A new family of 3-designs of degree 3*

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(joint work with Lucija Relić)

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* This work was partially supported by the Croatian Science Foundation under the project 9752.

Introduction

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The degree of a design is the number of distinct block intersection sizes:

$$d = |\{\, |B_1 \cap B_2| \;:\; B_1 \neq B_2 \text{ are blocks}\}|$$

Motivation

$d = 1$: **Symmetric designs** $(v = b,\ t = 2)$

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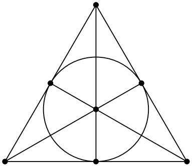

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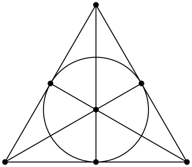

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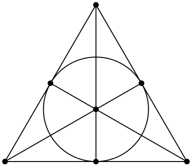

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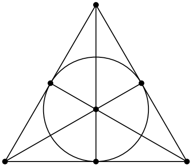

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$t = 4$:

V. Krčadinac, R. Vlahović Kruc, *Schematic 4-designs*, Discrete Math. **346** (2023), no. 7, Paper No. 113385, 7 pp.

Designs of degree $d = 3$ and strength $t = 4$

No.	v	k	λ	x	y	z	$\exists$
1	11	5	1	1	2	3	
2	23	8	4	0	2	4	
3	23	11	48	3	5	7	
4	24	8	5	0	2	4	
5	47	11	8	1	3	5	
6	71	35	264	14	17	20	
7	199	99	2328	44	49	54	
8	391	195	9264	90	97	104	
9	647	323	25680	152	161	170	
10	659	329	390874	153	164	175	
11	967	483	57720	230	241	252	

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No.	v	k	λ	x	y	z	$\exists$
1	11	5	1	1	2	3	✓
2	23	8	4	0	2	4	✓
3	23	11	48	3	5	7	✓
4	24	8	5	0	2	4	✓
5	47	11	8	1	3	5	✓
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$QR(11,3)$: $[11,6,5]_3$

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$QR(47,2)$: $[47,24,11]_2$

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3	23	11	48	3	5	7	✓	$QR(23,2)$: $[23,12,7]_2$
4	24	8	5	0	2	4	✓	$\widehat{QR(23,2)}$: $[24,12,8]_2$
5	47	11	8	1	3	5	✓	$QR(47,2)$: $[47,24,11]_2$
6	71	35	264	14	17	20	?	
7	199	99	2328	44	49	54	?	
8	391	195	9264	90	97	104	?	
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Designs of degree $d = 3$ and strength $t = 4$

Admissible parameters:

$$\begin{aligned} v &= 8n^2 - 1 \\ k &= 4n^2 - 1 = (2n-1)(2n+1) \\ \lambda &= 4n^4 - 7n^2 + 3 = (n-1)(n+1)(4n^2-3) \\ x &= 2n^2 - n - 1 = (n-1)(2n+1) \\ y &= 2n^2 - 1 \\ z &= 2n^2 + n - 1 = (n+1)(2n-1) \end{aligned}$$

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Theorem (Cameron, Delsarte, 1973)

In a design of degree d and strength $t \geq 2d - 2$, the blocks form a symmetric association scheme with d classes.

$\rightsquigarrow$ Schematic designs

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v	k	λ	x	y	z	$\exists$
16	4	1	0	1	2	✓
16	6	4	1	2	3	✓
32	8	7	0	2	4	✓
32	12	22	2	4	6	✓
64	16	35	0	4	8	✓
64	28	156	10	12	14	✓
128	32	155	0	8	16	✓
128	56	660	20	24	28	✓
256	64	651	0	16	32	✓
256	120	3304	52	56	60	✓
512	128	2667	0	32	64	✓
512	240	13384	104	112	120	✓

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v	k	λ	x	y	z	$\exists$	
16	4	1	0	1	2	✓	$AG_2(4,2)$, $RM(2,4)$: $[16,11,4]_2$
16	6	4	1	2	3	✓	
32	8	7	0	2	4	✓	$AG_3(5,2)$, $RM(2,5)$: $[32,16,8]_2$
32	12	22	2	4	6	✓	
64	16	35	0	4	8	✓	$AG_4(6,2)$, $RM(2,6)$: $[64,22,16]_2$
64	28	156	10	12	14	✓	
128	32	155	0	8	16	✓	$AG_5(7,2)$, $RM(2,7)$: $[128,29,32]_2$
128	56	660	20	24	28	✓	
256	64	651	0	16	32	✓	$AG_6(8,2)$, $RM(2,8)$: $[256,37,64]_2$
256	120	3304	52	56	60	✓	
512	128	2667	0	32	64	✓	$AG_7(9,2)$, $RM(2,9)$: $[512,46,128]_2$
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16	4	1	0	1	2	✓	$AG_2(4,2)$, $RM(2,4)$: $[16,11,4]_2$
16	6	4	1	2	3	✓	Nordstrom-Robinson: $(16, 2^8, 6)_2$
32	8	7	0	2	4	✓	$AG_3(5,2)$, $RM(2,5)$: $[32,16,8]_2$
32	12	22	2	4	6	✓	
64	16	35	0	4	8	✓	$AG_4(6,2)$, $RM(2,6)$: $[64,22,16]_2$
64	28	156	10	12	14	✓	Kerdock code: $(64, 2^{12}, 28)_2$
128	32	155	0	8	16	✓	$AG_5(7,2)$, $RM(2,7)$: $[128,29,32]_2$
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Commercial break

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Combinatorial Constructions Conference (CCC) will take place at the Centre for Advanced Academic Studies in Dubrovnik, Croatia.

