Hamiltonian graphs in Abelian 2-groups

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where H/T is a quotient group isomorphic to $E_{2^{m-t}}$ and $\begin{bmatrix} a \\ b \end{bmatrix}_2$ is a Gaussian coefficient.

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Ore's Theorem immediately yields that $(E_{2^2}[E_{2^3}], \mathcal{E}_3)$ and $(E_{2^2}[E_{2^4}], \mathcal{E}_4)$ are Hamiltonian.

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Therefore,

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Any automorphism $\alpha \in \mathrm{Aut}(E_{2^5})$ is represented by its action on a generators like

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The following is crucial for a construction of a Hamiltonian cycle in $(E_{2^2}[E_{2^5}], \mathcal{E}_5)$.

Lemma: Let $E_{2^5} = \langle a, b, c, d, e\rangle$ and let $\alpha \in \mathrm{Aut}(E_{2^5})$ be given by $\alpha = \begin{pmatrix} a & b & c & d & e \\ bc & cd & bcd & de & a \end{pmatrix}$, then $o(\alpha) = 31$ and $H^{\langle\alpha\rangle} = E_{2^4}[E_{2^5}]$ where $H = \langle a, b, c, d\rangle$. If $T = \langle a, b, c\rangle$ and $\Delta_i = T \cap T^{\alpha^i}$ for $i \in \mathbb{Z}_{31}$, then

$$\Delta_i = \begin{cases} \langle b, c\rangle, & \text{if } i = 1, 14 \\ \langle a, bc\rangle, & \text{if } i = 13, 30 \\ \langle ab, c\rangle, & \text{if } i = 17, 18 \\ \cong \mathbb{Z}_2 & \text{otherwise.} \end{cases}$$

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We can rewrite an automorphism α in a simplified form over $\mathbb{Z}_2$

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Back
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Following this process, after finite number of steps, we will construct a sequence

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