**Scrambling index and operators preserving its values**

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This talk is based on the joint work with A.M. Maksaev.

Scrambling index is a fundamental matrix invariant. Shortly, a matrix is said to be scrambling if for any two rows there is a column having positive intersections with both these rows. A scrambling index of a non-negative matrix $A$ is a minimal $k$ such that $A^k$ is a scrambling matrix, and 0, if such $k$ does not exist. Correspondingly, a scrambling index of a primitive directed graph $G$ is the smallest positive integer $k = k(G)$ such that for any pair of vertices $u, v$ of $G$ there exists a vertex $w$ of $G$ such that there are directed walks of length $k$ from $u$ to $w$ and from $v$ to $w$. If $G$ is not primitive, then it can appear that the integer $k$ described above does not exist. In this case we say that $k(G) = 0$, otherwise we define $k(G)$ as in the primitive case.

The scrambling index is important in several applications. In particular, if $A$ is an $n \times n$ non-negative primitive stochastic matrix with a non-unit eigenvalue $\lambda$, and $k$ is the scrambling index of $G(A)$, then $|\lambda| \leq (\tau_1(A^k))^{1/k} < 1$, where $\tau_1$ is a certain matrix invariant, usually called Dobrushin coefficient.

Also scrambling index provides lower bounds for the length of reset words for synchronizing automata, since it gives a lower bound for the exponent of the matrix representing this automata.

More applications are in the theory of memoryless communication systems and related areas. Scrambling index for primitive graphs was an object of intensive investigations starting from the works by Seneta, Paz, Akelbek, Kirkland, and others.

We prove that for non-primitive digraphs on $n$ vertices the following bound $k(G) \leq 1 + \left\lceil \frac{(n-2)^2+1}{2} \right\rceil$ is true. We characterize the graphs with the maximal scrambling index and characterize non-primitive graphs possessing positive scrambling index.

In addition we investigate linear transformations preserving scrambling index. In particular, we show that these maps are always bijective. The structure of linear maps preserving only several values of the scrambling index is also characterized.