April 7-13, 2024

Invited speakers:	Marco Buratti, Italy	Michael Kiermaier, Germany
	Eimear Byrne, Ireland	Patric Östergård, Finland
	Dean Crnković, Croatia	Kai-Uwe Schmidt, Germany
	Daniel Horsley, Australia	

https://web.math.pmf.unizg.hr/acco/meetings.php

Known series of 3-designs of degree 3

The known series:

$$\begin{aligned}
v &= 2^m \\
k &= 2^{m-1} - 2^{(m-2)/2} \\
\lambda &= 2^{(m-8)/2}\left(2^{m/2}-2\right)\left(2^m - 2^{m/2} - 4\right) \\
x &= 2^{(m-4)/2}\left(2^{m/2}-3\right) \\
y &= 2^{(m-4)/2}\left(2^{m/2}-2\right) \\
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$m \geq 4$ **even**

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$$RM(1,m)$$

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$$RM(1,m) \subset K(m) \subset RM(2,m)$$

Kerdock code $K(m)$: $\left(2^m,\, 2^{2m},\, 2^{m-1} - 2^{(m-2)/2}\right)$

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Weight (distance) distribution of $RM(1,m) \subset K(m)$:

wt	0	$2^{m-1} - 2^{(m-2)/2}$	2^{m-1}	$2^{m-1} + 2^{(m-2)/2}$	2^m
#	1	$2^m(2^{m-1}-1)$	$2^{m+1}-2$	$2^m(2^{m-1}-1)$	1

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$2^{m-1}-1$ LSSD(v,k,y)s

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k &= 2^{m-1} - 2^{(m-2)/2} \\
\lambda &= 2^{(m-8)/2}\left(2^{m/2}-2\right)\left(2^m - 2^{m/2} - 4\right) \\
x &= 2^{(m-4)/2}\left(2^{m/2}-3\right) \\
y &= 2^{(m-4)/2}\left(2^{m/2}-2\right) \\
z &= 2^{(m-4)/2}\left(2^{m/2}-1\right)
\end{aligned}
$$

Points: $AG(m, 2)$

$m \geq 4$ **even**

Schematic!

Weight (distance) distribution of $RM(1, m) \subset K(m)$:

wt	0	$2^{m-1} - 2^{(m-2)/2}$	2^{m-1}	$2^{m-1} + 2^{(m-2)/2}$	2^m
#	1	$2^m(2^{m-1} - 1)$	$2^{m+1} - 2$	$2^m(2^{m-1} - 1)$	1

Known series of 3-designs of degree 3 – wherefrom?

A. W. Nordstrom, J. P. Robinson, *An optimum nonlinear code*, Information and Control **11** (1967), 613–616.

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R. Noda, *On homogeneous systems of linked symmetric designs*, Math. Z. **138** (1974), 15–20.

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K. Yang, T. Helleseth, *Two new infinite families of 3-designs from Kerdock codes over $\mathbb{Z}_4$*, Des. Codes Cryptogr. **15** (1998), no. 2, 201–214.

$v = 2^m$, $k = 2^{m-1} + 2^{m-2} \pm 2^{(m-3)/2}$, $\lambda = k(k-1)(k-2)/(2^m-2)$, $m \geq 3$ **odd**

New series of 3-designs of degree 3

The new series:

$$
\begin{aligned}
v &= 2^m \\
k &= 2^{m-1} - 2^{(m-1)/2} \\
\lambda &= 2^{(m-7)/2}\left(2^{(m-1)/2} - 2\right)\left(2^m - 2^{(m+1)/2} - 2\right) \\
x &= 2^{(m-3)/2}\left(2^{(m-1)/2} - 3\right) \\
y &= 2^{(m-3)/2}\left(2^{(m-1)/2} - 2\right) \\
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Points: $AG(m, 2)$

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#	1	$2^{m-1}(2^m-1)$	$2^m(2^m+1)-2$	$2^{m-1}(2^m-1)$	1

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wt	0	$2^{m-1} - 2^{(m-1)/2}$	2^{m-1}	$2^{m-1} + 2^{(m-1)/2}$	2^m
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Kerdock sets

Quadratic forms over $GF(2)$:

$$B(x_1,\ldots,x_m)=\sum_{1\le i<j\le m} b_{ij}x_ix_j \quad\longleftrightarrow\quad B=\begin{bmatrix} 0 & & b_{ij} \\ & \ddots & \\ b_{ji} & & 0 \end{bmatrix}$$

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If m is **odd**, alternating matrices cannot be nonsingular (because their rank is even). Next best thing: take matrices B of rank $m-1$, i.e. $m = 2r+1$.

Kerdock sets in odd dimensions

For $rk(B) = m - 1$, the minimum weight of the coset $B + RM(1,2)$ is

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There are also $GF(2)$-linear codes with the same weight distribution (extended BCH codes) supporting non-isomorphic designs!

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For odd m, the Gray maps of these codes are not $\mathbb{Z}_4$-linear.

Numbers of non-isomorphic designs

v	k	λ	x	y	z	Nd	
16	4	1	0	1	2	≥ 45	$AG_2(4,2)$
16	6	4	1	2	3	$= 1$	Mathon 1981
32	8	7	0	2	4	≥ 3	$AG_3(5,2)$
32	12	22	2	4	6	≥ 3	
64	16	35	0	4	8	≥ 1	$AG_4(6,2)$
64	28	156	10	12	14	≥ 1	
128	32	155	0	8	16	≥ 1	$AG_5(7,2)$
128	56	660	20	24	28	≥ 4	
256	64	651	0	16	32	≥ 1	$AG_6(8,2)$
256	120	3304	52	56	60	≥ 1	
512	128	2667	0	32	64	≥ 1	$AG_7(9,2)$
512	240	13384	104	112	120	≥ 4	

The End

Thanks for your attention!