



# Combinatorial Constructions Conference

Centre for Advanced Academic Studies  
Dubrovnik, Croatia • April 7–13, 2024



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**Book of abstracts**

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Co-organized by the [Croatian Mathematical Society](#).

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Abstracts were prepared by the authors.

**Website:** <https://web.math.pmf.unizg.hr/acco/meetings.php>



# Contents

<b>Welcome</b>	<b>1</b>
<b>Venue</b>	<b>2</b>
<b>Conference program</b>	<b>3</b>
<b>Invited talks</b>	<b>8</b>
Marco Buratti: <i>Heffter spaces and additive designs</i> . . . . .	9
Eimear Byrne: <i>q-Matroids and related structures</i> . . . . .	10
Dean Crnković: <i>Constructions of LCD subspace codes</i> . . . . .	11
Daniel Horsley: <i>Minimising the number of comparable sets</i> . . . . .	12
Michael Kiermaier: <i>The degree of functions in the Johnson and q-Johnson schemes</i> . . . . .	13
Patric R. J. Östergård: <i>Counting Steiner triple systems of order 21</i> . . . . .	14
<b>Contributed talks</b>	<b>15</b>
Onur Ağırseven: <i>Grid-based graphs, linear realizations, and the Buratti-Horak-Rosa conjecture</i> . . . . .	16
Robert F. Bailey: <i>Distance-regular and strongly regular graphs from affine permutation groups</i> . . . . .	16
Sara Ban: <i>New extremal Type II <math>\mathbb{Z}_4</math>-codes of length 64</i> . . . . .	17
Anton Betten: <i>Quartic curves with 63 Kovalevski points</i> . . . . .	17
Zoltán L. Blázsik: <i>Hypergraphs are 2-colorable if the 1-intersection graph of their hyperedges is bipartite</i> . . . . .	18
Arne Botteldoorn: <i>The minimal blocking sets of <math>\text{PG}(2, 11)</math> with a non-trivial automorphism group</i> . . . . .	18
Patrick J. Browne: <i>Erdős-Ko-Rado type problems in root systems</i> . . . . .	19
Andrea Burgess: <i>Constructing solutions of the Hamilton-Waterloo Problem using row-sum matrices</i> . . . . .	19
Carl Johan Casselgren: <i>Brooks' theorem with forbidden colors</i> . . . . .	19
Peter Danziger: <i>Colouring Kirkman triple systems</i> . . . . .	20
Jan De Beule: <i>Existence and non-existence of Cameron-Liebler k-sets in projective spaces</i> . . . . .	20
Bart De Bruyn: <i>Characterising ovoidal cones by their intersection numbers</i> . . . . .	21
Doris Dumičić Danilović: <i>On Steiner systems <math>S(2, 6, 91)</math></i> . . . . .	21
Mathieu Dutour Sikirić: <i>High dimensional computation of fundamental domains</i> . . . . .	22
Christian Elsholtz: <i>New lower bounds on caps, and on progression-free sets in <math>\mathbb{F}_p^n</math> and the integers</i> . . . . .	22
Alena Ernst: <i>Transitivity in finite general linear groups</i> . . . . .	23
Raúl M. Falcón: <i>Heffter arrays over partial loops</i> . . . . .	23
Giovanni Falcone: <i>Goppa codes over hyperelliptic curves</i> . . . . .	24
Mohammad Ghebleh: <i>Reinforcement learning for graphs</i> . . . . .	24
Marcus Greferath: <i>Some thoughts on algebraic coding theory over the Boolean semifield</i> . . . . .	24
Dirk Hachenberger: <i>Ovoids and primitive normal bases for quartic extensions of Galois fields</i> . . . . .	25

Daniel Hawtin: <i>Transitive <math>(q - 1)</math>-fold packings of <math>PG_n(q)</math></i> . . . . .	25
Thomas Honold: <i>Maximum weight spectrum codes over finite fields and rings</i>	26
Lukas Klawuhn: <i>Designs in the generalised symmetric group</i> . . . . .	26
Vedran Krčadinac: <i>Some nice combinatorial objects</i> . . . . .	27
Denis Krotov: <i>Multispreads and the characterization of parameters of additive one-weight codes</i> . . . . .	27
Ivan Landjev: <i>Constructions of binary codes with two distances</i> . . . . .	28
Stefano Lia: <i>A note on strong blocking sets and higgledy-piggledy sets of lines</i>	29
Jonathan Mannaert: <i>Bounds for <math>m</math>-ovoids using combinatorial techniques</i>	29
Giuliamaria Menara: <i>Improving the lower bound for the order of correlation immune Boolean functions</i> . . . . .	30
Giuliamaria Menara: <i>Eulerian magnitude homology</i> . . . . .	30
Francesca Merola: <i>Banff designs: difference methods for coloring incidence graphs</i> . . . . .	31
Matteo Mravić: <i>Construction of extremal <math>\mathbb{Z}_4</math>-codes using a neighborhood search algorithm</i> . . . . .	31
Amela Muratović-Ribić: <i>On properties of matrix representation of quadratic planar polynomials over finite fields and its connection with Latin squares</i> . . . . .	32
Misha Muzychuk: <i>Constructing linked systems of relative difference sets via Schur rings</i> . . . . .	32
Gábor P. Nagy: <i>Switching equivalence of strongly regular polar graphs</i> . .	33
Vito Napolitano: <i>On <math>(4, r, 6)</math>-bipartite biregular cages</i> . . . . .	34
Padraig Ó Catháin: <i>Monomial Representations and combinatorics</i> . . . .	34
Péter Pál Pach: <i>Line-free sets</i> . . . . .	34
Silvia M. C. Pagani: <i>Ghost hunting</i> . . . . .	35
Mark Pankov: <i>Point-line geometries related to binary equidistant codes</i> . .	35
Kamal Lochan Patra: <i>On vertex connectivity of zero-divisor graphs of finite commutative rings</i> . . . . .	36
Marco Antonio Pellegrini: <i>Magic partially filled arrays on abelian groups</i> .	36
Safet Penjić: <i>On (di)graphs and doubly stochastic matrices</i> . . . . .	37
Valentina Pepe: <i>On subspaces defining linear sets of maximum rank</i> . . . .	38
David Pike: <i>2-block-intersection graphs of twofold triple systems</i> . . . . .	38
René Rodríguez Aldama: <i>Bending and plateauing: an approach to minimal <math>p</math>-ary codes</i> . . . . .	38
Assia Rousseva: <i>Constructions of affine <math>t</math>-fold blocking sets</i> . . . . .	39
Sanja Rukavina: <i>Self-dual (near-)extremal ternary codes and combinatorial 2-designs</i> . . . . .	40
Binod Kumar Sahoo: <i>Blocking sets of external, tangent and secant lines to an elliptic quadric in <math>PG(3, q)</math></i> . . . . .	40
Patrick Solé: <i>Hadamard matrices and spherical designs</i> . . . . .	41
Dragan Stevanović: <i>Searching for regular, triangle-distinct graphs</i> . . . . .	41
Leo Storme: <i>The minimum distance of the code of intersecting lines in <math>PG(3, q)</math></i> . . . . .	42
Andrea Švob: <i>On some structures related to intriguing sets of strongly regular graphs</i> . . . . .	42
Tommaso Traetta: <i>Near alternating sign matrices with an application</i> . . .	43
Edwin van Dam: <i>Amorphic association schemes and fusing pairs</i> . . . . .	43

Draženka Višnjić: <i>The distance function on Coxeter like graphs</i> . . . . .	44
Konstantin Vorob'ev: <i>Binary codes with distances <math>d</math> and <math>d + 2</math></i> . . . . .	44
Alfred Wassermann: <i>Higher incidence matrices and tactical decomposition matrices of designs</i> . . . . .	45
Charlene Weiß: <i>Existence of <math>t</math>-designs in polar spaces for all <math>t</math></i> . . . . .	45
Qing Xiang: <i>Cameron-Liebler line classes, tight sets and strongly regular Cayley graphs</i> . . . . .	46
E. Şule Yazıcı: <i>Non-extendable partial Latin hypercubes and maximal orthogonal partial Latin squares</i> . . . . .	46
Sjanne Zeijlemaker: <i>On the existence of small strictly Neumaier graphs</i> .	47
Tin Zrinski: <i>Applications of genetic algorithms for constructions of SRGs and DSRGs</i> . . . . .	47
<b>List of participants</b>	<b>48</b>

## Welcome from the Organizing committee

We wish you a warm welcome to Dubrovnik, one of Croatia's top tourist destinations! Between the 14th and 19th centuries, Dubrovnik was a maritime republic with a strong merchant fleet. Because of its skilled diplomacy it remained largely independent from the superpowers of the day, such as Venice and the Ottoman Empire. Dubrovnik's love of freedom is reflected in its flag featuring the word *Libertas*:



Our *Combinatorial Constructions Conference* takes place at the [Centre for Advanced Academic Studies \(CAAS\)](#), very near the Old City of Dubrovnik. Talks are scheduled from Monday, April 8 to Friday, April 12, 2024. We have 6 invited and 64 contributed talks, resulting in a program which cannot avoid parallel sessions. We miss a lot our deceased colleague and friend Kai-Uwe Schmidt who was also invited to present his work.

The plenary and “A” sessions are in the big conference hall, while the “B” sessions are in one of the smaller lecture rooms. Lunches will be served in the CAAS Restaurant from 13:00 to 14:30. On Wednesday, April 10 we will have a guided city tour from 16:30, and conference dinner from 19:00 at the restaurant “[Posat](#)”. Every participant will get the [Dubrovnik 3-Day Pass](#) covering free entrance to the city walls and other sights, as well as local transport. We suggest that you visit the walls on Wednesday after lunch, when no talks are scheduled.

Our conference is supported by the [Croatian Science Foundation](#) in scope of the project *Algorithmic Constructions of Combinatorial Objects*, and co-organized by the [Croatian Mathematical Society](#). We gratefully acknowledge support from these organizations and help from the CAAS staff in organizing this event. We hope that you will find the scientific program interesting and enjoy your stay in Dubrovnik!

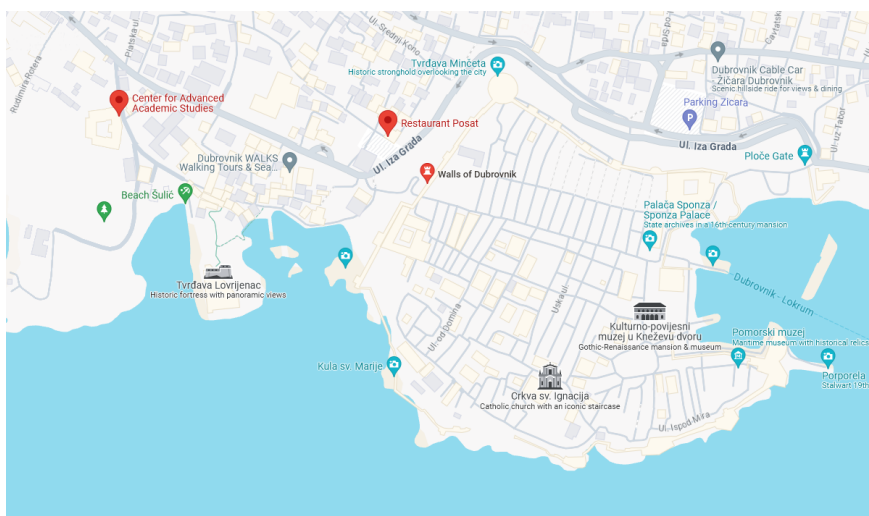
Mario, Vedran, Anamari, Renata, Kristijan, Lucija and Filip

## Venue

The conference takes place at

### **Centre for Advanced Academic Studies (CAAS) Don Frana Bulića 4, Dubrovnik**

The CAAS Residence and the CAAS Restaurant are at the same address.

