Lacing designs in PAG*

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29.6.2022.

* This work was fully supported by the Croatian Science Foundation under the project 9752.

PAG

Prescribed Automorphism Groups

0.1.2

Abstract

PAG is a GAP package for constructing combinatorial objects with prescribed automorphism groups.

PAG 0.1.2 – Manual and installation

The PAG manual is available at:

https://web.math.pmf.unizg.hr/acco/publications.php

Contents

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Contents

The installation files can be obtained from the authors. Please write to vedran.krcadinac@math.hr.

PAG 0.1.2 – Implemented so far

Constructing combinatorial designs by the Kramer-Mesner method:

- Generating G-orbits of k-subsets of V [GAP code]
  - $\rightsquigarrow$ Orderly algorithm using GAP package **images**
  - $\rightsquigarrow$ Algorithm for short orbits
- Computing the Kramer-Mesner matrix [GAP code]
- Solving 0-1 systems by A. Wassermann's LLL solver [interface to C program]
- Transforming solutions to GAP package **DESIGN** format [GAP code]
- Command *KramerMesnerSearch* that does everything automatically

PAG 0.1.2 – To do list

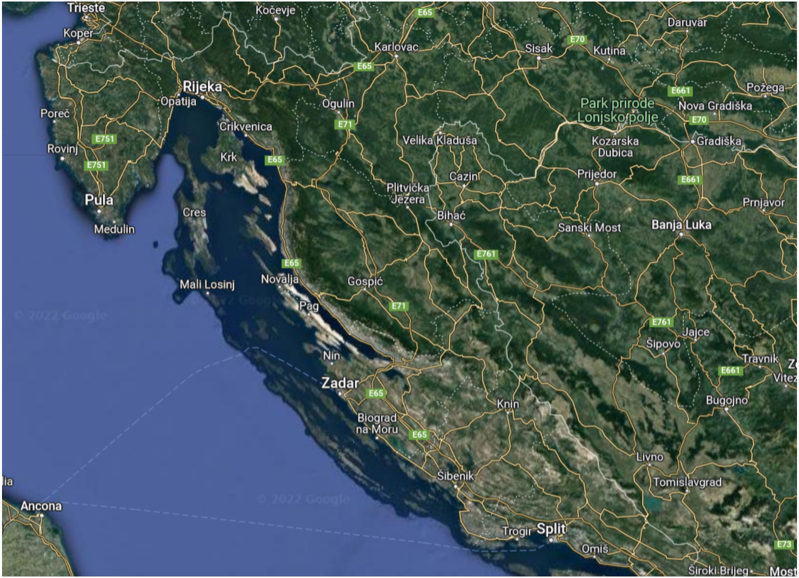
Enhancements of the Kramer-Mesner method:

- Tactical decomposition matrices
- Quasi-symmetric designs: good orbits, compatibility matrices
- More solvers: Gurobi, Minion...

Other construction methods and types of objects:

- Quasi-symmetric designs by clique search
- Configurations
- Strongly regular graphs
- Latin squares

