

# Heffter arrays: origins, variants and applications

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The notion of a Heffter array has been introduced by Archdeacon [1]. Given a positive integer  $v = 2nk + 1$ , a *Heffter array*  $H(n; k)$  is an  $n \times n$  partially filled array with entries in  $\mathbb{Z}_v$  satisfying the following conditions: 1) each row and each column contains exactly  $k$  filled cells; 2) for every  $x \in \mathbb{Z}_v \setminus \{0\}$ , either  $x$  or  $-x$  appears in the array; 3) the sum of the elements in every row and column is 0 (mod  $v$ ). In this talk, besides presenting the most important existence results on this topic (see [2], [3] and [5]), I will propose recent variants [4] and generalizations [6] emphasizing their applications to difference families, (orthogonal) graph decompositions, and biembeddings.

## References

- [1] D.S. Archdeacon, *Heffter arrays and biembedding graphs on surfaces*, Electron. J. Combin., **22** #P1.74, 2015.
- [2] D.S. Archdeacon, J.H. Dinitz, D.M. Donovan, E.S. Yazıcı, *Square integer Heffter arrays with empty cells*, Des. Codes Cryptogr. **77** (2015).
- [3] N.J. Cavenagh, J. Dinitz, D. Donovan, E.S. Yazıcı, *The existence of square non-integer Heffter arrays*, Ars Math. Contemp. **17** (2019).
- [4] S. Costa, S. Della Fiore, A. Pasotti, *Non-zero sum Heffter arrays and their applications*, to appear in Discrete Math..
- [5] J.H. Dinitz, I.M. Wanless, *The existence of square integer Heffter arrays*, Ars Math. Contemp. **13** (2017).
- [6] S. Costa, A. Pasotti, *On  $\lambda$ -fold relative Heffter arrays and biembedding multigraphs on surfaces*, Europ. J. Combin. **97** (2021).