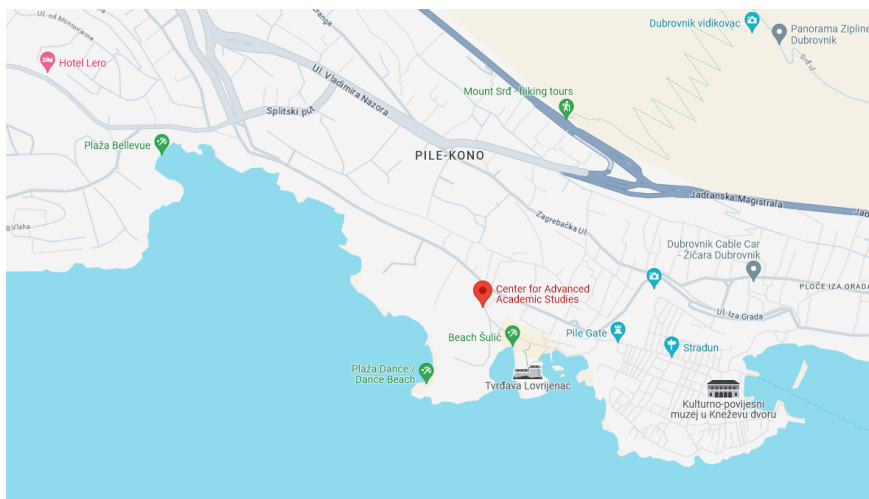


Entrance to the City walls is near the “Pile” gate, about 5 minutes from CAAS. The restaurant where we will have conference dinner is also nearby:

### **Restaurant Posat Uz Posat 1, Dubrovnik**

Hotel Lero is some 10–15 minutes on foot from CAAS:

### **Hotel Lero Iva Vojnovića 14, Dubrovnik**





## Conference program

Sunday, April 7

19:00 - 20:00 REGISTRATION

Monday, April 8

8:45 REGISTRATION

9:15 Opening

9:25 Charlene Weiß  
*Existence of  $t$ -designs in polar spaces for all  $t$*

9:50 Alena Ernst  
*Transitivity in finite general linear groups*

10:15 Lukas Klawuhn  
*Designs in the generalised symmetric group*

10:40 COFFEE

11:15 Pádraig Ó Catháin  
*Monomial representations and combinatorics*

11:40 Patrick Solé  
*Hadamard matrices and spherical designs*

12:05 Jan De Beule  
*Existence and non-existence of Cameron-Liebler  $k$ -sets in projective spaces*

12:30 Qing Xiang  
*Cameron-Liebler line classes, tight sets and strongly regular Cayley graphs*

13:00 LUNCH

14:30 **INVITED TALK**  
Michael Kiermaier  
*The degree of functions in the Johnson and  $q$ -Johnson schemes*

15:30 Misha Muzychuk  
*Constructing linked systems of relative difference sets via Schur rings*

15:55 Edwin van Dam  
*Amorphic association schemes and fusing pairs*

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 Tuesday, April 9
 

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	SECTION A	SECTION B
9:00	Peter Danziger <i>Colouring Kirkman triple systems</i>	Giuliamaria Menara <i>Improving the lower bound for the order of correlation immune Boolean functions</i>
9:25	Francesca Merola <i>Banff designs: difference methods for coloring incidence graphs</i>	René Rodríguez Aldama <i>Bending and plateauing: an approach to minimal <math>p</math>-ary codes</i>
9:50	Alfred Wassermann <i>Higher incidence matrices and tactical decomposition matrices of designs</i>	Matteo Mravić <i>Construction of extremal <math>\mathbb{Z}_4</math>-codes using a neighborhood search algorithm</i>
10:15	Vito Napolitano <i>On <math>(4, r, 6)</math>-bipartite biregular cages</i>	Sara Ban <i>New extremal Type II <math>\mathbb{Z}_4</math>-codes of length 64</i>
10:40	COFFEE	
	SECTION A	SECTION B
11:15	Leo Storme <i>The minimum distance of the code of intersecting lines in <math>PG(3, q)</math></i>	Onur Ağırseven <i>Grid-based graphs, linear realizations, and the Buratti-Horak-Rosa conjecture</i>
11:40	Sanja Rukavina <i>Self-dual (near-)extremal ternary codes and combinatorial 2-designs</i>	Zoltán L. Blázsik <i>Hypergraphs are 2-colorable if the 1-intersection graph of their hyperedges is bipartite</i>
12:05	Thomas Honold <i>Maximum weight spectrum codes over finite fields and rings</i>	Safet Penjić <i>On <math>(di)</math>graphs and doubly stochastic matrices</i>
12:30	Marcus Greferath <i>Some thoughts on algebraic coding theory over the Boolean semifield</i>	Draženka Višnjić <i>The distance function on Coxeter like graphs</i>
13:00	LUNCH	
14:30	<b style="color: red;">INVITED TALK</b> Patric R. J. Östergård <i>Counting Steiner triple systems of order 21</i>	
15:30	<b style="color: red;">INVITED TALK</b> Daniel Horsley <i>Minimising the number of comparable sets</i>	

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**Wednesday, April 10**


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	SECTION A	SECTION B
9:00	Valentina Pepe <i>On subspaces defining linear sets of maximum rank</i>	Mathieu Dutour Sikirić <i>High dimensional computation of fundamental domains</i>
9:25	Bart De Bruyn <i>Characterising ovoidal cones by their intersection numbers</i>	Tin Zrinski <i>Applications of genetic algorithms for constructions of SRGs and DSRGs</i>
9:50	Dirk Hachenberger <i>Ovoids and primitive normal bases for quartic extensions of Galois fields</i>	Dragan Stevanović <i>Searching for regular, triangle-distinct graphs</i>
10:15	Daniel Hawtin <i>Transitive <math>(q - 1)</math>-fold packings of <math>PG_n(q)</math></i>	Mohammad Ghebleh <i>Reinforcement learning for graphs</i>
10:40	COFFEE	
	SECTION A	SECTION B
11:15	Raúl M. Falcón <i>Heffter arrays over partial loops</i>	Assia Rousseva <i>Constructions of affine <math>t</math>-fold blocking sets</i>
11:40	Andrea Burgess <i>Constructing solutions of the Hamilton-Waterloo Problem using row-sum matrices</i>	Binod Kumar Sahoo <i>Blocking sets of external, tangent and secant lines to an elliptic quadric in <math>PG(3, q)</math></i>
12:05	Tommaso Traetta <i>Near alternating sign matrices with an application</i>	Stefano Lia <i>A note on strong blocking sets and higgledy-piggledy sets of lines</i>
12:30	Marco Antonio Pellegrini <i>Magic partially filled arrays on abelian groups</i>	Arne Botteldoorn <i>The minimal blocking sets of <math>PG(2, 11)</math> with a non-trivial automorphism group</i>
13:00	LUNCH	
14:30	INDIVIDUAL TOUR OF THE CITY WALLS	
16:30	GUIDED CITY TOUR	
19:00	CONFERENCE DINNER – RESTAURANT “POSAT”	



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 Thursday, April 11
 

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	SECTION A	SECTION B
9:00	E. Şule Yazıcı <i>Non-extendable partial Latin hypercubes and maximal orthogonal partial Latin squares</i>	Mark Pankov <i>Point-line geometries related to binary equidistant codes</i>
9:25	Amela Muratović-Ribić <i>On properties of matrix representation of quadratic planar polynomials over finite fields and its connection with Latin squares</i>	Carl Johan Casselgren <i>Brooks' theorem with forbidden colors</i>
9:50	Christian Elsholtz <i>New lower bounds on caps, and on progression-free sets in <math>\mathbb{F}_p^n</math> and the integers</i>	Kamal Lochan Patra <i>On vertex connectivity of zero-divisor graphs of finite commutative rings</i>
10:15	Péter Pál Pach <i>Line-free sets</i>	Giuliamaria Menara <i>Eulerian magnitude homology</i>
10:40	COFFEE	
	SECTION A	SECTION B
11:15	Robert F. Bailey <i>Distance-regular and strongly regular graphs from affine permutation groups</i>	Denis Krotov <i>Multispreads and the characterization of parameters of additive one-weight codes</i>
11:40	Gábor P. Nagy <i>Switching equivalence of strongly regular polar graphs</i>	
12:05	Andrea Švob <i>On some structures related to intriguing sets of strongly regular graphs</i>	
12:30	Sjanne Zeijlemaker <i>On the existence of small strictly Neumaier graphs</i>	
13:00	LUNCH	
14:30	INVITED TALK Eimear Byrne <i>q-Matroids and related structures</i>	
15:30	INVITED TALK Marco Buratti <i>Heffter spaces and additive designs</i>	

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## Friday, April 12

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9:00	Silvia M. C. Pagani <i>Ghost hunting</i>
9:25	Jonathan Mannaert <i>Bounds for <math>m</math>-ovoids using combinatorial techniques</i>
9:50	Anton Betten <i>Quartic curves with 63 Kovalevski points</i>
10:15	Patrick J. Browne <i>Erdős–Ko–Rado type problems in root systems</i>
10:40	COFFEE
11:15	Ivan Landjev <i>Constructions of binary codes with two distances</i>
11:40	Konstantin Vorob'ev <i>Binary codes with distances <math>d</math> and <math>d + 2</math></i>
12:05	Giovanni Falcone <i>Goppa codes over hyperelliptic curves</i>
12:30	David Pike <i>2-block-intersection graphs of twofold triple systems</i>
13:00	LUNCH
14:30	<b>INVITED TALK</b> Dean Crnković <i>Constructions of LCD subspace codes</i>
15:30	Doris Dumičić Danilović <i>On Steiner systems <math>S(2, 6, 91)</math></i>
15:55	Vedran Krčadinac <i>Some nice combinatorial objects</i>
16:20	CLOSING

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## Invited talks



## Heffter spaces and additive designs

Marco Buratti

Sapienza University of Rome (Italy)

A *half-set* of an additive group  $G$  of odd order is a complete system of representatives for the set of all pairs  $\{g, -g\}$  of opposite elements of  $G \setminus \{0\}$ .

In this talk I will mainly speak about a new combinatorial design that Anita Pasotti and I have called a  $(v, k; r)$  *Heffter space* [3]. This is a resolvable partial linear space of degree  $r$  whose point set is a half set of an abelian group  $G$  of order  $2v + 1$  and whose blocks are zero-sum  $k$ -subsets of  $G$ . When  $r$  is just 1 or 2, it is equivalent to a *Heffter system* or a *Heffter array* on  $G$ , respectively, a topic widely investigated in the last decade [7].

Among the motivations of studying Heffter spaces, there is the fact that every  $(v, k; r)$  Heffter space with suitable properties gives rise to  $r$  mutually orthogonal  $k$ -cycle decompositions of the complete graph on  $2v + 1$  vertices, a topic recently studied in [4, 6].

Even though, for  $k$  odd, we have been able to construct elementary abelian Heffter spaces of arbitrarily large degree, many questions remain open; first among them is whether this result can be obtained also for  $k$  even. In my opinion, however, the most intriguing question is whether there exists a Heffter space which is also a Steiner 2-design. An attempt to answer this very hard question made me “stumbling” again onto *additive designs*, an interesting topic introduced in [5] and recently studied in [1, 2].

### References

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5. A. Caggegi, G. Falcone, M. Pavone, *On the additivity of block designs*, J. Algebr. Comb. **45** (2017), 271–294.
6. S. Kukukcifici, E.Ş. Yazıcı, *Orthogonal cycle systems with cycle length less than 10*, J. Combin. Des. **32** (2024), 31–45.
7. A. Pasotti, J.H. Dinitz, *A survey of Heffter arrays*, to appear in: Stinson 66 – New Advances in Designs, Codes and Cryptography, Colbourn, C.J., Dinitz, J.H. (eds.).

## $q$ -Matroids and related structures

Eimear Byrne

University College Dublin (Ireland)

Matroids have natural connections to graphs, geometry and linear codes. Several invariants of these objects turn out to be special cases of matroid invariants.  $q$ -Matroids, which are  $q$ -analogues of matroids, have gathered much interest in recent years, especially from researchers in rank-metric codes. While a matroid may be defined in terms of a semi-modular function on a finite Boolean lattice, a  $q$ -matroid comprises a rank function defined on the lattice of subspaces of a finite dimensional vector space. Matroids and their generalisations have many equivalent axiomatic descriptions, a fact that has often been exploited. A ( $q$ )-matroid is determined by its lattice of flats and similarly by its cycles. It is also determined by its lattice of cyclic flats and their ranks. However, the lattice of cyclic flats is often very small compared with the lattices of flats and cycles, It also behaves very well with respect to different forms of decomposition. In this talk we will give an introduction to  $q$ -matroids and their structural properties, especially in relation to their cyclic flats. We will describe the free product of a  $q$ -matroid and look at the question of representability in relation to this product.

