Heffter spaces and additive designs

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A *half-set* of an additive group G of odd order is a complete system of representatives for the set of all pairs $\{g, -g\}$ of opposite elements of $G \setminus \{0\}$.

In this talk I will mainly speak about a new combinatorial design that Anita Pasotti and I have called a $(v, k; r)$ *Heffter space* [3]. This is a resolvable partial linear space of degree r whose point set is a half set of an abelian group G of order $2v + 1$ and whose blocks are zero-sum k-subsets of G. When r is just 1 or 2, it is equivalent to a *Heffter system* or a *Heffter array* on G, respectively, a topic widely investigated in the last decade [7].

Among the motivations of studying Heffter spaces, there is the fact that every $(v, k; r)$ Heffter space with suitable properties gives rise to r mutually orthogonal k-cycle decompositions of the complete graph on $2v + 1$ vertices, a topic recently studied in [4, 6].

Even though, for k odd, we have been able to construct elementary abelian Heffter spaces of arbitrarily large degree, many questions remain open; first among them is whether this result can be obtained also for k even. In my opinion, however, the most intriguing question is whether there exists a Heffter space which is also a Steiner 2-design. An attempt to answer this very hard question made me "stumbling" again onto *additive designs*, an interesting topic introduced in [5] and recently studied in [1, 2].

References

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