Images of noncommutative polynomials

Matej Brešar
University of Ljubljana and University of Maribor

Let F be a field, let $f = f(X_1, \ldots, X_m)$ be a noncommutative polynomial with coefficients in F, and let A be an F-algebra. We will discuss various questions concerning the *image of* f (in A), which is defined to be the set $f(A) = \{f(a_1, \ldots, a_m) \,|\, a_1, \ldots, a_m \in A\}$. A special emphasis will be on the Waring type problem, asking about the existence of a positive integer N (independent of f, provided that f satisfies some natural restrictions) such that every element in A is a linear combination of N elements from $f(A)$. Our methods are algebraic and we are primarily interested in the case where $A = M_n(F)$, but the case where $A = B(H)$ will also be considered.