### References

1. G. Alfarano, E. Byrne, *The Cyclic Flats of a  $q$ -Matroid*, <https://doi.org/10.48550/arXiv.2204.02353>, 2022.
2. E. Byrne, A. Fulcher, *The Cyclic Flats of  $\mathcal{L}$ -Polymatroids*, <https://doi.org/10.48550/arXiv.2312.05522>, 2023.
3. M. Ceria, R. Jurrius, *The direct sum of  $q$ -matroids*, *Journal of Algebraic Combinatorics*, 2024.
4. H. Gluesing-Luerssen, B. Jany, *Decompositions of  $q$ -Matroids Using Cyclic Flats*, <https://doi.org/10.48550/arXiv.2302.02260>, 2023.



## Constructions of LCD subspace codes

Dean Crnković

University of Rijeka (Croatia)

The dual code  $C^\perp$  of a code  $C$  is the orthogonal complement of  $C$  under the standard inner product  $(\cdot, \cdot)$ . Linear codes with complementary duals, or LCD codes, are linear codes whose intersection with their duals are trivial. LCD codes were introduced by Massey in [5] and have been widely applied in information protection, electronics and cryptography.

In 2000, Ahlswede, Cai, Li and Yeung (see [1]) introduced network coding, and in 2006 random network coding was introduced by Ho, Médard, Kötter, Karger, Effros, Shi and Leong (see [3]). Further, in 2008, Kötter and Kschischang introduced subspace codes and propose their applications in error correction for random network coding (see [4]).

LCD subspace codes were introduced recently in [2]. In this talk, we will give some properties and constructions of LCD subspace codes.

### References

1. R. Ahlswede, N. Cai, S.-Y. R. Li, R. W. Yeung, *Network information flow*, IEEE Trans. Inform. Theory 46 (2000), 1204–1216.
2. D. Crnković, A. Švob, *LCD subspace codes*, Des. Codes Cryptogr. 91 (2023), 3215–3226.
3. T. Ho, M. Médard, R. Kötter, D. R. Karger, M. Effros, J. Shi, B. Leong, *A random linear network coding approach to multicast*, IEEE Trans. Inform. Theory 52 (2006), 4413–4430.
4. R. Kötter, F. Kschischang, *Coding for errors and erasures in random network coding*, IEEE Trans. Inform. Theory 54 (2008) 3579–3591.
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## Minimising the number of comparable sets

**Daniel Horsley**

Monash University (Australia)

Joint work with Adam Gowty and Adam Mammoliti

Consider the subsets of a ground set  $\{1, \dots, n\}$ . Over all the families composed of a given number of  $k$ -sets, what is the minimum number of  $(k-1)$ -sets that are subsets of at least one set in the family? The Kruskal-Katona theorem famously answers this question. Bashov considered a two-sided variant of this problem in which one attempts to minimise the total number of  $(k-1)$ -sets and  $(k+1)$ -sets that are subsets or supersets of at least one set in the family. In this talk we discuss a different two sided variant: over all the families  $\mathcal{F}$  composed of a given number of sets (of any sizes), what is the minimum number of sets that are subsets or supersets of at least one set in the family? The results we obtain have parallels with the Kruskal-Katona theorem and also with Harper's theorem on isoperimetry in hypercubes.

## The degree of functions in the Johnson and $q$ -Johnson schemes

**Michael Kiermaier**

Universität Bayreuth (Germany)

Joint work with Jonathan Mannaert and Alfred Wassermann

*Cameron-Liebler line classes* have been introduced in 1982 in an article by Cameron and Liebler as “special” sets  $\mathcal{L}$  of lines in the projective space  $\text{PG}(3, q)$ , which can be defined by several equivalent properties. By the *algebraic property*, the characteristic function  $\chi_{\mathcal{L}}$  of  $\mathcal{L}$  is contained in the row space of the point-line incidence matrix of  $\text{PG}(3, q)$  over  $\mathbb{R}$ , and by the *geometric property*,  $\mathcal{L}$  has constant intersection with each line spread of  $\text{PG}(3, q)$ . In the literature, various generalizations of Cameron-Liebler line classes have been considered. For example, the set case  $q = 1$  has been studied, and the point-line incidence matrix has been generalized to the incidence matrices of  $d$ -spaces vs.  $k$ -spaces. In this talk, a coherent theory of these generalizations will be discussed.

For  $q = 1$ , let  $V$  be a set of finite size  $n$ , and for a prime power  $q \geq 2$ , let  $V$  be an  $\mathbb{F}_q$ -vector space of finite dimension  $n$ . Denoting the set of all  $k$ -subsets (or  $k$ -subspaces) of  $V$  by  $\binom{V}{k}$ , the *degree* of a function  $f : \binom{V}{k} \rightarrow \mathbb{R}$  will be defined by the generalization of the above algebraic property. We will investigate fundamental properties of the degree. The question for the counterpart of the geometric property is related to Delsarte’s concept of *design orthogonality* and leads to a connection to the theory of combinatorial designs and subspace designs.

Of particular interest are Boolean functions  $\binom{V}{k} \rightarrow \{0, 1\}$ , which correspond to subsets  $\mathcal{F}$  of  $\binom{V}{k}$  via characteristic functions. A new divisibility condition on the size of sets  $\mathcal{F}$  of degree  $d$  will be presented. Moreover, we discuss the computer classification of sets of degree 2 in the case  $q = 1$ . Also, for  $q = 1$  a general construction of sets of low degree will be given.

## Counting Steiner triple systems of order 21

**Patric R. J. Östergård**

Aalto University (Finland)

Joint work with Daniel Heinlein

The classification of Steiner triple systems (STSs) has proceeded by centuries: the two STS(13)s were known in the 19th century, the 80 STS(15)s in the 20th, and the 11,084,874,829 STS(19)s in the 21st. We might have to wait until the 22nd century to get the STS(21)s classified and stored, but perhaps they could be *counted* even earlier? Computational approaches for counting STSs will here be discussed. These lead to an algorithm that has been used to obtain (in 82 core-years) the number of isomorphism classes of STS(21)s, 14,796,207,517,873,771, as well as the total number of STS(21)s, 755,952,181,048,907,354,964,715,609,522,176,000. The issue of correctness of these numbers is also addressed.

## Contributed talks



## Grid-based graphs, linear realizations, and the Buratti-Horak-Rosa conjecture

**Onur Ağırseven**

USA/Turkey

Joint work with M. A. Ollis (Emerson College, USA)

Label the vertices of the complete graph  $K_v$  with the integers  $\{0, 1, \dots, v-1\}$  and define the *length*  $\ell$  of the edge between distinct vertices labeled  $x$  and  $y$  by  $\ell(x, y) = \min(|y-x|, v-|y-x|)$ . A *realization* of a multiset  $L$  of size  $v-1$  is a Hamiltonian path through  $K_v$  whose edge labels are  $L$ . The *Buratti-Horak-Rosa (BHR) Conjecture* is that there is a realization for a multiset  $L$  if and only if for any divisor  $d$  of  $v$  the number of multiples of  $d$  in  $L$  is at most  $v-d$ .

We use “grid-based graphs” which are a useful tool for constructing particular types of realizations, called “linear realizations,” especially when the multiset in question has three distinct elements. We focus on multisets whose underlying set has one of the following forms for small  $k$ :  $\{1, x, x+k\}$ ,  $\{1, k, x\}$  and  $\{1, x, kx \pm 1\}$ . These constructions considerably extend the parameters for which BHR Conjecture is known to hold.

## Distance-regular and strongly regular graphs from affine permutation groups

**Robert F. Bailey**

Grenfell Campus, Memorial University (Canada)

Recently, with the assistance of several students, I have been cataloguing distance-regular and strongly regular graphs arising from the libraries of primitive permutation groups. In the case where the group  $G$  has *affine type*, i.e.  $G = V \rtimes H$  where  $V$  is the additive group of a vector space and  $H \leq \Gamma L(V)$ , the possibilities are too numerous to make much sense of.

However, when the dimension of  $V$  is small, there are connections with other interesting combinatorial objects, including Cayley graphs, mutually orthogonal Latin squares, partial difference sets and affine planes. In this talk, we will explain some of these connections, and present a conjecture which aims to completely describe such graphs when  $\dim(V) \leq 3$ .

## New extremal Type II $\mathbb{Z}_4$ -codes of length 64

**Sara Ban**

University of Rijeka (Croatia)  
Joint work with Sanja Rukavina

Extremal Type II  $\mathbb{Z}_4$ -codes are a class of self-dual  $\mathbb{Z}_4$ -codes with Euclidean weights divisible by eight and the largest possible minimum Euclidean weight for a given length. A small number of such codes is known for lengths greater than or equal to 48. The doubling method is a method for constructing Type II  $\mathbb{Z}_4$ -codes from a given Type II  $\mathbb{Z}_4$ -code. The subject of this talk is a construction of new extremal Type II  $\mathbb{Z}_4$ -codes of length 64 by the doubling method.

We develop a method to construct new extremal Type II  $\mathbb{Z}_4$ -codes starting from an extremal Type II  $\mathbb{Z}_4$ -code of type  $4^k$  with an extremal residue code and length 48, 56 or 64. Using this method, we construct three new extremal Type II  $\mathbb{Z}_4$ -codes of length 64 and type  $4^{31}2^2$ . Extremal Type II  $\mathbb{Z}_4$ -codes of length 64 of this type were not known before. Moreover, the residue codes of the constructed extremal  $\mathbb{Z}_4$ -codes are new best known [64, 31] binary codes and the supports of the minimum weight codewords of the residue code and the torsion code of one of these codes form self-orthogonal 1-designs.

## Quartic curves with 63 Kovalevski points

**Anton Betten**

Kuwait University (Kuwait)

Quartic curves with 28 bitangents are related to cubic surfaces with 27 lines with a point of the surface not on any line. Bitangents can meet at most 4 at a time, in a point that we wish to call Kovalevski point, in honor of Kovalevski's contributions to the field. The number of Kovalevski points is at most 63, and if it is, the associated incidence structure gives rise to the classical unital. We wish to describe a family of curves with 63 Kovalevski points over a finite field containing the field with nine elements as a subfield. The associated cubic surface has 9 Eckardt points. Further properties will be described in the talk.

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## Hypergraphs are 2-colorable if the 1-intersection graph of their hyperedges is bipartite

**Zoltán L. Blázsik**

HUN-REN Alfréd Rényi Institute of Mathematics, MTA-ELTE GAC, SZTE Bolyai  
Institute (Hungary)

Joint work with Nathan W. Lemons

For two edges  $e, f \in E$  in a hypergraph  $\mathcal{H}(V, E)$ , we say edge  $e$  has a 1-intersection with another edge  $f$  if  $|e \cap f| = 1$ . The 1-intersection graph  $\mathcal{H}^{[1]}$  of the hypergraph  $\mathcal{H}(V, E)$  is defined as follows. The vertices of  $\mathcal{H}^{[1]}$  corresponds to the hyperedges  $E$  of  $\mathcal{H}$  and two vertices are adjacent if and only if they have a 1-intersection in  $\mathcal{H}$ . In 2015, Gyárfás et al proved that for 3-uniform hypergraphs if  $\mathcal{H}^{[1]}$  is  $k$ -colorable then  $\mathcal{H}$  is  $k$ -colorable, too. They conjectured that the same conclusion might hold if one consider  $\ell$ -uniform hypergraphs for  $\ell > 3$ . We generalize their results in a different way, because we omit the uniformity condition and consider arbitrary hypergraphs and prove the same conclusion but only for  $k = 2$ .

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## The minimal blocking sets of $\text{PG}(2, 11)$ with a non-trivial automorphism group

**Arne Botteldoorn**

Ghent University (Belgium)

Joint work with Kris Coolsaet and Veerle Fack

We used a computer search to generate, up to equivalence, all minimal blocking sets of  $\text{PG}(2, 11)$  with a non-trivial automorphism group.

Experiments have shown that exhaustive isomorph-free generation of all minimal blocking sets of  $\text{PG}(2, 11)$  (including those with a trivial automorphism group) is infeasible because there are simply too many of them. For this reason, special techniques are needed to generate only those blocking sets whose automorphism group is non-trivial.

We shall discuss the core mathematical ideas used in the generator algorithms we developed for this purpose, report on the results we have obtained and give explicit constructions of some of the blocking sets with a fairly large automorphism group.

We have found minimal blocking sets of every size in the interval  $[18, 36]$ , proving, together with the known bounds, that this is the full spectrum of minimal blocking set sizes in  $\text{PG}(2, 11)$ .



