Construction of q-analogs of combinatorial objects with prescribed symmetries

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The search for combinatorial objects like designs, error-correcting codes or incidence structures from finite geometry is limited very often by a phenomenon called "combinatorial explosion". This describes the problem that the size of a search tree grows exponentially with the increase of parameters of the combinatorial structure. A successful approach to tame the combinatorial explosion to a certain extend is to restrict the search to objects with symmetry, i.e. to prescribe an automorphism group.

In the first part of this mini-course we will study the Kramer-Mesner method, which is one possible way to search for objects with prescribed symmetry. Up to now, it has been very successful in the search for combinatorial t-designs and many other combinatorial structures. In this method the search is reduced to solving a Diophantine linear system of equations. We will discuss available algorithms to solve such linear systems and especially highlight the connection to lattice algorithms.

In the second part, q-analogs of combinatorial structures are introduced and the search for those objects using the Kramer-Mesner approach is discussed. In particular, we will have a look at subspace designs, q-analogs of group divisible designs, MRD codes and designs in polar spaces.