17,19,18)(20,22,21)(23,25,24)
  (28,30,29) ])
```

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

$$2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,1,0)$$

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

   $$2\text{-}(21, 5, 1) \oplus 2\text{-}(21, 5, 1) \oplus 2\text{-}(21, 5, 1) \oplus 2\text{-}(21, 5, 1) \oplus 2\text{-}(21, 1, 0)$$

2. Postoje li "planarna" popločavanja grupa za $q = 9$ i veće redove?

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

   $$2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,1,0)$$

2. Postoje li “planarna” popločavanja grupa za $q = 9$ i veće redove?

3. Može li se Hasse-Minkowskijeva teorija primijeniti na ovaj problem?

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

   $$2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,1,0)$$

2. Postoje li “planarna” popločavanja grupa za $q = 9$ i veće redove?

3. Može li se Hasse-Minkowskijeva teorija primijeniti na ovaj problem?

Kramer-Mesnerova metoda za mozaike

- Za automorfizme s $\gamma = id$: blokovi su particije v-skupa umjesto k-podskupova v-skupa

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

$$2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,1,0)$$

2. Postoje li "planarna" popločavanja grupa za $q = 9$ i veće redove?

3. Može li se Hasse-Minkowskijeva teorija primijeniti na ovaj problem?

Kramer-Mesnerova metoda za mozaike

- Za automorfizme s $\gamma = id$: blokovi su particije v-skupa umjesto k-podskupova v-skupa

- Problemi: G-orbita particija ima više,

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

   $$2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,1,0)$$

2. Postoje li "planarna" popločavanja grupa za $q = 9$ i veće redove?

3. Može li se Hasse-Minkowskijeva teorija primijeniti na ovaj problem?

Kramer-Mesnerova metoda za mozaike

- Za automorfizme s $\gamma = id$: blokovi su particije v-skupa umjesto k-podskupova v-skupa

- Problemi: G-orbita particija ima više, mozaici "ne vole" automorfizme

Mozaici – TO DO lista

Pitanja o mozaicima projektivnih ravnina:

1. Postoji li mozaik za $q = 4$?

   $$2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,5,1) \oplus 2\text{-}(21,1,0)$$

2. Postoje li "planarna" popločavanja grupa za $q = 9$ i veće redove?

3. Može li se Hasse-Minkowskijeva teorija primijeniti na ovaj problem?

Kramer-Mesnerova metoda za mozaike

- Za automorfizme s $\gamma = id$: blokovi su particije v-skupa umjesto k-podskupova v-skupa
- Problemi: G-orbita particija ima više, mozaici "ne vole" automorfizme
- Kako raditi za autotopije s $\gamma \neq id$?

Mozaici – TO DO lista

Diferencijske familije za mozaike

Nehomogene mozaike 2-$(13,3,1) \oplus$ 2-$(13,4,2) \oplus$ 2-$(13,6,5)$ dobivamo od ovakvih uređenih diferencijskih familija u $\mathbb{Z}_{13}$:

$$\mathcal{F}_1 = (\{0,1,4\},\ \{0,2,7\})$$

$$\mathcal{F}_2 = (\{2,6,7,9\},\ \{1,3,10,11\})$$

$$\mathcal{F}_3 = (\{3,5,8,10,11,12\},\ \{4,5,6,8,9,12\})$$

Mozaici – TO DO lista

Diferencijske familije za mozaike

Nehomogene mozaike 2-$(13, 3, 1) \oplus$ 2-$(13, 4, 2) \oplus$ 2-$(13, 6, 5)$ dobivamo od ovakvih uređenih diferencijskih familija u $\mathbb{Z}_{13}$:

$$\mathcal{F}_1 = (\{0, 1, 4\}, \ \{0, 2, 7\})$$

$$\mathcal{F}_2 = (\{2, 6, 7, 9\}, \ \{1, 3, 10, 11\})$$

$$\mathcal{F}_3 = (\{3, 5, 8, 10, 11, 12\}, \ \{4, 5, 6, 8, 9, 12\})$$

M. Buratti, J. Yan, C. Wang, *From a 1-rotational RBIBD to a partitioned difference family*, Electron. J. Combin. **17** (2010), no. 1, R139, 23 pp.