## Erdős–Ko–Rado type problems in root systems

**Patrick J. Browne**

Technological University of the Shannon (Ireland)

Joint work with Qëndrim R. Gashi, University of Prishtina

Given a Lie algebra, two roots are strongly orthogonal if neither their sum nor difference is a root. In this talk, we investigate sets of mutually strongly orthogonal roots. In particular, those such that any two such sets have the property that the difference between their sums can itself be expressed as the sum of a strongly orthogonal set of roots. We discuss this property and its relationship to Erdős–Ko–Rado type problems and finally discuss applications in terms of the existence of finite projective planes of certain orders. This is joint work with Qëndrim R. Gashi, University of Prishtina.

## Constructing solutions of the Hamilton-Waterloo Problem using row-sum matrices

**Andrea Burgess**

University of New Brunswick (Canada)

Joint work with Peter Danziger, Adrián Pastine and Tommaso Traetta

A *row-sum matrix* over a group  $G$  is a matrix whose columns are permutations of a given subset of elements of  $G$  and whose rows sum to specified elements of  $G$ . Such matrices have recently proved useful in constructing 2-factorizations of complete and complete multipartite graphs. In this talk, we describe how row-sum matrices can be used to find solutions of the uniform Hamilton-Waterloo Problem, which asks for a factorization of the complete graph into  $\alpha$   $C_m$ -factors and  $\beta$   $C_n$ -factors. In particular, by constructing row-sum matrices over a generalized dihedral group, we are able to make significant progress on the case where the cycle lengths have opposite parity.

## Brooks' theorem with forbidden colors

**Carl Johan Casselgren**

Linköping University (Sweden)

Brooks' classic theorem on graph coloring states that the number of colors needed for a proper coloring of a graph  $G$  is at most the maximum degree of  $G$ , unless  $G$  is a complete graph or an odd cycle. I shall discuss some variations on this theorem, including variants proved for list coloring, precoloring extension and a recent variation where some colors cannot be used on certain vertices.

## Colouring Kirkman triple systems

Peter Danziger

Toronto Metropolitan University (Canada)

Joint work with Andrea Burgess, Nicholas Cavenagh and David Pike

A weak  $\delta$ -colouring of a block design is an assignment of  $\delta$  colours to the point set so that no block is monochromatic. The *weak chromatic number*  $\chi(S)$  of a block design  $S$  is the smallest integer  $\delta$  such that  $S$  has a weak  $\delta$ -colouring. It has previously been shown that any Steiner Triple System has weak chromatic number at least 3 and that for each  $v \equiv 1$  or  $3 \pmod{6}$  there exists a Steiner triple system on  $v$  points that has weak chromatic number 3. Moreover, for each integer  $\delta \geq 3$  there exist infinitely many Steiner triple systems with weak chromatic number  $\delta$ .

In this talk we consider colourings of the subclass of Steiner triple systems which are resolvable, namely Kirkman Triple Systems. We show that for each  $v \equiv 3 \pmod{6}$  we construct a Kirkman Triple System on  $v$  points with weak chromatic number 3. We also show that for each integer  $\delta \geq 3$ , there exist infinitely many Kirkman triple systems with weak chromatic number  $\delta$ .

## Existence and non-existence of Cameron-Liebler $k$ -sets in projective spaces

Jan De Beule

Vrije Universiteit Brussel (Belgium)

Joint work with Jonathan Mannaert and Leo Storme

Let  $\text{PG}(n, q)$  be the  $n$ -dimensional projective space over the finite field  $\mathbb{F}_q$ . Let  $P_n$  denote its point/ $k$ -space incidence matrix. A Cameron-Liebler  $k$ -set is a collection  $\mathcal{L}$  of  $k$ -spaces of  $\text{PG}(n, q)$  for which its characteristic vector  $\chi_{\mathcal{L}} \in \text{Im}(P_n^T)$ . For  $n = 3$  and  $k = 1$  such objects have been well studied and are known as Cameron-Liebler line classes.

Cameron-Liebler  $k$ -sets have a parameter  $x$ . When  $\text{PG}(n, q)$  allows  $k$ -spreads, the parameter  $x$  is a natural number which is simply the number of elements that the Cameron-Liebler  $k$ -set and any spread have in common, indeed this parameter is constant and depends only on the Cameron-Liebler  $k$ -set.

At the time of their introduction by Cameron and Liebler in 1982, it was conjectured that only trivial Cameron-Liebler line classes exist in  $\text{PG}(3, q)$ . Right now, there are both constructions of non-trivial examples (with a particular parameter), and many non-existence results for other possible values of the parameter  $x$ .

For  $n \geq 4$ , the situation is quite different. So far, no non-trivial examples are known. In this talk, an overview of recent non-existence results will be discussed.

## Characterising ovoidal cones by their intersection numbers

**Bart De Bruyn**

Ghent University (Belgium)

Joint work with Geertrui Van de Voorde

Consider a solid  $\Pi$  in the projective space  $\text{PG}(4, q)$ , an ovoid  $O$  in  $\Pi \cong \text{PG}(3, q)$  and a point  $x \notin \Pi$ . The set of points obtained by joining  $x$  with the points of  $O$  is called an *ovoidal cone*. We will characterise ovoidal cones by their intersection numbers. We will show that a set of points of  $\text{PG}(4, q)$  which blocks all planes and intersects solids in  $q+1$ ,  $q^2+1$  or  $q^2+q+1$  points is a plane or an ovoidal cone, and determine all examples that arise when the blocking condition is omitted.

## On Steiner systems $S(2, 6, 91)$

**Doris Dumičić Danilović**

University of Rijeka (Croatia)

Joint work with Dean Crnković

There are only four known Steiner 2–designs  $S(2, 6, 91)$ . The designs have been found by C. J. Colbourn, M. J. Colbourn and W. H. Mills. Each design is cyclic, i.e. having a cyclic automorphism group acting transitively on points. In 1991, Z. Janko and V. D. Tonchev showed that any point-transitive 2– $(91, 6, 1)$  design with an automorphism group of order larger than 91 is one of the four known designs. It is a difficult open problem to classify Steiner 2–designs with these parameters. In this talk we show that any Steiner 2–design  $S(2, 6, 91)$  with a non-abelian automorphism group of order 26 (i.e. the Frobenius group  $\text{Frob}_{26}$ ) is isomorphic to one of the known designs. Thus, it remains an open question whether there exists a 2– $(91, 6, 1)$  design that admits an automorphism group of order less than 91.

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## High dimensional computation of fundamental domains

**Mathieu Dutour Sikirić**

MSM Programming (Croatia)

Joint work with Paul Gunnells

Over the years we have developed open-source software in C++ for computing with polyhedra, lattices, and related algebraic structures. We will shortly explain its design. Then we will explain how it was used for computing the dual structure of the  $W(H_4)$  polytope.

Then we will consider another application to finding the fundamental domain of cocompact subgroups  $G$  of  $SL_n(\mathbb{R})$ . The approach defines a cone associated with the group and a point  $x \in \mathbb{R}^n$ . It is a generalization of Venkov reduction theory for  $GL_n(\mathbb{Z})$ . We recall the Poincaré Polyhedron Theorem which underlies these constructions.

We give an iterative algorithm that allows computing a fundamental domain. The algorithm is based on linear programming, the Shortest Group Element (SGE) problem and combinatorics. We apply it to the Witte cocompact subgroup of  $SL_3(\mathbb{R})$  defined by Witte for the cubic ring of discriminant 49.

## New lower bounds on caps, and on progression-free sets in $\mathbb{F}_p^n$ and the integers

**Christian Elsholtz**

Graz University of Technology (Austria)

We give new lower bounds for high-dimensional caps (affine and projective) over  $\mathbb{F}_p$ , for the primes 5, 11, 17, 23, 29, 41. This improves bounds of Bose, Edel and Bierbrauer. We then give new constructions for sets in  $\mathbb{F}_p^n$  (or  $\mathbb{Z}_m^n$ ) without any arithmetic progression of length 3, of size  $(cp)^n$ , where  $c > \frac{1}{2}$ . (Previous methods did not go beyond 1/2.) Finally, we mention how this can be used to give a new lower bound for the frequently studied size of progression-free sets in the set of integers. These results improve work of Salem and Spencer (1942) and Behrend (1946).

(Corresponding upper bounds for the progression-free sets are by Ellenberg and Gijswijt, by Croot, Lev and Pach, and by Roth, Szemerédi, Bourgain, Sanders, Bloom and Sisask, and eventually Kelley and Meka.)

The results in this talk are based on joint work with P.P. Pach (Des. Codes Cryptogr., 88 (2020), 2133–2170), Gabriel Lipnik (Exponentially Larger Affine and Projective Caps, Mathematika 69 (2023), 232–249), and two very recent manuscripts, one with Laura Proske and Lisa Sauermann, and the second one by Zach Hunter. (I will update on this in the talk.)

## Transitivity in finite general linear groups

**Alena Ernst**

Paderborn University (Germany)

Joint work with Kai-Uwe Schmidt

This talk is about subsets of the finite general linear group  $\mathrm{GL}(n, q)$  acting transitively on flag-like structures, which are common generalisations of  $t$ -dimensional subspaces of  $\mathbb{F}_q^n$  and bases of  $t$ -dimensional subspaces of  $\mathbb{F}_q^n$ . We discuss structural characterisations of transitive subsets of  $\mathrm{GL}(n, q)$  using the character theory of  $\mathrm{GL}(n, q)$  and interpret such subsets as designs in the conjugacy class association scheme of  $\mathrm{GL}(n, q)$ . We also show that, for all fixed  $t$ , there exist nontrivial subsets of  $\mathrm{GL}(n, q)$  that are transitive on linearly independent  $t$ -tuples of  $\mathbb{F}_q^n$ , which also shows the existence of nontrivial subsets of  $\mathrm{GL}(n, q)$  that are transitive on more general flag-like structures. These results can be interpreted as  $q$ -analogs of corresponding results for the symmetric group.

## Heffter arrays over partial loops

**Raúl M. Falcón**

Universidad de Sevilla (Spain)

Joint work with Lorenzo Mella

A Heffter array over an abelian group  $G$  is any partially filled array  $A$  satisfying that: (1) each one of its rows and columns sum to zero in  $G$ , and (2) if  $i \in G \setminus \{0\}$ , then either  $i$  or  $-i$  appears exactly once in  $A$ . In this talk, we introduce a natural generalization of this concept so that summations are defined over partial loops instead that groups, and other block sums apart from row and column ones are considered. The general lack of commutativity and associativity in partial loops makes us to be very careful with the way in which these summations are described. Non-associative polynomials constitute a useful approach to formalize this aspect. In this regard, some illustrative examples of non-associative polynomials that give rise to zero sums over non-associative partial loops are shown. The existence of this kind of examples enables us to introduce the notion of  $\mathcal{P}$ -Heffter array over a partial loop, where  $\mathcal{P}$  is a set of block-sum polynomials over an affine 1-design on the set of entries of the array. The usual concept of Heffter array over an abelian group is a particular case of this new notion. Some constructive examples are described.

## Goppa codes over hyperelliptic curves

**Giovanni Falcone**

University of Palermo (Italy)

Joint work with Giuseppe Filippone

Both the Mumford representation of a divisor  $\Delta$  of degree zero on a hyperelliptic curve and the Riemann-Roch space  $\mathcal{L}(D)$ , where  $D = \Delta + m\Omega$ , are the subject of a large number of papers, also due to their applications in Coding theory.

But it has not been indicated in the literature that a basis of the latter can be directly found from the former, and in this talk we give an explicit basis of  $\mathcal{L}(D)$ , stressing the meaning of the Mumford representation of  $\Delta$  in this context.

Using this basis, one constructs directly a generating matrix of a Goppa code over a hyperelliptic curve defined over a Galois field of characteristic  $p \geq 2$ . Although the reduction of a divisor  $D$  to its reduced Mumford form might be an inconvenient task, involving the application of the Cantor algorithm, this difficulty does not occur in the construction of Goppa codes, because in that case one can directly take  $D$  in the reduced form  $D = \Delta + m\Omega$ .

## Reinforcement learning for graphs

**Mohammad Ghebleh**

Kuwait University (Kuwait)

Joint work with Ali Kanso, Salem Al-Yakoob,  
and Dragan Stevanović

We discuss here a versatile reimplementaion of a recently proposed approach of using reinforcement learning to construct (counter)examples in graph theory. We showcase our reimplementaion on a handful of conjectures from literature for which reinforcement learning was able to suggest proper structure of counterexamples.

## Some thoughts on algebraic coding theory over the Boolean semifield

**Marcus Greferath**

University College Dublin (Ireland)

Group Testing is a well-established field within Mathematics of Communications. A recent paper of the author drew its connections to Residuation Theory and a form of Linear Algebra on the Boolean Semifield. In this respect it is not surprising that Group Testing, and particularly its error-correcting version, is a type of Algebraic Coding Theory over that Boolean alphabet. In the present talk, I plan to commence pursuing this line of research and present a few further insights.

