q-Matroids and related structures

Eimear Byrne
University College Dublin (Ireland)

Matroids have natural connections to graphs, geometry and linear codes. Several invariants of these objects turn out to be special cases of matroid invariants. q-Matroids, which are q-analogues of matroids, have gathered much interest in recent years, especially from researchers in rank-metric codes. While a matroid may be defined in terms of a semi-modular function on a finite Boolean lattice, a q-matroid comprises a rank function defined on the lattice of subspaces of a finite dimensional vector space. Matroids and their generalisations have many equivalent axiomatic descriptions, a fact that has often been exploited. A (q)-matroid is determined by its lattice of flats and similarly by its cycles. It is also determined by its lattice of cyclic flats and their ranks. However, the lattice of cyclic flats is often very small compared with the lattices of flats and cycles, It also behaves very well with respect to different forms of decomposition. In this talk we will give an introduction to q-matroids and their structural properties, especially in relation to their cyclic flats. We will describe the free product of a q-matroid and look at the question of representability in relation to this product.

References

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