The island Pag



Source: https://www.google.com/maps/place/Pag

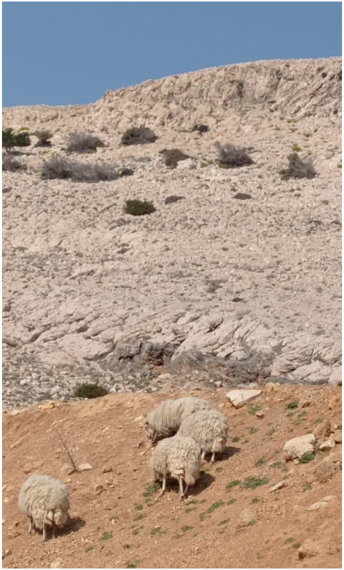
The island Pag

Source: https://www.google.com/maps/place/Pag

The island Pag – North shore

The island Pag – North shore

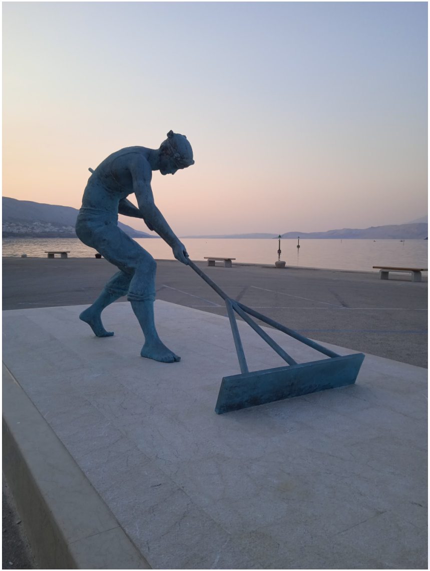
The island Pag – Sheep



The island Pag – Cheese

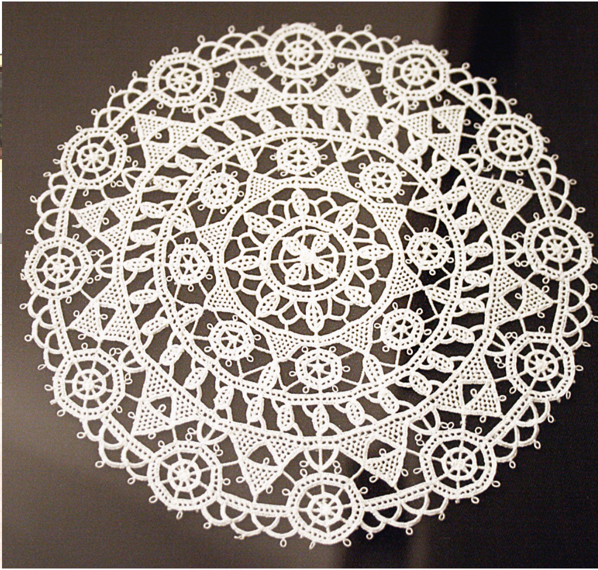
Source: https://www.paskasirana.com

The island Pag – Salt



Source: https://solana-pag.hr

The island Pag – Lacemaking



Source: https://en.wikipedia.org/wiki/Pag_(town)

The island Pag – Lacemaking

United Nations Educational, Scientific and Cultural Organization · Intangible Cultural Heritage

Intangible cultural heritage

Connection · EN · FR · ES

Search ICH website

Webpages, elements, decisions...

NEWS · EVENTS · CONVENTION · LISTS · SAFEGUARDING · ACTORS · THEMES · COUNTRY

UNESCO » Culture » Intangible Heritage » Lists » Lacemaking in Croatia

0:00 / 0:12

Nomination file No. 00245

- Nomination form: **English**|**French**
- Consent of communities: **English/Croatian**

Decision

Inscription: **4.COM 13.32**

Periodic reporting

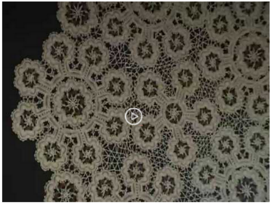
The report on the implementation of the Convention, due by States Parties every 6 years, includes a section on the elements inscribed on the

Lacemaking in Croatia

Croatia

Inscribed in 2009 (**4.COM**) on the Representative List of the Intangible Cultural Heritage of Humanity

At least three distinct traditions of Lacemaking in Croatia persist today, centred on the towns of Pag on the Adriatic, Lepoglava in northern Croatia and Hvar on the Dalmatian island of the same name. Pag needle-point lace was originally used to make ecclesiastical garments, tablecloths and ornaments for clothing. The process involves embellishing a spider web pattern with geometrical motifs and is transmitted today by older women who offer year-long courses. Lepoglava bobbin lace is made by braiding thread wound on spindles, or bobbins; it is often used to make lace ribbons for folk costumes or is sold at village fairs. An International Lace Festival in Lepoglava celebrates the art every year. Aloe lace is made in Croatia only by Benedictine nuns in the town of Hvar. Thin, white threads are obtained from the core of fresh aloe leaves and woven into a net or other pattern on a cardboard background. The resulting pieces are a symbol of Hvar. Each variety of lace has long been created by rural women as a source of additional income and has left a permanent mark on the culture of its region. The craft both produces an important component of traditional clothes and is itself testimony to a living cultural tradition.



© 2008 by Ministry of Culture :

Source: https://ich.unesco.org/en/RL/lacemaking-in-croatia-00245

How to choose a group?

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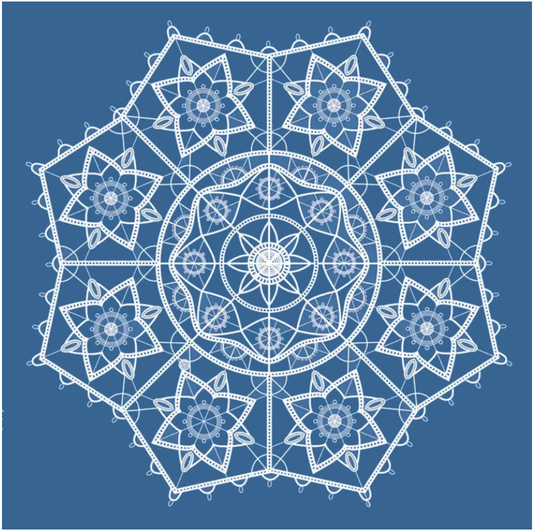