## Ovoids and primitive normal bases for quartic extensions of Galois fields

**Dirk Hachenberger**

University of Augsburg (Germany)

By a celebrated result of H. W. Lenstra, Jr. and R. J. Schoof (1987), any extension  $E/F$  of Galois fields admits a generator of the multiplicative group of  $E$  whose conjugates under the Galois group are linearly independent over  $F$ . Any such element is called a *primitive normal basis generator* for  $E/F$ . For the case where  $E = \text{GF}(q^4)$  is the quartic extension over  $F = \text{GF}(q)$  we present the lower bound

$$\varphi(q^4 - 1) - \omega(q) \cdot \varphi(q^2 - 1)$$

for the number of such elements. There,  $\varphi$  denotes the Euler function and  $\omega(q)$  is equal to  $4(q-1)$ , if  $q \equiv 1 \pmod{4}$ , to  $2(q-1)$ , if  $q \equiv 3 \pmod{4}$ , and to  $q$ , if  $q \equiv 0 \pmod{2}$ , respectively.

Our approach is geometric: Considering  $E$  as the three-dimensional projective space  $\Gamma = \text{PG}(3, q)$ , the points of that space are distinguished into primitive and non-primitive ones. The structure of the multiplicative group of  $E$  gives rise to a partition of the point set of  $\Gamma$  into  $q+1$  ovoids. The bound is derived by studying the intersection of those ovoids which cover the primitive points with the non-normal configuration; the latter is the collection of points of  $\Gamma$  which do *not* give rise to normal elements of  $E/F$ .

Given that  $q^2 + 1$  is a prime number when  $q$  is even, or that  $\frac{1}{2}(q^2 + 1)$  is a prime number when  $q$  is odd, we actually achieve the exact number of all primitive normal elements for the quartic extension over  $\text{GF}(q)$ . Moreover, the proportion of all primitive normal elements among all primitive elements converges to 1 as  $q$  tends to infinity. For instance, when  $q \geq 79$ , then at least 95 percent of all primitive elements of  $\text{GF}(q^4)$  are normal over  $\text{GF}(q)$ .

## Transitive $(q-1)$ -fold packings of $\text{PG}_n(q)$

**Daniel Hawtin**

University of Rijeka (Croatia)

A *t-fold packing* of a projective space  $\text{PG}_n(q)$  is a collection  $\mathcal{P}$  of line-spreads such that each line of  $\text{PG}_n(q)$  occurs in precisely  $t$  spreads in  $\mathcal{P}$ . A *t-fold packing*  $\mathcal{P}$  is *transitive* if a subgroup of  $\text{P}\Gamma\text{L}_{n+1}(q)$  preserves and acts transitively on  $\mathcal{P}$ . We give a construction for a transitive  $(q-1)$ -fold packing of  $\text{PG}_n(q)$ , where  $q = 2^k$ , for any odd positive integers  $n$  and  $k$ , such that  $n \geq 3$ . This generalises a construction of Baker from 1976 for the case  $q = 2$ .

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## Maximum weight spectrum codes over finite fields and rings

**Thomas Honold**

ZJU-UIUC Institute, Zhejiang University (China)

Joint work with M. Kiermaier, M.J. Shi, P. Solé, T.T. Tong

A linear code over a finite field or ring  $R$  is called a maximum weight spectrum (MWS) code if it has the maximum number  $s$  of distinct (nonzero) weights among all linear codes of a fixed module type. The underlying weight function is usually taken as the Hamming weight if  $R = \mathbb{F}_q$  and as the homogeneous weight if  $R$  is a chain ring. In these cases  $s$  has been determined in [1], resp., [2]. In my talk I will explain these results and also discuss what is known about the smallest possible lengths of MWS codes.

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## Designs in the generalised symmetric group

**Lukas Klawuhn**

Paderborn University (Germany)

Joint work with Kai-Uwe Schmidt

It is known that the notion of a transitive subgroup of a permutation group  $G$  extends naturally to the subsets of  $G$ . We study transitive subsets of the wreath product  $C_r \wr S_n$  of generalised permutations acting on subsets of  $\{1, \dots, n\}$  whose elements are coloured with one of  $r$  possible colours. This includes the symmetric group for  $r = 1$  and the hyperoctahedral group for  $r = 2$ . The group  $C_r \wr S_n$  can also be interpreted as the symmetry group of a regular polytope for every  $r$  and this gives rise to an intuitively accessible definition of transitivity. We consider different notions of transitivity in  $C_r \wr S_n$  and interpret these algebraically as designs in the conjugacy class association scheme of  $C_r \wr S_n$  using representation theory. We also give constructions showing that there exist transitive subsets of  $C_r \wr S_n$  that are small compared to the size of the group. Many of these results extend results previously known for the symmetric group  $S_n$ .



## Some nice combinatorial objects

**Vedran Krčadinac**

University of Zagreb (Croatia)

In this talk I will present some nice finite objects constructed during the course of the project *Algorithmic Constructions of Combinatorial Objects*. I will also remember [Rudi Mathon](#) (1940–2022), who was a master at constructing combinatorial objects that are true gems.

## Multispreads and the characterization of parameters of additive one-weight codes

**Denis Krotov**

Sobolev Institute of Mathematics (Russia)

Joint work with Ivan Mogilnykh

Let  $S$  be a collection (multiset) of subspaces of dimension at most  $t$  of an  $m$ -dimensional space over  $\text{GF}(q)$ . Each subspace  $X$  of dimension  $s$  from  $S$  is treated as the multiset of cardinality  $q^t - 1$  where every nonzero vector of  $X$  has multiplicity  $q^{t-s}$  and the zero vector has multiplicity  $q^{t-s} - 1$ . Such  $S$  is called a  $(\lambda, \mu)$ -multispread (more specifically, a  $(\lambda, \mu)_q^{t,m}$ -multispread) if the union of the multisets corresponding to the subspaces from  $S$  contains the zero vector with multiplicity  $\lambda$  and each nonzero vector of the space with multiplicity  $\mu$ .

Ordinary spreads correspond to  $(0, 1)$ -multispreads, and  $\mu$ -fold spreads correspond to  $(0, \mu)$ -multispreads. An example of  $(\lambda, \mu)$ -multispread with nonzero  $\lambda = q^{m'-m} - 1$  and  $\mu = q^{m'-m}$  can be obtained from a spread of an  $m'$ -dimensional space,  $m' > m$ , by projection onto an  $m$ -dimensional space (we consider the projection that respects the multiplicity and preserves the cardinality of a multiset of vectors).

Multispreads over a field  $\text{GF}(q)$  of prime order  $q$  are equivalent to additive one-weight codes over  $\text{GF}(q^t)$ . The current work is devoted to the characterization of the parameters of multispreads, which is equivalent (for prime  $q$ ) to the characterization of the parameters of additive one-weight codes over  $\text{GF}(q^t)$ . We characterize these parameters for the case  $t = 2$  and make a partial characterization for  $t = 3$  and  $t = 4$  (including a complete characterization for  $q^t = 2^3, 3^3$ , and  $2^4$ , where several key cases are solved computationally).

The work is funded by the Russian Science Foundation (22-11-00266).

## Constructions of binary codes with two distances

Ivan Landjev

Institute of Mathematics and Informatics, BAS (Bulgaria)

Joint work with Konstantin Vorob'ev

We investigate bounds on the size of a binary (non-linear) codes of length  $n$  and size  $M$  with two distances  $d_1$  and  $d_2$ . It was proved in [3] that

$$M \leq \binom{n+2}{2}.$$

This bound was improved in [1] to

$$M \leq \binom{n}{2} + 1, n \geq 6.$$

Equality in this bound can be achieved, e.g. for  $d_1 = 2, d_2 = 4$ . For special values of  $d_1$  and  $d_2$  more precise bounds can be obtained. For instance, if  $d_2 > 2d_1$  or  $d \not\equiv d_2 \pmod{2}$ ,  $M$  is upperbounded by a linear function of  $n$  [3]. In this talk, we give a brief survey on some of these bounds.

This research was supported by the Bulgarian NSF under Grant KP-06-N72/6-2023.

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## A note on strong blocking sets and higgledy-piggledy sets of lines

**Stefano Lia**

University College Dublin (Ireland)  
Joint work with Geertrui Van De Voorde

A  $t$ -fold blocking set  $S$  in the  $N$ -dimensional finite projective space  $\text{PG}(N, q)$  is a set  $S$  meeting each hyperplane  $\pi$  of  $\text{PG}(N, q)$  in at least  $t$  points. A 1-fold blocking set is simply called blocking set. If the blocking set  $S$  has the further property that, for each hyperplane  $\pi$ , the set of points  $S \cap \pi$  spans  $\pi$ , then  $S$  is called *strong blocking set*. Strong blocking sets are in particular  $N$ -fold blocking sets. Since  $t$ -fold blocking sets can also be obtained as the union of  $t$  distinct blocking sets, it is natural to look for strong blocking set given by the union of lines. A set of lines forming a strong blocking set it's called a *higgledy-piggledy set of lines*. In this talk, after reviewing the state of the art, I will show that certain unions of blocking sets cannot form strong blocking sets, which leads to a new lower bound on the size of a strong blocking set in  $\text{PG}(N, q)$ . I will also show that, for  $q > \frac{2}{\ln(2)}(N + 1)$ , there exists a higgledy-piggledy subset of  $2N - 2$  lines of a Desarguesian line spread in  $\text{PG}(N, q)$ ,  $N$  odd; thus giving rise to a strong blocking set of size  $(2N - 2)(q + 1)$ .

## Bounds for $m$ -ovoids using combinatorial techniques

**Jonathan Mannaert**

Vrije Universiteit Brussel (Belgium)  
Joint work with Jan De Beule and Valentino Smaldore

An  $m$ -ovoid in a finite classical polar space is a set  $\mathcal{O}$  of points such that each generator meets  $\mathcal{O}$  in exactly  $m$  points. These objects are quite rare and hence non-existence conditions on the parameter  $m$  remains one of the main research questions. Using combinatorial arguments and known techniques with the required major adjustments, we can improve the lower bound on  $m$  for the hermitian polar space  $H(2r, q)$  ( $q$  square,  $e = \frac{3}{2}$ ), the symplectic polar space  $W(2r + 1, q)$  ( $e = 1$ ) and the elliptic quadric  $Q^-(2r + 1, q)$  ( $e = 2$ ) found in [1]. In [2], we show for  $q > 2$  and  $r \geq 3$  (or  $r > 3$  in some very particular cases) that

$$m \geq \frac{-r(1 + \frac{2}{q^{r-e-1}} + \frac{1}{q^{r-2}}) + \sqrt{r^2(1 + \frac{1}{q^{r-e-1}})^2 + 4(q-2)(r-1)(q^{e+1} \frac{q^{r-2}-1}{q-1} + q^e + 1)}}{2(q-1)}.$$

This result is a direct improvement of all known bounds so far. More interesting is that previously known bounds can also be derived from the same result. Hence our approach unifies many known non-existence results. In this talk we will focus on the proof and techniques described in [2].

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## Improving the lower bound for the order of correlation immune Boolean functions

**Giuliamaria Menara**

University of Trieste (Italy)

Joint work with Luca Manzoni and Luca Mariot

In this talk, I will present a possible approach to an open problem posed by Manzoni and Mariot in [1] about correlation immune Boolean functions. In their work, the authors investigate a method to construct correlation immune functions through families of mutually orthogonal cellular automata (MOCA). They show that the orthogonal array (OA) associated to a family of MOCA can be expanded to a binary OA of strength at least 2, and then they use the resulting binary OA to define the support of a second-order correlation immune function. They further perform some computational experiments to construct all such functions up to  $n = 12$  variables, and observe that their correlation immunity order is actually greater, always at least 3. In this talk, I propose a construction that improves the lower bound proved in [1] for the correlation immunity order.

### References

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## Eulerian magnitude homology

**Giuliamaria Menara**

University of Trieste (Italy)

Joint work with Chad Giusti

Magnitude was first introduced by Leinster in 2008 [1]. It is a notion analogous to the Euler characteristic of a category, and it captures the structure and complexity of a metric space. Magnitude homology was defined in 2014 by Hepworth and Willerton [2] as a categorification of magnitude in the context of simple undirected graphs, and although the construction of the boundary map suggests that magnitude homology groups are strongly influenced by the graph substructures, it is not straightforward to detect such subgraphs. In this talk, I introduce eulerian magnitude homology. I will do this by defining the eulerian magnitude chain complex, a subcomplex of the magnitude chain complex exhibiting a more explicit connection to the combinatorics of the graph. I will illustrate how eulerian magnitude homology enables a more

accurate analysis of graph substructures and then apply these results to Erdős-Rényi random graphs and random geometric graphs and obtain an asymptotic estimate for the Betti numbers of the eulerian magnitude homology groups on the diagonal.

