Positive solutions of the operator equation $AXB = C$

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We will discuss the existence of a positive solution of the equation $AXB = C$. This problem was considered in different settings but only under additional conditions including that of regularity, as well as under certain range conditions such as $\mathbb{R}(B) \subseteq \overline{\mathbb{R}(A^*)}$. We will answer this question of the existence of a positive solution of the operator equation $AXB = C$ without any additional range or regularity assumptions using two well-known results of Douglas and Zoltán. Also we will give a general form of a positive solution and consider some possible applications.

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