B-spline interpolation problem in Hilbert C^*-modules

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The B-spline interpolation problem corresponding to a C^*-valued sesquilinear form on a Hilbert C^*-module is introduced and its basic properties are studied. Fiirst, the problem is investigated in the case when the Hilbert C^*-module is self-dual. Extending a bounded C^*-valued sesquilinear form on a Hilbert C^*-module to a sesquilinear form on its second dual, some necessary and sufficient conditions for the B-spline interpolation problem to have a solution are provided.

Passing to the setting of Hilbert W^*-modules, the main result is presented by characterizing when the spline interpolation problem for the extended C^*-valued sesquilinear to the dual $\mathcal{X}'$ of the Hilbert W^*-module $\mathcal{X}$ has a solution. As a consequence, a sufficient condition is given characterizing orthogonally complemented submodules of a self-dual Hilbert W^*-module $\mathcal{X}$ that are orthogonally complemented with respect to another C^*-inner product on $\mathcal{X}$. Finally, solutions of the B-spline interpolation problem for Hilbert C^*-modules over C^*-ideals of W^*-algebras are extensively discussed. Several examples are provided to illustrate the existence or lack of a solution for the problem.

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