The degree of functions in the Johnson and q-Johnson schemes

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Cameron-Liebler line classes have been introduced in 1982 in an article by Cameron and Liebler as "special" sets $\mathcal{L}$ of lines in the projective space $\mathrm{PG}(3, q)$, which can be defined by several equivalent properties. By the *algebraic* property, the characteristic function $\chi_{\mathcal{L}}$ of $\mathcal{L}$ is contained in the row space of the point-line incidence matrix of $\mathrm{PG}(3, q)$ over $\mathbb{R}$, and by the *geometric property*, $\mathcal{L}$ has constant intersection with each line spread of $\mathrm{PG}(3, q)$. In the literature, various generalizations of Cameron-Liebler line classes have been considered. For example, the set case $q = 1$ has been studied, and the point-line incidence matrix has been generalized to the incidence matrices of d-spaces vs. k-spaces. In this talk, a coherent theory of these generalizations will be discussed.

For $q = 1$, let V be a set of finite size n, and for a prime power $q \geq 2$, let V be an $\mathbb{F}_q$-vector space of finite dimension n. Denoting the set of all k-subsets (or k-subspaces) of V by $\begin{bmatrix} V \\ k \end{bmatrix}$, the *degree* of a function $f : \begin{bmatrix} V \\ k \end{bmatrix} \to \mathbb{R}$ will be defined by the generalization of the above algebraic property. We will investigate fundamental properties of the degree. The question for the counterpart of the geometric property is related to Delsarte's concept of *design orthogonality* and leads to a connection to the theory of combinatorial designs and subspace designs.

Of particular interest are Boolean functions $\begin{bmatrix} V \\ k \end{bmatrix} \to \{0, 1\}$, which correspond to subsets $\mathcal{F}$ of $\begin{bmatrix} V \\ k \end{bmatrix}$ via characteristic functions. A new divisibility condition on the size of sets $\mathcal{F}$ of degree d will be presented. Moreover, we discuss the computer classification of sets of degree 2 in the case $q = 1$. Also, for $q = 1$ a general construction of sets of low degree will be given.