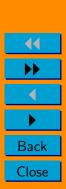
Cubes of designs

Kristijan Tabak Rochester Institute of Technology, Zagreb Campus Croatia e-mail: kxtcad@rit.edu Seminar za geometriju Seminar je održan u sklopu HRZZ projekata 6732 i 9752 (joint work with M.O. Pavčević)





An incidence structure $\mathcal{D}=(\mathcal{P},\mathcal{B})\text{, such that}$

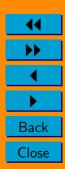




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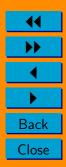


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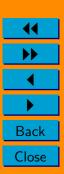
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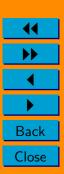
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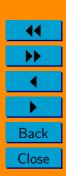




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write group G in a group ring $\mathbb{Z}[G]$ as $G = \sum_{s=1}^{\infty} g_s$ where $g_1 = 1$ (unit

in a group G)

Image: A transmission of the sector of t

If $A \in \mathcal{M}_{v \times v}$ is a matrix then *t*-th row shall be denoted by $[A_t]$

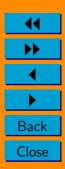








represents $[A]_t$ using group ring notation as $[A]_t = \sum_{s=1}^{r} A_{ts}g_s$.



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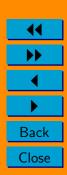
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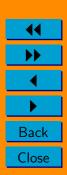


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$$\delta_X(x) = \begin{cases} 1, & \text{if } x \in X \\ 0, & \text{otherwise.} \end{cases}$$

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Theorem: Let $G = \sum_{s=1}^{v} g_s$, $g_1 = 1$ be a group of order v with a (v, k, λ) difference set D. Let $A = [a_{ijm}] \in \mathcal{M}_{v \times v \times v}$ be a 3-dimensional matrix defined by $a_{ijm} = \delta_{g_jg_iD}(g_m)$ for all $i, j, m \in [v]$. Then the following holds:

1. A is a cube of a (v, k, λ) symmetric design i.e. $A \in \mathcal{C}(v, k, \lambda)$, 2. A_x^i is an incidence matrix of a symmetric design $(G, \mathcal{D}ev(g_iD))$, 3. $[A_y^m]_t = g_t^{-1}g_mD^{(-1)}$

4. A_y^m is an incidence matrix of a symmetric design $(G, \sum_{t=1}^{n} g_t^{-1} g_m D^{(-1)}),$

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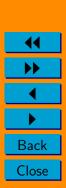
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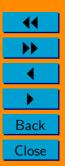
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Proposition: Let $A \in \mathcal{C}(v, k, \lambda)$ be a cube constructed via difference set $D \subseteq G$. Let ψ be a permutation of G. Let A^{ψ} be a 3-dimensional matrix such that $(A^{\psi})_{ijm} = \delta_{g_j^{\psi}g_iD}(g_m)$. Then A is a cube, i.e. $A^{\psi} \in \mathcal{C}(v, k, \lambda)$ and $[(A^{\psi})_y^m]_t = (g_t^{-1})^{\psi}g_m D^{(-1)}$ and $[(A^{\psi})_z^m]_t = g_m^{\psi}g_t D$.



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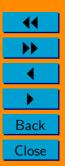
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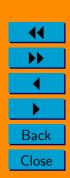
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 $\sum_{s=1}^{v} \delta_{B_{x,m}^t}(p_s)p_s$ for all $m, t \in [v]$, where $[A_y^m]_t$ and $[A_z^m]_t$ are t-th blocks
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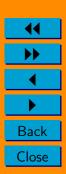
Theorem: Let $A \in \mathcal{C}(v, k, \lambda)$. Then for every $i, m \in [v]$ designs A_x^i , A_z^m and A_y^m satisfy the following:

1.
$$A_x^i = (\mathcal{P}, \sum_{t=1}^v B_{x,t}^i) = (\mathcal{P}, \mathcal{B}_x^i),$$

2. $A_z^m = (\mathcal{P}, \sum_{t=1}^{\circ} B_{x,m}^t) = (\mathcal{P}, \mathcal{B}_z^m)$, meaning that the set of blocks of a design A_z^m is a set of m-th blocks of designs A_x^t for all $t \in [v]$,



3. $A_y^m = (\mathcal{P}_y^m, \sum_{t=1}^{v} \langle p_m \rangle_{A_z^t}) = (\mathcal{P}_y^m, \mathcal{B}_y^m)$, meaning that the *t*-th block of a design A_y^m is *m*-th dual block of a design A_z^t .



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Definition: Let $G = \sum_{i=1}^{v} g_i$ where $g_1 = 1$, be a group of order v. Let (G, \mathcal{B}) be a (v, k, λ) symmetric design where $\mathcal{B} = \sum_{i=1}^{v} B_i$. Let A_x^1 be an incidence matrix of a design (G, \mathcal{B}) . Let A_x^m be an incidence matrix of an incidence structure $(G, g_m \mathcal{B})$, where $g_m \mathcal{B} = \sum_{i=1}^{v} g_m B_i$. A cyclic cube (generated by a symmetric design (G, \mathcal{B})) is a 3-dimensional matrix $A = (a_{ijm})$ such that $a_{ijm} = (A_x^i)_{jm}$ for all $i, j, m \in [v]$.



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Thank you! Any Q's?

