

Thoughts on the classification problem for amenable C^* -algebras

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Thirty years ago—thirty years after Glimm’s classification of uniformly hyperfinite (UHF) C^* -algebras—and fifty years after Gelfand and Naimark classified commutative C^* -algebras by their spectrum, an invariant that also is fundamental in the non-commutative case—, the prospect of classifying all (separable) amenable (= nuclear) C^* -algebras seemed perhaps to hover on the horizon.

Now, after thousands of pages of painstaking calculations, coupled with numerous conceptual advances, the restricted but robust Toms-Winter class of especially well-behaved simple separable amenable C^* -algebras (assumed to be Jiang-Su stable—not automatic but examples arising everywhere—and to satisfy the Universal Coefficient Theorem (UCT)—perhaps automatic) has been classified by means of a simple invariant (K-theoretic in nature), the possible values of which are also very simple.

(The classification is perhaps reminiscent of that of finite simple groups, although that is not formulated in terms of an invariant. One hopes that the C^* -algebra classification could also be fruitful.)

Given the additional information contained in more subtle invariants, such as the Cuntz semigroup, it does not seem unreasonable to continue to envisage the original goal.