Sequence dominance in shift-invariant spaces

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For a given function $\psi \in L^2(\mathbb{R})$ we study the system of integer translates $B_\psi = \{T_k\psi : k \in \mathbb{Z}\}$, where T_k is the translation operator. Numerous properties of B_ψ can be described via its periodization function $p_\psi(\xi) = \sum_{k\in\mathbb{Z}} \left|\widehat{\psi}(\xi + k)\right|^2$. For ψ we define its associated coefficient space Cof_ψ as the set of all the sequences $(c_k)_{k\in\mathbb{Z}}$ for which $\sum c_k T_k \psi$ converges in the L^2 norm (with respect to the ordering $0, 1, -1, 2, -2, \ldots$ of $\mathbb{Z}$). There are two important special cases: when Cof_ψ contains $\ell^2(\mathbb{Z})$, in which case we say that B_ψ has the (H)–property, and when Cof_ψ is contained in $\ell^2(\mathbb{Z})$, when we say that B_ψ has the (B)–property. Characterization of the (H)–property via the periodization function is already known. The (B)–property seems to be much more difficult to characterize and we will characterize it in two important special cases: when the periodization function has a certain degree of smoothness and when the system B_ψ has the (H)–property alongside the (B)–property.

This is joint work with Hrvoje Šikić.