Minimising the number of comparable sets

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Consider the subsets of a ground set $\{1, \dots, n\}$. Over all the families composed of a given number of k-sets, what is the minimum number of $(k-1)$-sets that are subsets of at least one set in the family? The Kruskal-Katona theorem famously answers this question. Bashov considered a two-sided variant of this problem in which one attempts to minimise the total number of $(k-1)$-sets and $(k+1)$-sets that are subsets or supersets of at least one set in the family. In this talk we discuss a different two sided variant: over all the families $\mathcal{F}$ composed of a given number of sets (of any sizes), what is the minimum number of sets that are subsets or supersets of at least one set in the family? The results we obtain have parallels with the Kruskal-Katona theorem and also with Harper's theorem on isoperimetry in hypercubes.