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## Banff designs: difference methods for coloring incidence graphs

**Francesca Merola**

Roma Tre University (Italy)

Joint work with Marco Buratti, Anamari Nakić, Christian Rubio-Montiel

A harmonious coloring of a graph is a proper vertex-coloring such that every pair of colors appears on at most one pair of adjacent vertices, and the harmonious chromatic number  $h(G)$  of a graph  $G$  is then the minimum number of colors needed for a harmonious coloring of  $G$ . It is easily seen that the harmonious chromatic number of the incidence graph of a  $2$ -( $v, k, \lambda$ )-design must be at least  $v$ , the number of points of the design.

In this talk, I will present some applications of difference methods to study and construct *Banff designs*, that is designs having harmonious chromatic number of the incidence graph exactly equal to the lower bound  $v$ .

## Construction of extremal $\mathbb{Z}_4$ -codes using a neighborhood search algorithm

**Matteo Mravić**

University of Rijeka (Croatia)

Joint work with Dean Crnković and Sanja Rukavina

A  $\mathbb{Z}_4$ -code of length  $n$  is a  $\mathbb{Z}_4$ -submodule of the  $\mathbb{Z}_4$ -module  $\mathbb{Z}_4^n$ . The dual code of a  $\mathbb{Z}_4$ -code is defined as its orthogonal complement with respect to the usual inner product on the module  $\mathbb{Z}_4^n$ . A  $\mathbb{Z}_4$ -code is self-dual if it is equal to its dual code. For  $x \in \mathbb{Z}_4^n$ , Euclidean weight of  $x$  is defined as  $wt_E(x) = n_1(x) + 4n_2(x) + n_3(x)$ , where  $n_i(x)$  denotes the number of coordinates in  $x$  equal to  $i$ , for  $i = 1, 2, 3$ . A self-dual  $\mathbb{Z}_4$ -code can have codewords of Euclidean weight divisible by 4 or 8. If all codewords have Euclidean weight divisible by 8, then it is a Type II code. It is known that such codes can only exist for lengths divisible by 8. If the self-dual

$\mathbb{Z}_4$ -code is not a Type II code, it is a Type I code. The minimum Euclidean weight of Type II codes is at most  $8 \lfloor \frac{n}{24} \rfloor + 8$ . The same bound holds for Type I codes of length  $n \not\equiv 23 \pmod{4}$ . If minimum Euclidean weight of a self-dual  $\mathbb{Z}_4$ -code is equal to that bound, it is an extremal  $\mathbb{Z}_4$ -code. All self-dual  $\mathbb{Z}_4$ -codes are classified up to length 20. Also, extremal  $\mathbb{Z}_4$ -codes of length 24 are classified. Therefore, length 32 is the smallest length for which new extremal Type II codes can be found.

In this talk, we present a method for constructing extremal  $\mathbb{Z}_4$ -codes based on random neighborhood search. This method is used to find new extremal Type I and Type II  $\mathbb{Z}_4$ -codes of lengths 32 and 40.

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## On properties of matrix representation of quadratic planar polynomials over finite fields and its connection with Latin squares

**Amela Muratović-Ribić**

University of Sarajevo (Bosnia and Hercegovina)

To quadratic planar polynomial we can assign a symmetric bilinear form  $F(x, y) = x^T A y$  where  $A$  is a symmetric matrix. For a given basis over  $\mathbb{F}_q$  we can define a matrix representation  $M$  of  $F(x, y)$ . Here we will specify condition of  $M$  in order  $F(x, x)$  to be a planar polynomial and show how to construct a symmetric Latin square using  $M$ . The interesting question is for a given set of permutation polynomials when every nontrivial linear combination is also a permutation polynomial. Using matrix  $M$ , we can find such maximal sets of linearized permutation polynomials.

## Constructing linked systems of relative difference sets via Schur rings

**Misha Muzychuk**

Ben-Gurion University of the Negev (Israel)

Joint work with G. Ryabov

Linked systems of symmetric block designs were introduced by P. Cameron in 1972 as combinatorial objects associated to inequivalent 2-transitive representations of a given group. It turned out that these objects have close connections to association schemes, coding theory, and other parts of combinatorics. D. Higman generalized Cameron's idea and introduced uniformly linked strongly regular designs in 1995. A particular case of this object named linked system of divisible symmetric block designs was studied in details by H. Kharaghani and S. Suda in 2018.

In 2014 J. Davis, W. Martin, and J. Polhill proposed to use difference sets to construct linked block designs – they introduced a concept of linked systems of difference sets. A few years later this idea was extended to relative difference sets by Davis, Polhill and Smith.

In my talk it will be shown how one can use Schur rings in order to construct systems of linked relative difference sets (LRDSs). As a result of this approach, we obtain two constructions of LRDSs. The first one is based on cyclotomic Schur rings over the extraspecial groups of odd order. It provides systems of linked relative difference sets with new intersection numbers. The corresponding systems of group divisible block designs with these intersection numbers were mentioned as open cases by Kharaghani and Suda. The second construction generalizes the Davis-Polhill-Smith systems of LRDSs.

## Switching equivalence of strongly regular polar graphs

**Gábor P. Nagy**

University of Szeged (Hungary)

Joint work with Valentino Smaldore (Padova, Italy)

We investigate the two-graphs associated with strongly regular polar graphs that belong to three infinite classes. Specifically, we examine the strongly regular graph  $\Gamma(O^\pm(2m, 2))$ , which has vertices representing points of a nondegenerate hyperbolic or elliptic quadric  $Q^\pm(2m - 1, 2)$  in the projective space  $PG(2m - 1, 2)$ . The set of vertices for  $NO^\pm(2m, 2)$  is the complement of  $Q^\pm(2m - 1, 2)$ . Additionally, we consider the vertices of the third graph  $NO^\pm(2m + 1, q)$ , where  $q$  is even, which correspond to hyperplanes of  $PG(2m, q)$  that intersect the nondegenerate parabolic quadric in a nondegenerate hyperbolic or elliptic quadric.

Our main result is the proof of switching equivalence for the strongly regular polar graphs  $NO^\pm(4m, 2)$ ,  $NO^\mp(2m + 1, 4)$ , and  $\Gamma(O^\mp(4m, 2))$  with an isolated vertex. We establish this by providing an analytic description for these graphs and their corresponding two-graphs.

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## On $(4, r, 6)$ –bipartite biregular cages

**Vito Napolitano**

Università degli Studi della Campania Luigi Vanvitelli (Italy)

Joint work with Suliman Khan

Let  $k < r$  be two positive integers and  $g$  be an even integer. A *bipartite biregular*  $(k, r; g)$ –graph  $G$  is a bipartite graph of even girth  $g$  with degree set  $\{r, k\}$  such that vertices in the same partite set have the same degree. A *bipartite biregular cage* is a  $(k, r; g)$ –biregular graph of minimum order. In [1] the authors study bipartite biregular graphs related to Steiner systems and generalized polygons to present new families of bipartite biregular cages, and also they determine the order of all  $(3, r; 6)$ –cages,  $r \geq 4$ . In this talk, I will present a result concerning the order of  $(4, r; 6)$ –cages.

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## Monomial Representations and combinatorics

**Padraig Ó Catháin**

Dublin City University (Ireland)

Joint work with Ronan Egan, Heiko Dietrich and Santiago Barrera-Acevedo

D. G. Higman observed that the centraliser algebra of a rank 3 permutation groups is spanned by the incidence matrix of a strongly regular graph on which the permutation group acts by automorphisms. Quite generally, incidence matrices of combinatorial structures invariant under a permutation group  $G$  live in the centraliser algebra of the corresponding permutation representation. In this talk, we will consider instead monomial representations and their centraliser algebras, and discuss computational techniques for deciding whether such an algebra contains a complex Hadamard matrix.

## Line-free sets

**Péter Pál Pach**

Budapest University of Technology (Hungary)

In this talk we discuss some bounds on the possible size of sets avoiding certain arithmetic or geometric configurations in  $\mathbb{F}_p^n$  (or more generally, in  $\mathbb{Z}_m^n$ ). In particular, we will consider the following forbidden configurations:  $p$ -term arithmetic progressions (lines) in  $\mathbb{F}_p^3$ , right angles in  $\mathbb{F}_p^n$  and 6-term arithmetic progressions in  $\mathbb{Z}_6^n$ .



## Ghost hunting

**Silvia M. C. Pagani**

Università Cattolica del Sacro Cuore, Brescia (Italy)

Joint work with Marco L. Della Vedova (Chalmers) and Silvia Pianta (UniCatt)

An active area of research in combinatorial geometry is the investigation of substructures of a projective space which are characterised by incidence properties or other features. We define ghosts, also known as generalised Vandermonde sets, as those (multi-)subsets of  $\text{PG}(n, q = p^h)$  having constant intersection size (modulo  $p$ ) with hyperplanes.

In this talk we focus on the planar case and give some results about the characterisation of ghosts, together with some examples. In particular, a link to coding theory is exploited.

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## Point-line geometries related to binary equidistant codes

**Mark Pankov**

University of Warmia and Mazury (Poland)

Joint work with Krzysztof Petelczyc, Mariusz Żynel

We investigate point-line geometries whose singular subspaces correspond to binary equidistant codes. The main result is a description of automorphisms of these geometries. In some important cases (for example, the geometry related to simplex codes), automorphisms induced by non-monomial linear automorphisms surprisingly arise.

## On vertex connectivity of zero-divisor graphs of finite commutative rings

**Kamal Lochan Patra**

National Institute of Science Education and Research (India)  
Joint work with Sriparna Chattopadhyay and Binod Kumar Sahoo

Let  $R$  be a finite commutative ring with identity. We denote by  $Z(R)$  the set of all zero-divisors of  $R$  and by  $Z^*(R)$  the set of all nonzero zero-divisors of  $R$ . The *zero-divisor graph* of  $R$ , denoted by  $\Gamma(R)$ , is the simple graph with vertex set  $Z^*(R)$  in which two distinct vertices  $x$  and  $y$  are adjacent if and only if  $xy = 0$ . We study the structure of the zero-divisor graph of  $R$  and then determine its vertex connectivity when: (i)  $R$  is a local principal ideal ring, and (ii)  $R$  is a finite direct product of local principal ideal rings. For such rings  $R$ , we also characterize the vertices of minimum degree and the minimum cut-sets of the zero-divisor graph of  $R$ .

## Magic partially filled arrays on abelian groups

**Marco Antonio Pellegrini**

Università Cattolica del Sacro Cuore (Italy)  
Joint work with Fiorenza Morini

In this talk I will speak about some results and constructions we recently obtained about a special class of partially filled arrays. A magic partially filled array  $\text{MPF}_\Omega(m, n; s, k)$  on a subset  $\Omega$  of an abelian group  $(\Gamma, +)$  is a partially filled array of size  $m \times n$  with entries in  $\Omega$  such that (i) every  $\omega \in \Omega$  appears once in the array; (ii) each row contains  $s$  filled cells and each column contains  $k$  filled cells; (iii) there exist (not necessarily distinct) elements  $x, y \in \Gamma$  such that the sum of the elements in each row is  $x$  and the sum of the elements in each column is  $y$ . In particular, if  $x = y = 0_\Gamma$ , we have a zero-sum magic partially filled array  ${}^0\text{MPF}_\Omega(m, n; s, k)$ .

Examples of these objects are

- a magic rectangle  $\text{MR}(m, n; s, k)$ , that is an  $\text{MPF}_\Omega(m, n; s, k)$  where  $\Omega = [0, nk - 1] \subset \mathbb{Z}$ ;
- a signed magic array  $\text{SMA}(m, n; s, k)$ , that is a  ${}^0\text{MPF}_\Omega(m, n; s, k)$  where  $\Omega = \left[-\frac{nk-1}{2}, +\frac{nk-1}{2}\right] \subset \mathbb{Z}$  if  $nk$  is odd or  $\Omega = \left[-\frac{nk}{2}, -1\right] \cup \left[1, \frac{nk}{2}\right] \subset \mathbb{Z}$  if  $nk$  is even;
- an integer Heffter array  $\text{H}(m, n; s, k)$ , that is a  ${}^0\text{MPF}_\Omega(m, n; s, k)$  where  $\{\Omega, -\Omega\}$  is a partition of  $[-nk, -1] \cup [1, nk] \subset \mathbb{Z}$ ;
- a relative Heffter array  $\text{H}_t(m, n; s, k)$ , that is a  ${}^0\text{MPF}_\Omega(m, n; s, k)$  where  $\{\Omega, -\Omega\}$  is a partition of  $\mathbb{Z}_{2nk+t} \setminus J$ , with  $J$  being the subgroup of  $\mathbb{Z}_{2nk+t}$  of size  $t$ .