Source: https://www.plakati.com.hr

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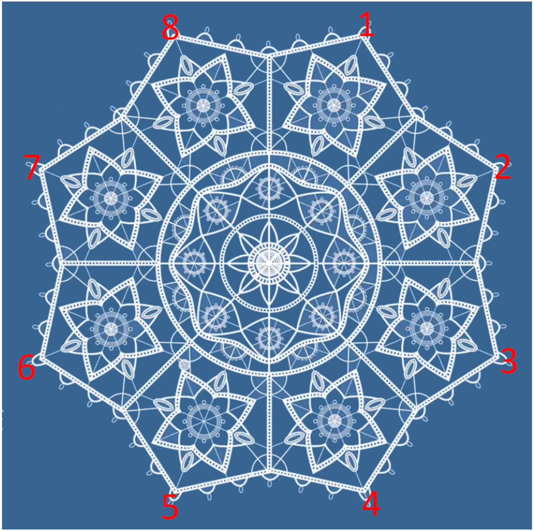

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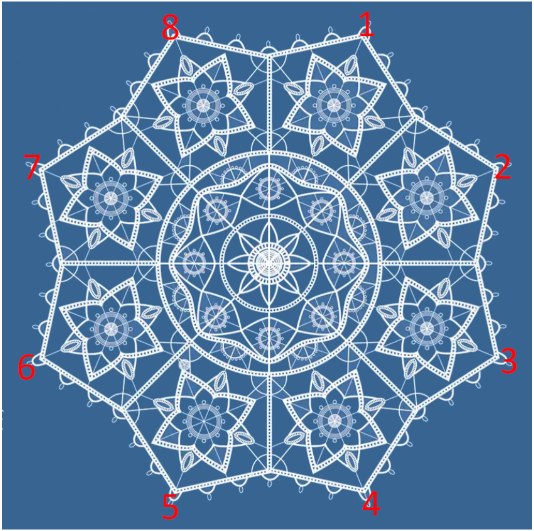

Source: https://www.plakati.com.hr

```
a:=(1,2,3,4,5,6,7,8);
b:=(1,8)(2,7)(3,6)(4,5);
g:=Group(a,b);
```

Suitable design parameters?

t-(v, k, λ)

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$v =$ degree of the permutation group

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$k = t + 1, t + 2, \dots, \lfloor v/2 \rfloor$

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$v = 8$

$t = 3$

Suitable design parameters?

t-(v, k, λ)

v = degree of the permutation group

$t = 2, 3, 4, 5 \dots$

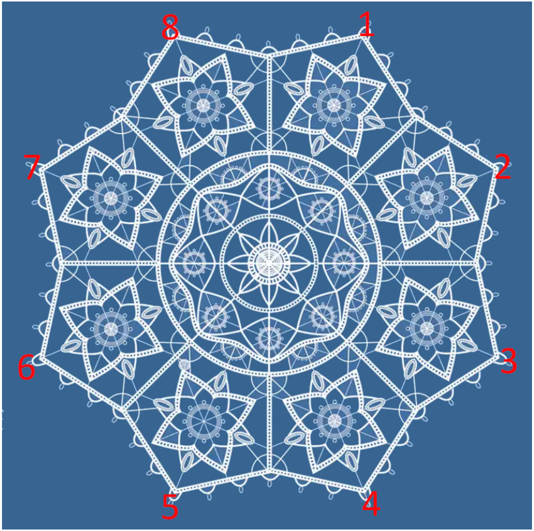
$k = t + 1, t + 2, \dots, \lfloor v/2 \rfloor$

λ = smallest number such that t-(v, k, λ) are admissible (λ_{min})

$v = 8$

$t = 3 \quad k = 4 \rightsquigarrow \lambda_{\text{min}} = 1 \rightsquigarrow$ 3-(8, 4, 1) [extended Fano plane]

Action of D_{16} on 16 points



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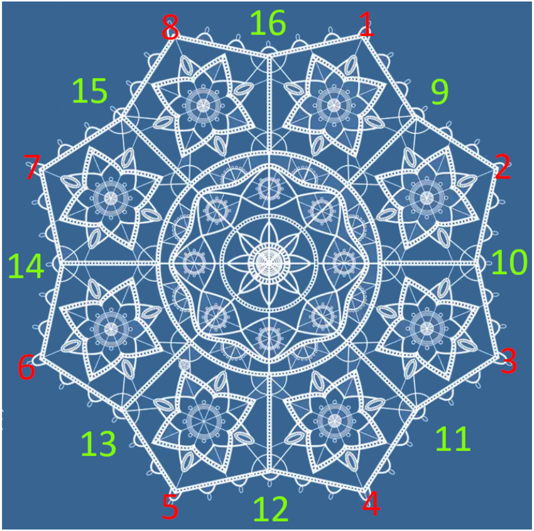