M. Buratti, *On disjoint* $(v, k, k-1)$ *difference families*, Des. Codes Cryptogr. **87** (2019), no. 4, 745–755.

Mozaici – TO DO lista

M. Buratti, D. Jungnickel, *Partitioned difference families: the storm has not yet passed*, Adv. Math. Commun. **17** (2023), no. 4, 928–934.

M. Buratti, D. Jungnickel, *Partitioned difference families and harmonious linear spaces*, Finite Fields Appl. **92** (2023), 102274, 21 pp.

M. Buratti, D. Jungnickel, *Partitioned difference families: the storm has not yet passed*, Adv. Math. Commun. **17** (2023), no. 4, 928–934.

M. Buratti, D. Jungnickel, *Partitioned difference families and harmonious linear spaces*, Finite Fields Appl. **92** (2023), 102274, 21 pp.

Obojeni dizajni i $\mathbb{Z}_4$-kodovi

I. Duursma, T. Helleseth, C. Rong, K. Yang, *Split weight enumerators for the Preparata codes with applications to designs*, Des. Codes Cryptogr. **18** (1999), no. 1–3, 103–124.

A. Bonnecaze, E. Rains, P. Solé, 3-*colored* 5-*designs and* $\mathbb{Z}_4$-*codes*, J. Statist. Plann. Inference **86** (2000), no. 2, 349–368.

Mozaici – TO DO lista

M. Buratti, D. Jungnickel, *Partitioned difference families: the storm has not yet passed*, Adv. Math. Commun. **17** (2023), no. 4, 928–934.

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A. Bonnecaze, E. Rains, P. Solé, *3-colored 5-designs and $\mathbb{Z}_4$-codes*, J. Statist. Plann. Inference **86** (2000), no. 2, 349–368.

$$5\text{-}(24,8,800) \oplus 5\text{-}(24,8,800) \oplus 5\text{-}(24,8,800)$$

$$5\text{-}(24,6,54) \oplus 5\text{-}(24,8,504) \oplus 5\text{-}(24,10,2268)$$

Mozaici – TO DO lista

M. Buratti, D. Jungnickel, *Partitioned difference families: the storm has not yet passed*, Adv. Math. Commun. **17** (2023), no. 4, 928–934.

M. Buratti, D. Jungnickel, *Partitioned difference families and harmonious linear spaces*, Finite Fields Appl. **92** (2023), 102274, 21 pp.

Obojeni dizajni i $\mathbb{Z}_4$-kodovi

I. Duursma, T. Helleseth, C. Rong, K. Yang, *Split weight enumerators for the Preparata codes with applications to designs*, Des. Codes Cryptogr. **18** (1999), no. 1–3, 103–124.

A. Bonnecaze, E. Rains, P. Solé, 3-*colored* 5-*designs and* $\mathbb{Z}_4$-*codes*, J. Statist. Plann. Inference **86** (2000), no. 2, 349–368.

$$5\text{-}(24,8,800) \oplus 5\text{-}(24,8,800) \oplus 5\text{-}(24,8,800)$$

$$5\text{-}(24,6,54) \oplus 5\text{-}(24,8,504) \oplus 5\text{-}(24,10,2268)$$

$$5\text{-}(24,(6,8,10),382\,536)$$

M. Buratti, D. Jungnickel, *Partitioned difference families: the storm has not yet passed*, Adv. Math. Commun. **17** (2023), no. 4, 928–934.

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Obojeni dizajni i $\mathbb{Z}_4$-kodovi

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$$5\text{-}(24,8,800) \oplus 5\text{-}(24,8,800) \oplus 5\text{-}(24,8,800)$$

$$5\text{-}(24,6,54) \oplus 5\text{-}(24,8,504) \oplus 5\text{-}(24,10,2268)$$

$$5\text{-}(24,(6,8,10),382\,536) \qquad \binom{24}{6} = 134\,596$$

Kraj

Hvala na pažnji!