In particular, we gave necessary and sufficient conditions for the existence of a magic rectangle  $\text{MR}(m, n; s, k)$  and of a signed magic array  $\text{SMA}(m, n; s, k)$  when  $nk$  is odd. We also generalized Froncek's results about tight magic rectangle sets and

proved that, writing  $\Gamma^* = \Gamma \setminus \{0\}$  where  $\Gamma$  is an abelian group of order  $2n + 1 \geq 5$ , there exists a tight  ${}^0\text{MPF}_{\Gamma^*}(2, n; n, 2)$  if and only if  $\Gamma \notin \{\mathbb{Z}_5, \mathbb{Z}_3 \oplus \mathbb{Z}_3\}$ .

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## On (di)graphs and doubly stochastic matrices

**Safet Penjić**

University of Primorska (Slovenia)

Joint work with Giusy Monzillo.

The Hoffman polynomial  $h(t)$  of a (di)graph  $\Gamma = \Gamma(A)$  is the unique polynomial of smallest degree satisfying  $h(A) = J$ , where  $J$  denotes the all-ones matrix and  $A$  denotes adjacency matrix of a (strongly) connected regular (di)graph  $\Gamma$ . Let  $X$  denote a nonempty finite set. A nonnegative matrix  $B \in \text{Mat}_X(\mathbb{R})$  is called  $\lambda$ -doubly stochastic matrix if  $\sum_{z \in X} (B)_{yz} = \sum_{z \in X} (B)_{zy} = \lambda$  for each  $y \in X$ . In this talk we show connections between the Hoffman polynomial  $h(t)$  of  $B$  and a  $\lambda$ -doubly stochastic irreducible matrix.

Now, let  $B \in \text{Mat}_X(\mathbb{R})$  denote a normal irreducible nonnegative matrix, and  $\mathcal{B} = \{p(B) \mid p \in \mathbb{C}[t]\}$  denote the vector space of all polynomials in  $B$ . For the moment, define a 01-matrix  $A$  in the following way:  $(A)_{xy} = 1$  if and only if  $(B)_{xy} > 0$  ( $x, y \in X$ ). Let  $\Gamma = \Gamma(A)$  denote a digraph with adjacency matrix  $A$  and diameter  $D$ . We present some combinatorial and algebraic properties of  $\Gamma$  under the assumption that  $\mathcal{B}$  is the Bose–Mesner algebra of a commutative  $D$ -class association scheme.

## On subspaces defining linear sets of maximum rank

**Valentina Pepe**

Sapienza, University of Rome (Italy)

Let  $V$  denote an  $r$ -dimensional  $\mathbb{F}_{q^n}$ -vector space. Let  $U$  and  $W$  be  $\mathbb{F}_q$ -subspaces of  $V$ ,  $L_U$  and  $L_W$  the projective points of  $\text{PG}(V, q^n)$  defined by  $U$  and  $W$  respectively. We determine when  $L_W = L_U$  under the hypothesis that  $U$  and  $W$  have maximum dimension, i.e.,  $\dim_{\mathbb{F}_q} W = \dim_{\mathbb{F}_q} U = rn - n$ .

## 2-block-intersection graphs of twofold triple systems

**David Pike**

Memorial University of Newfoundland (Canada)

Joint work with Benjamin Stanley

A twofold triple system (TTS) is a combinatorial design for which every block has exactly three points, and each pair of points occurs together in precisely two blocks. The 2-block-intersection graph (2-BIG) of a TTS is the graph having the blocks of the TTS as its vertices, and vertices are adjacent if their corresponding blocks have exactly two elements in common. We discuss several properties of the 2-BIGs of TTSs, including recent observations about planarity and vertex-transitivity.

## Bending and plateauing: an approach to minimal $p$ -ary codes

**René Rodríguez Aldama**

University of Primorska (Slovenia)

Joint work with Enes Pasalic, Fengrong Zhang and Yongzhuang Wei.

In this talk, I aim to present some ways to obtain infinite families of minimal linear codes over the prime field  $\mathbb{F}_p$  using bent and plateaued functions. In the first part of the talk, I present the first construction, which uses the direct sum of an arbitrary function  $f: \mathbb{F}_{p^r} \rightarrow \mathbb{F}_p$  and a bent function  $g: \mathbb{F}_{p^s} \rightarrow \mathbb{F}_p$  to induce minimal codes with parameters  $[p^{r+s} - 1, r + s + 1]$  and minimum distance larger than  $p^r(p - 1)(p^{s-1} - p^{s/2} - 1)$ . This result leads to the first known general construction of linear codes from a subclass of non-weakly regular plateaued functions, which partially answers an open problem posed by Li and Mesnager in 2020. In the second part of the talk, I will provide a construction method which deals with a bent function  $g: \mathbb{F}_{p^m} \rightarrow \mathbb{F}_p$  and a suitable subspace of derivatives of  $g$ , i.e., functions of the form  $g(y + a) - g(y)$  for some  $a \in \mathbb{F}_{p^m}^*$ . If time allows, I will mention a sound generalization of the recently introduced notion of *non-covering permutations*, a concept closely related to minimal codes from functions, and attempt to pinpoint some of their most important structural properties. I will then say a word about our last construction, which combines the previous methods yielding non-equivalent minimal codes with larger dimensions.

## Constructions of affine $t$ -fold blocking sets

**Assia Rousseva**

Sofia University (Bulgaria)

Joint work with Ivan Landjev

A set of points  $\mathcal{B}$  in  $AG(n, q)$ ,  $q = p^h$ , is called a  $t$ -fold blocking set, or a  $t$ -fold intersection set, if  $|\mathcal{B} \cap H| \geq t$  for every hyperplane  $H$  in  $AG(n, q)$  and there exists a hyperplane  $H_0$  with  $|\mathcal{B} \cap H_0| = t$ . A  $t$ -fold blocking set  $\mathcal{B}$  in  $AG(n, q)$  with  $|\mathcal{B}| = N$  is called an  $(N, t)$ -blocking set.

A lower bound on the size  $N$  of an affine blocking set with  $t = 1$  was obtained independently by Jamison [5] and Brouwer and Schrijver [3]. They proved that for an 1-fold blocking set  $\mathcal{B}$  in  $AG(n, q)$ :

$$|\mathcal{B}| \geq n(q - 1) + 1.$$

This has been generalized by Bruen in [4] for  $t$ -fold blocking sets to

$$|\mathcal{B}| \geq (n + t - 1)(q - 1) + 1.$$

Further generalizations were obtained by Ball [1] and Ball-Blokhuis [2]. There are just a few constructions of good affine blocking sets meeting the above mentioned bounds [1,6,7].

In this talk we present a new construction for affine  $t$ -fold blocking sets that are optimal or lie close to the existing lower bounds.

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## Self-dual (near-)extremal ternary codes and combinatorial 2-designs

**Sanja Rukavina**

University of Rijeka (Croatia)

Joint work with Vladimir D. Tonchev (Michigan Technological University, USA)

The number of known self-dual extremal ternary codes for the lengths  $n \equiv 0 \pmod{12}$  is quite small and their classification is only completed for the lengths  $n = 12$  and  $n = 24$ . The sparseness of extremal codes has recently aroused interest in self-dual near-extremal ternary codes.

In this talk we consider the cases  $n = 36$  and  $n = 48$ . We report on the classification of all symmetric 2-(36, 15, 6) designs which admit an automorphism of order 2 and whose incidence matrices span an extremal or near-extremal ternary self-dual code of length 36. Furthermore, we use symmetric 2-(47, 23, 11) designs with an automorphism group of order 6 to construct new self-dual near-extremal ternary codes of length 48.

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## Blocking sets of external, tangent and secant lines to an elliptic quadric in $\text{PG}(3, q)$

**Binod Kumar Sahoo**

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Joint work with B. De Bruyn and P. Pradhann

Let  $\text{PG}(n, q)$  denote the  $n$ -dimensional projective space defined over a finite field of order  $q$ . For a given set  $\mathcal{L}$  of lines of  $\text{PG}(n, q)$ , a set  $B$  of points of  $\text{PG}(n, q)$  is called an  $\mathcal{L}$ -blocking set if each line of  $\mathcal{L}$  contains at least one point of  $B$ . The first step in the study of blocking sets is to determine the smallest cardinality of a blocking set and then to characterize, if possible, all blocking sets of that cardinality. With respect to an elliptic quadric  $Q^-(3, q)$  in  $\text{PG}(3, q)$ , denote by  $\mathcal{E}$  (respectively,  $\mathcal{T}$ ,  $\mathcal{S}$ ) the set of lines of  $\text{PG}(3, q)$  that are external (respectively, tangent, secant) to  $Q^-(3, q)$ . In this talk, we shall discuss the minimum size  $\mathcal{L}$ -blocking sets in  $\text{PG}(3, q)$ , where the line set  $\mathcal{L}$  is one of  $\mathcal{E}$ ,  $\mathcal{T}$ ,  $\mathcal{S}$ ,  $\mathcal{E} \cup \mathcal{T}$ ,  $\mathcal{E} \cup \mathcal{S}$  and  $\mathcal{T} \cup \mathcal{S}$ .

## Hadamard matrices and spherical designs

Patrick Solé

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Joint work with Minjia Shi, Danni Lu, A. Armario, R. Egan, F. Ozbudak

Hadamard codes over finite rings are explored wrt their covering radius for the chinese euclidean distance. Generalized bent sequences, when they exist, provide a lower bound. Upper bounds are obtained by considering a spherical code attached to Hadamard matrices of Butson type. Under mild hypotheses, this code is spherical 2-design, which yields upper bound on its covering radius on the sphere. This bound, in turn, gives an upper bound on the covering radius of the Hadamard codes.

## Searching for regular, triangle-distinct graphs

Dragan Stevanović

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Joint work with Mohammad Ghebleh, Gilles Caporossi, Sanja Stevanović and Ambat Vijayakumar

For a graph  $G$  and a vertex  $u$  of  $G$ , the triangle-degree of  $u$  is a number of triangles in  $G$  that contain  $u$ . A graph  $G$  is triangle-distinct if triangle-degrees of all vertices in  $G$  are mutually distinct. Such graph is necessarily asymmetric with only trivial automorphisms, as an automorphism mapping  $u$  to  $v$  implies that  $u$  and  $v$  also have equal triangle-degrees. As expected, there is an abundant number of triangle-distinct graphs among small graphs with up to 11 vertices. Berikkyzy et al. [1] even gave a construction of an infinite family of triangle-distinct graphs, and posed an interesting question whether there exists a regular, triangle-distinct graph?

The degrees and triangle-degrees are directly related to the numbers of closed walks of lengths two and three, respectively. Although there does not appear to be any reasonable barrier to the existence of regular triangle-distinct graphs, they turn out to be very hard to find. We will describe here various computational methods used in our search for regular, triangle-distinct graphs, as well as the lessons learned along the way.

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## The minimum distance of the code of intersecting lines in $\text{PG}(3, q)$

**Leo Storme**

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Joint work with S. Adriaensen, M. Datta and R. Simoens

Let  $\mathcal{L}$  denote the set of all lines in  $\text{PG}(3, q)$ , with  $q = p^h$ ,  $p$  prime. For every subset  $S \subseteq \mathcal{L}$ , define the characteristic function of  $S$  as

$$\chi_S : \mathcal{L} \rightarrow \mathbb{F}_p : \ell \mapsto \begin{cases} 1 & \text{if } \ell \in S, \\ 0 & \text{otherwise.} \end{cases}$$

For each line  $\ell$ , we denote by  $\chi_\ell$  the characteristic function of the set  $\{\ell' \in \mathcal{L} \mid \ell' \cap \ell \neq \emptyset\}$  of lines that intersect  $\ell$ . Consider the linear code  $C(q) = \langle \chi_\ell \mid \ell \in \mathcal{L} \rangle$  of  $\mathbb{F}_p^{\mathcal{L}}$ .