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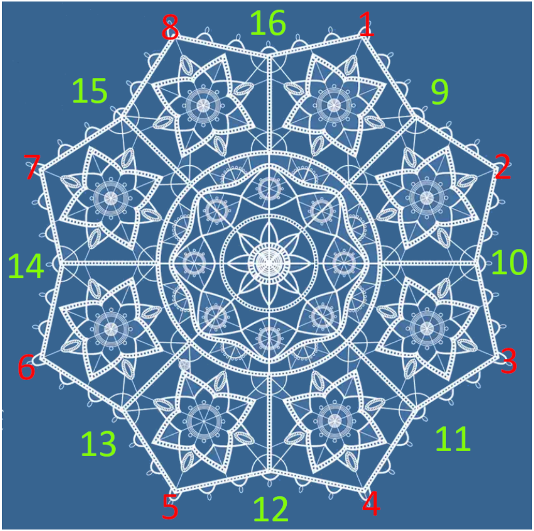
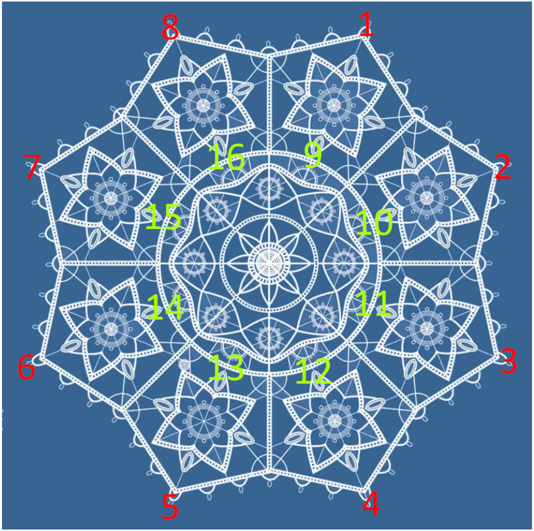


Source: https://www.plakati.com.hr

```
a:=(1,2,3,4,5,6,7,8)
(9,10,11,12,13,14,15,16);
b:=(1,8)(2,7)(3,6)(4,5)
(9,15)(10,14)(11,13);
g1:=Group(a,b);
```

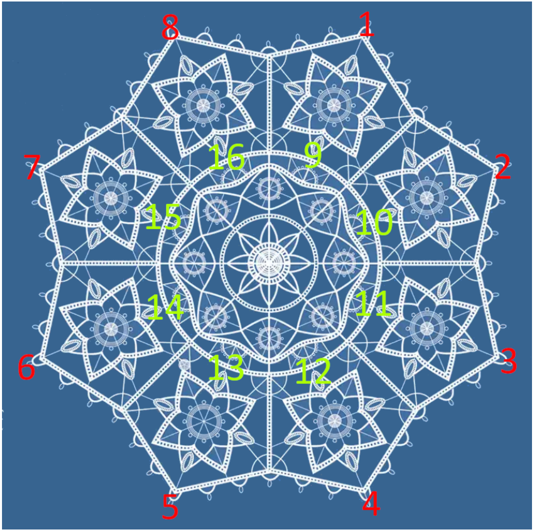
Action of D_{16} on 16 points



Source: https://www.plakati.com.hr

```
a:=(1,2,3,4,5,6,7,8)
(9,10,11,12,13,14,15,16);
b:=(1,8)(2,7)(3,6)(4,5)
(9,16)(10,15)(11,14)(12,13);
g2:=Group(a,b);
```

Action of D_{16} on 16 points



Source: https://www.plakati.com.hr

```
a:=(1,2,3,4,5,6,7,8)
(9,10,11,12,13,14,15,16);

b:=(1,8)(2,7)(3,6)(4,5)
(9,16)(10,15)(11,14)(12,13);

g2:=Group(a,b);

c:=(1,9)(2,10)(3,11)(4,12)
(5,13)(6,14)(7,15)(8,16);

g22:=Group(a,b,c);
```

