Polar decomposition of the linear pencil

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For a pair of operators $A, B \in \mathcal{B}(H)$, the operator $A - \lambda B$, $\lambda \in \mathbb{C}$, is called the *linear pencil* of an ordered pair (A, B). The *spectrum* of a linear pencil is the set

$$\sigma(A, B) = \{\lambda \in \mathbb{C} : A - \lambda B \text{ is not invertible}\},$$

and $\rho(A, B) = \mathbb{C} \setminus \sigma(A, B)$ is called the *resolvent set* of the linear pencil (A, B).

We recall that the polar decomposition of any bounded linear operator T between complex Hilbert spaces is a canonical factorization as the product of a partial isometry and a non-negative operator, that is, $T = U|T|$, where U is a partial isometry and $|T| = \sqrt{T^*T}$ is a non-negative operator.

In this talk we introduct *the (spherical) polar decomposition* of a linear pencil of an ordered pair (A, B) and we give some properties of it. Using this way of decomposition of a pair of operators, we are able to define several type of Aluthge transforms for a linear pencil of operators.

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