We call this linear code  $C(q)$  the *code of intersecting lines in  $\text{PG}(3, q)$* .

We determine the minimum weight of this code. For  $q$  even, this minimum weight is  $q^3 + q^2 + q + 1$ , and the minimum weight codewords are the incidence vectors of the sets of totally isotropic lines of a symplectic polarity in  $\text{PG}(3, q)$ . For  $q$  odd, the minimum weight is  $q^3 + 2q^2 + q + 1$ , and the minimum weight codewords are the non-zero scalar multiples of the vectors  $\chi_\ell$ ,  $\ell \in \mathcal{L}$ . We also prove that this type of incidence vectors are equal to the codewords of second weight in the code of intersecting lines in  $\text{PG}(3, q)$ ,  $q$  even.

## On some structures related to intriguing sets of strongly regular graphs

**Andrea Švob**

University of Rijeka (Croatia)

Joint work with Dean Crnković and Francesco Pavese

In this talk, we will be interested in a connection among some finite incidence structures - partial geometric designs, special partially balanced incomplete block designs, directed strongly regular graphs and strongly regular graphs. Further, we give a technique for constructing directed strongly regular graphs by using strongly regular graphs that have a nice family of intriguing sets.



## Near alternating sign matrices with an application

**Tommaso Traetta**

University of Brescia (Italy)

A *near alternating sign matrix* (NASM) is an  $m \times n$  array with entries from  $\{0, \pm 1\}$  such that, ignoring 0s, the +1s and -1s alternate in each row and each column; in addition, we require such a matrix to have arbitrarily prescribed weights for each row and column. These arrays, which generalize *alternating sign matrices*, were studied for the first time by Brualdi and Kim [1], although they focused on constructing them, leaving out the weights, but with arbitrarily prescribed first and last nonzero entries in each row and column. It is well known that these matrices have connections with partitions, tilings, and statistical physics.

In this talk, we describe an application of NASMs to construct (generalized) Heffter arrays [2] satisfying further properties that allow us to obtain orthogonal cycle decompositions and biembeddings of Cayley graphs. Partly based on joint work with Lorenzo Mella.

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## Amorphic association schemes and fusing pairs

**Edwin van Dam**

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Joint work with Jack Koolen and Yanzhen Xiong

Association schemes are colorings of the edges of the complete graph satisfying many combinatorial regularity conditions. Extremal examples are colorings given by distance in a distance-regular graph, the so-called P-polynomial schemes. Dual to this are Q-polynomial schemes. Typical of such schemes is that they have few fusion schemes, where we speak of a fusion scheme of a scheme if joining some of the colors gives rise to another association scheme. At the other end of the spectrum are the so-called amorphic schemes. In such a scheme, any fusion of relations gives rise to a fusion scheme.

In this talk, we will present some new results on amorphic schemes. We focus on fusions of pairs of relations, and give sufficient conditions for amorphicness in terms of such fusions of pairs. Essential methods to obtain these results are dual versions (concerning idempotents in the Bose-Mesner algebra) of some results on (negative) Latin square graphs and strongly regular decompositions of the complete graph.

## The distance function on Coxeter like graphs

**Draženka Višnjić**

University of Primorska (Slovenia)

Joint work with Marko Orel

Let  $S_n(\mathbb{F}_2)$  be the set of all  $n \times n$  symmetric matrices with coefficients from the binary field  $\mathbb{F}_2 = \{0, 1\}$ , and let  $SGL_n(\mathbb{F}_2)$  be the subset of all invertible matrices. Let  $\tilde{\Gamma}_n$  be the graph with the vertex set  $S_n(\mathbb{F}_2)$ , where two matrices  $A, B \in S_n(\mathbb{F}_2)$  form an edge if and only if  $\text{rank}(A - B) = 1$ . Let  $\Gamma_n$  be the subgraph in  $\tilde{\Gamma}_n$ , which is induced by the set  $SGL_n(\mathbb{F}_2)$ . In particular  $\Gamma_3$  is well known-Coxeter graph. The distance function on  $\tilde{\Gamma}_n$  is given by

$$d(A, B) = \begin{cases} \text{rank}(A - B), & \text{if } A - B \text{ is nonalternate or zero,} \\ \text{rank}(A - B) + 1, & \text{if } A - B \text{ is alternate and nonzero.} \end{cases}$$

Even the Coxeter graph shows that the distance in  $\Gamma_n$  must be different. The main goal is to describe the distance function on this graph.

## Binary codes with distances $d$ and $d + 2$

**Konstantin Vorob'ev**

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Joint work with Ivan Landjev

We are interested in the value  $A_2(n, \{d_1, d_2\})$  defined as the maximal size of a binary code of length  $n$  with two distances  $d_1$  and  $d_2$ . It was recently proved[1] by Barg et al. that  $A_2(n, \{d_1, d_2\}) \leq 1 + \binom{n}{2}$ . The goal of this work is to find this value for the case  $d_2 = d_1 + 2$ , when  $d_1$  is even. The 2-packings of an  $n$ -element set by blocks of cardinality  $\frac{d}{2} + 1$  is a good example of constant-weight codes with distances  $d$  and  $d + 2$  and size  $\sim \frac{n^2}{\frac{d}{2}(\frac{d}{2}+1)}$ , which is far from the Barg's bound.

In this work we prove that for every fixed even  $d$  there exists an integer  $n_0(d)$  such that for every  $n \geq n_0(d)$  any optimal code of length  $n$  with distances  $d$  and  $d + 2$  is isomorphic to a constant weight code. We also provide upper bounds on  $n_0(d)$  for  $d = 4$  and  $d = 6$ .

The author was supported by the NSP P. Beron project CP-MACT.

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## Higher incidence matrices and tactical decomposition matrices of designs

Alfred Wassermann

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Joint work with Michael Kiermaier

In 1985, Janko and Tran Van Trung published an algorithm for constructing symmetric 2-designs with prescribed automorphisms. The idea of the algorithm is to use equations by Dembowski (1958) for tactical decompositions of point-block incidence matrices for the classification of these tactical decomposition matrices. In the sequel, this algorithm has been generalized and improved in many articles.

In 1982, Wilson introduced higher incidence matrices for  $t$ -designs with arbitrary strength  $t \geq 2$ . These matrices have proven useful for obtaining several restrictions on the existence of designs. For example, a short proof of the *generalized Fisher's inequality* makes use of these incidence matrices.

In this talk we present a unified approach to tactical decompositions and incidence matrices. It works for both combinatorial and subspace designs alike. As a result, we obtain a generalized Fisher's inequality for tactical decompositions of combinatorial and subspace designs. Moreover, our approach is explored for the construction of combinatorial and subspace designs of arbitrary strength.

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## Existence of $t$ -designs in polar spaces for all $t$

Charlene Weiß

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A finite classical polar space of rank  $n$  consists of the totally isotropic subspaces of a finite vector space over  $\mathbb{F}_p$  equipped with a nondegenerate form such that  $n$  is the maximal dimension of such a subspace. A  $t$ - $(n, k, \lambda)$  design in a finite classical polar space of rank  $n$  is a collection  $Y$  of totally isotropic  $k$ -spaces such that each totally isotropic  $t$ -space is contained in exactly  $\lambda$  members of  $Y$ . Nontrivial examples are currently only known for  $t \leq 2$ . We show that  $t$ - $(n, k, \lambda)$  designs in polar spaces exist for all  $t$  and  $p$  provided that  $k > \frac{21}{2}t$  and  $n$  is sufficiently large enough. The proof is based on a probabilistic method by Kuperberg, Lovett, and Peled, and it is thus nonconstructive.

## Cameron-Liebler line classes, tight sets and strongly regular Cayley graphs

**Qing Xiang**

Southern University of Science and Technology (China)

Joint work with Tao Feng, Koji Momihara, Morgan Rodgers, and Hanlin Zou

Cameron-Liebler line classes are sets of lines in  $\text{PG}(3, q)$  having many interesting combinatorial properties. These line classes were first introduced by Cameron and Liebler in their study of collineation groups of  $\text{PG}(3, q)$  having the same number of orbits on points and lines of  $\text{PG}(3, q)$ . During the past decade, Cameron-Liebler line classes have received considerable attention from researchers in both finite geometry and algebraic combinatorics. In the original paper [1] by Cameron and Liebler, the authors gave several equivalent conditions for a set of lines of  $\text{PG}(3, q)$  to be a Cameron-Liebler line class; later Penttila gave a few more of such characterizations. We will use one of these characterizations as the definition of Cameron-Liebler line class. Let  $\mathcal{L}$  be a set of lines of  $\text{PG}(3, q)$  with  $|L| = x(q^2 + q + 1)$ ,  $x$  a positive integer. We say that  $\mathcal{L}$  is a Cameron-Liebler line class with parameter  $x$  if every spread of  $\text{PG}(3, q)$  contains  $x$  lines of  $\mathcal{L}$ . It turned out that Cameron-Liebler line classes are closely related to certain subsets of points (tight sets) of the Klein quadric. We will talk about a recent construction in [2] of a new infinite family of Cameron-Liebler line classes with parameter  $x = (q + 1)^2/3$  for  $q \equiv 2 \pmod{3}$ . When  $q$  is an odd power of 2, this family of Cameron-Liebler line classes represents the first infinite family of Cameron-Liebler line classes ever constructed in  $\text{PG}(3, q)$ ,  $q$  even.

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## Non-extendable partial Latin hypercubes and maximal orthogonal partial Latin squares

**E. Şule Yazıcı**

Koç University (Turkey)

Joint work with D. M. Donovan and M. J. Grannell

A Latin hypercube is a generalisation of a Latin square to higher dimensions. A *maximal or non-extendable* partial Latin hypercube is a partial Latin Hypercube that cannot be extended to another partial Latin Hypercube with more filled cells by inserting any element of the entry set into any empty cell. A lower bound is presented for the minimal number of filled cells in a maximal partial Latin hypercube of dimension  $d$  and order  $n$ . The result generalises and extends previous results for

$d = 2$  (Latin squares) and  $d = 3$  (Latin cubes). Explicit constructions show that this bound is near-optimal for large  $n > d$ . For  $d > n$ , a connection with Hamming codes shows that this lower bound gives a related upper bound for the same quantity. We will also present the close relation of non-extendable partial Latin hypercubes to independent dominating sets in certain graphs, and codes that have covering radius 1 and minimum distance at least 2.

We will also introduce maximal orthogonal partial Latin squares, the structures whose classification is crucial for improving bounds for embeddings of orthogonal partial Latin squares. We will present some examples of these structures and a conjecture for the smallest possible number of filled cells in a pair of maximal orthogonal partial Latin squares.

## On the existence of small strictly Neumaier graphs

**Sjanne Zeijlemaker**

Eindhoven University of Technology (Netherlands)  
Joint work with Aida Abiad and Maarten De Boeck

A Neumaier graph is a non-complete edge-regular graph containing a regular clique. These graphs were first studied by Neumaier, who raised the question whether they must be strongly-regular. This has since been proven false, sparking a search for strictly (i.e., not strongly-regular) Neumaier graphs. In this presentation, we discuss several new results on the existence of small strictly Neumaier graphs. In particular, we present a new strictly Neumaier graph with parameters  $(25, 12, 5; 2, 5)$  and disprove the existence of Neumaier graphs with parameters  $(25, 16, 9; 3, 5)$ ,  $(28, 18, 11; 4, 7)$ ,  $(33, 24, 17; 6, 9)$ ,  $(35, 22, 12; 3, 5)$  and  $(55, 34, 18; 3, 5)$ . Our proofs use combinatorial techniques and a novel application of integer programming methods.

## Applications of genetic algorithms for constructions of SRGs and DSRGs

**Tin Zrinski**

University of Rijeka (Croatia)  
Joint work with Dean Crnković

Genetic algorithms are search methods used in computing whose objective is to find exact or approximate solutions to optimization and search problems. A genetic algorithm mimics natural evolution, that is, it is based on optimizing a population (a subset of the entire search space). As in nature, the population consists of individuals that can reproduce and that can be affected by certain mutations, thus creating new individuals with better or worse properties than the previous ones. The goal of the algorithm is to direct the population towards creating better individuals, which can result in finding optimal solutions to a given problem.

In this talk, we will describe the use of a genetic algorithm for the construction of strongly regular graphs and directed strongly regular graphs from equitable partitions with a prescribed automorphism group.

## List of participants



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- Sara **Ban**, University of Rijeka, Croatia.
- Anton **Betten**, Kuwait University, Kuwait.
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