Action of D_{16} on 16 points

```
g3:=Image(IsomorphismPermGroup(DihedralGroup(16)));
```

```
Group([ (1,2)(3,16)(4,14)(5,8)(6,15)(7,11)(9,13)(10,12),
(1,3,4,9,5,10,11,15)(2,6,7,12,8,13,14,16),
(1,4,5,11)(2,7,8,14)(3,9,10,15)(6,12,13,16),
(1,5)(2,8)(3,10)(4,11)(6,13)(7,14)(9,15) (12,16) ])
```

Action of D_{16} on 16 points

```
g3:=Image(IsomorphismPermGroup(DihedralGroup(16)));
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```
Group([ (1,2)(3,16)(4,14)(5,8)(6,15)(7,11)(9,13)(10,12),
(1,3,4,9,5,10,11,15)(2,6,7,12,8,13,14,16),
(1,4,5,11)(2,7,8,14)(3,9,10,15)(6,12,13,16),
(1,5)(2,8)(3,10)(4,11)(6,13)(7,14)(9,15) (12,16) ])
```

```
StructureDescription(g1);

StructureDescription(g2);

StructureDescription(g3);
```

```
"D16"
```

```
StructureDescription(g22);
```

```
"C2 x D16"
```

Action of D_{16} on 16 points

```
g3:=Image(IsomorphismPermGroup(DihedralGroup(16)));

Group([ (1,2)(3,16)(4,14)(5,8)(6,15)(7,11)(9,13)(10,12),
(1,3,4,9,5,10,11,15)(2,6,7,12,8,13,14,16),
(1,4,5,11)(2,7,8,14)(3,9,10,15)(6,12,13,16),
(1,5)(2,8)(3,10)(4,11)(6,13)(7,14)(9,15) (12,16) ])

StructureDescription(g1);
StructureDescription(g2);
StructureDescription(g3);

"D16"

StructureDescription(g22);

"C2 x D16"

Orbits(g1);  Orbits(g2);

[ [ 1, 2, 8, 3, 7, 4, 6, 5 ], [ 9, 10, 15, 11, 14, 16, 12, 13 ] ]

Orbits(g22);  Orbits(g3);

[ [ 1, 2, 3, 4, 5, 6, 7, 8, 16, 9, 10, 14, 11, 15, 12, 13 ] ]
```

Parameters of designs with 16 points

$v = 16$

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$t = 2 \quad k = 3 \rightsquigarrow \lambda_{\min} = 2 \rightsquigarrow$ 2-(16, 3, 2)

$k = 4 \rightsquigarrow \lambda_{\min} = 1 \rightsquigarrow$ 2-(16, 4, 1) [affine plane of order 4]

$k = 5 \rightsquigarrow \lambda_{\min} = 4 \rightsquigarrow$ 2-(16, 5, 4)

$k = 6 \rightsquigarrow \lambda_{\min} = 1 \rightsquigarrow$ ~~2-(16, 6, 1)~~ 2-(16, 6, 2) [biplane of order 4]

$k = 7 \rightsquigarrow \lambda_{\min} = 14 \rightsquigarrow$ 2-(16, 7, 14)

$k = 8 \rightsquigarrow \lambda_{\min} = 7 \rightsquigarrow$ 2-(16, 8, 7)

Parameters of designs with 16 points

$v = 16$

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$k = 6 \rightsquigarrow \lambda_{\min} = 1 \rightsquigarrow$ ~~2-(16, 6, 1)~~ 2-(16, 6, 2) [biplane of order 4]

$k = 7 \rightsquigarrow \lambda_{\min} = 14 \rightsquigarrow$ 2-(16, 7, 14)

$k = 8 \rightsquigarrow \lambda_{\min} = 7 \rightsquigarrow$ 2-(16, 8, 7)

$t = 3$ $k = 4 \rightsquigarrow \lambda_{\min} = 1 \rightsquigarrow$ 3-(16, 4, 1) [Steiner quadruple system]

$k = 5 \rightsquigarrow \lambda_{\min} = 6 \rightsquigarrow$ 3-(16, 5, 6)

$k = 6 \rightsquigarrow \lambda_{\min} = 2 \rightsquigarrow$ ~~3-(16, 6, 2)~~ 3-(16, 6, 4)

$k = 7 \rightsquigarrow \lambda_{\min} = 5 \rightsquigarrow$ 3-(16, 7, 5)

$k = 8 \rightsquigarrow \lambda_{\min} = 3 \rightsquigarrow$ 3-(16, 8, 3)

Parameters of designs with 16 points

$v = 16$

$t = 5 \quad k = 6 \rightsquigarrow \lambda_{\min} = 1 \rightsquigarrow$ ~~5-(16, 6, 1)~~

5-(16, 6, 2) ?

5-(16, 6, 3)

The End

Thanks for your attention!